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REPRESENTATION THEOREM FOR SOME FUZZY ALGEBRAIC HYPERSTRUCTURES

ABSTRACT: In this paper, we study the Representation Theorem in fuzzy set theory for some fuzzy algebraic hyperstructures, specially for fuzzy (weak) hyper BCK-ideals of a hyper BCK-algebra, fuzzy subhypergroups, fuzzy subhyperrings and fuzzy hyperideals.

1. INTRODUCTION

The hyperstructure theory was introduced by F. Marty [4] at the 8th congress of Scandinavian Mathematicians. Around the 40's, several authors worked on hypergroups, especially in France and in the United States, but also in Italy, Russia and Japan. Hyperstructures have many applications to several sectors of both pure and applied sciences. In 2000, Y. B. Jun, M. M. Zahedi and et. al applied the hyperstructure theory to BCK-algebras, which is a generalization of set-theoretic dierence and propositional calcului, and introduced the notion of hyper BCK-algebras [3], as a generalization of BCK-algebras.

The fuzzy set theory introduced by L. A. Zadeh [8] in 1965. One of important problems in fuzzy set theory is to determine a method to fuzzification of concepts. First of all, C. V. Negoita and D. A. Ralescu [5] start to solve this problem and then D. A. Ralescu gave some available results in [7]. Indeed, by using this method, one can determine a fuzzy set of a nonempty set by a class of ordinary subsets, with the same type, of it. Representation theorem answers to this question that if $\{B_\alpha\}_{\alpha\in[0,1]}$ is a class of subsets of a nonempty set X, then whether there exists a fuzzy subset μ of X such that for all $\alpha \in [0, 1]$, $\mu_\alpha = B_\alpha$ or not. In this paper we use Representation theorem on some algebraic hyperstructures to improve this question.

2. PRELIMINARIES

We first give some definitions and results in fuzzy set theory and some (fuzzy) algebraic hyperstructures.

Theorem 2.1. [6] (Representation Theorem) Let $\{B_\alpha\}_{\alpha\in[0,1]}$ be a class of subsets of a nonempty set X. The necessary and sufficient condition for that there exists a fuzzy subset A of X such that $A_\alpha = B_\alpha$, for all $\alpha \in [0, 1]$ is that

$$B_{\underset{\alpha\in M}{\vee}\alpha} = \bigcap_{\alpha\in M} B_\alpha \tag{1}$$

for every $M \subseteq [0, 1]$.

Remark 2.2. [6] If $\{B_\alpha\}_{\alpha\in[0,1]}$ is a class of subsets of a nonempty set X such that for every $M \subseteq [0, 1]$ the condition (1) of Theorem 2.1 holds, then:

(i) $B_0 = X$,

(ii) if $\alpha, \gamma \in [0, 1]$ be such that $\alpha \leq \gamma$, then $\mathrm{B}_\gamma \subseteq \mathrm{B}_\alpha$.

Definition 2.3. (i)[3] By a hyper BCK-algebra we mean a nonempty set H endowed with a hyperoperation "o" and a constant 0 satisfies the following axioms:

(HK1) $(x \circ z) \circ (y \circ z) << x \circ y$,

(HK2) $(x \circ y) \circ z = (x \circ z) \circ y$,

(HK3) $x \circ H << \{x\}$,

(HK4) $x << y$ and $y << x$ imply $x = y$,

for all $x, y, z \in H$, where $x << y$ is defined by $0 \in x \circ y$ and for nonempty subsets A and B of H, $A << B$ means that for all $a \in A$ there exists $b \in B$ such that $a << b$. In such case, we call "<<" the hyperorder in H.

Let H be a hyper BCK-algebra and I be a nonempty subset of H. Then I is said to be a:

(a) hyper subalgebra if $x \circ y \subseteq I$, for all $x, y \in I$,

(b) weak hyper *BCK*-ideal if $x \circ y \subseteq I$ and $y \in I$ imply $x \in I$, for all $x, y \in H$.

(c) hyper *BCK*-ideal if $x \circ y << I$ and $y \in I$ imply $x \in I$, for all $x, y \in I$.

(ii)[2] let μ be a fuzzy subset of H. Then μ is called a

(a) fuzzy hyper subalgebra if $\bigwedge_{a \in x \circ y} \mu(a) \geq \mu(x) \wedge \mu(y)$, for all $x, y \in H$.

(b) fuzzy weak hyper BCK-ideal if $\mu(0) \geq \mu(x)$ and $\mu(x) \geq \bigwedge_{a \in x \circ y} \mu(a) \wedge \mu(y)$, for all $x, y \in H$.

(c) fuzzy hyper BCK-ideal if $x << y$ implies that $\mu(x) \geq \mu(y)$ and $\mu(x) \geq \bigwedge_{a \in x \circ y} \mu(a) \wedge \mu(y)$, for all $x, y \in H$.

Definition 2.4. [1] Let (H, o) be a hypergroupoid (i.e. a nonempty set H together with a binary hyperoperation "o"). Then:

(i) (H, o) is said to be a hypergroup if "o" is associative and satisfies the reproductive property: $x \text{ o } H = H \text{ o } x = H$, for all $x \in H$,

Let H be a hypergroup. Then:

(a) an element $e \in H$ is called an identity if for all $a \in H$, $a \in a \text{ o } e \cap e \text{ o } a$.

(b) let H has an identity e and $a \in H$. The element a' is called an inverse of a if $e \in a \text{ o } a' \cap a' \text{ o } a$.

(c) an element $a \in H$ is called scalar identity if $|a \text{ o } x| = |x \text{ o } a| = 1$.

(ii) H is called regular if it has at least one identity and every element has at least one inverse.

(iii) if H is regular, then it is called reversible if for all $x, y, z, a \in H$:

(a) if $y \in a \text{ o } x$, then there exists an inverse a' of a, such that $x \in a' \text{ o } y$,

(b) if $y \in x \text{ o } a$, then there exists an inverse a'' of a such that $x \in y \text{ o } a''$.

(iv) H is called canonical if it is commutative (i.e. $a \text{ o } b = b \text{ o } a$ for all $a, b \in H$) has a scalar identity, every element has a unique inverse and it is reversible.

(v) a hyperring is a hyperstructure $(R, +, \cdot, 0)$ where,

(a) $(R, +)$ is a canonical hypregroup,

(b) $(R, \cdot)$ is a semigroup endowed with a two-sided absorbing element 0.

(c) the product is associative on addition.

(iv) a nonempty subset S of a hyperring $(R,+, \cdot)$ is said to be a subhyperring if $(S,+, \cdot)$ is itself a hyperring.

(v) a subhyperring I of a hyperring R is called a (left) right hyperideal of R if $\forall x \in I$ and $\forall r \in R$ we have $rx \in I$ $(xr \in I)$. If I is both a left and right hyperideal, then it is called a hyperideal.

(vi) let R be a hyperring. Then $(M, +, \mathrm{o})$ is said to be a right R-hypermodule if $(M, +)$ is a canonical hypergroup and the function $\mathrm{o} : M \times R \to M$ satisfies the following axioms: $\forall x, y \in M, \forall a, b \in R$,

(a) $(x + y) \mathrm{o}\, a = x \mathrm{o}\, a + y \mathrm{o}\, a$,

(b) $x \mathrm{o} (a + b) = x \mathrm{o}\, a + x \mathrm{o}\, b$,

(c) $x \mathrm{o} (a \cdot b) = (x \mathrm{o}\, a) \cdot b$,

(d) $x \mathrm{o}\, 0 = \{0\}$.

A left R-hypermodule is defined similarly. An R-hypermodule, is a left and right R-hypermodule.

Definition 2.5. [1] Let μ be a fuzzy subset of a hyperring $(R,+, \cdot)$. Then:

(i) μ is called a fuzzy subhyperring of R if

(a) $\bigwedge_{z \in x+y} \mu(z) \geq \mu(x) \wedge \mu(y)$,

(b) $\mu(-x) \geq \mu(x)$,

(c) $\mu(x \cdot y) \geq \mu(x) \wedge \mu(y)$,

for all $x, y \in R$.

(ii) μ is said to be a fuzzy hyperideal of R if

(a) $\bigwedge_{z \in x+y} \mu(z) \geq \mu(x) \wedge \mu(y)$,

(b) $\mu(-x) \geq \mu(x)$,

(c) $\mu(x \cdot y) \geq \mu(x) \vee \mu(y)$,

for all x, y 2 R.

(iii) Let ν be a fuzzy hyperideal of R and μ be a fuzzy subset of R-hypermodule M. Then μ is said to be a ν-fuzzy subhypermodule of M if

(a) $\bigwedge_{z \in x+y} \mu(z) \geq \mu(x) \wedge \mu(y)$,

(b) $\nu(-x) \geq \mu(x)$,

(c) $\mu(rx) \geq \nu(r) \wedge \mu(x)$,

for all $x, y \in M$ and $r \in R$.

3. MAIN RESULTS

In this section, we investigate the representstion theorem for fuzzy (weak) hyper *BCK*-ideals, fuzzy subhypergroups, fuzzy subhyperrings and so on.

3.1. Representation Theorem for fuzzy (weak) hyper BCK-ideals of a hyper BCKalgebra

Theorem 3.1. Let $\{I_\alpha\}_{\alpha\in[0,1]}$ be a class of weak hyper *BCK*-ideals of hyper *BCK*-algebra H. The necessary and sufficient condition for there exists a fuzzy weak hyper BCK-ideal μ of H such that for all $\alpha \in [0, 1]$ we have $\mu_\alpha = I_\alpha$ is that

$$I_{\bigvee_{\alpha\in M} \alpha} = \bigcap_{\alpha\in M} I_\alpha$$

for all $M \subseteq [0, 1]$.

Proof. Let fuzzy subset μ of H is defined by

$$\mu(x) = \bigvee_{x\in I_\alpha} \alpha \tag{2}$$

for all $x \in H$. We show that μ is a fuzzy weak hyper *BCK*-ideal of H and $\mu_\alpha = I_\alpha$, for all $\alpha \in [0, 1]$. First of all we observe that μ is well-defined, for uniqueness of $\vee$.

Now, since I_α is a weak hyper *BCK*-ideal of *H* and $0 \in I_\alpha$, for all $\alpha \in [0, 1]$, then $\{\alpha \in [0, 1] : x \in I_\alpha\} \subseteq \{\alpha \in [0, 1] : 0 \in I_\alpha\}$. Hence $\mu(x) = \bigvee_{x\in I_\alpha} \alpha \leq \bigvee_{0\in I_\alpha} \alpha = \mu(0)$.

Now, let

$$t = \bigwedge_{a\in x\circ y} \mu(a) \wedge \mu(y).$$

Hence, $\bigvee_{y\in I_\alpha} \alpha = \mu(y) \geq t$ and for all $a \in x \circ y$, $\bigvee_{a\in I_\alpha} \alpha = \mu(a) \geq t$. So, for $M = \{\alpha \in [0, 1] : a \in I_\alpha\}$, by Remark 2.2 we have

$$a \in \bigcap_{\alpha\in M} I_\alpha = I_{\bigvee_{\alpha\in M}\alpha} \subseteq I_t$$

and so $x \circ y \subseteq I_t$. Similarly, we can show that $y \in I_t$. Since I_t is a weak hyper *BCK*-ideal of H, then $x \in I_t$ i.e.

$$\mu(x) = \bigvee_{x\in I_\alpha} \alpha \geq t = \bigwedge_{a\in x\circ y} \mu(a) \wedge \mu(y)$$

This implies that μ is a fuzzy weak hyper BCK-ideal of H. Now, let $x \in I_\beta$, for $\beta \in [0, 1]$. Thus $\mu(x) = \bigvee_{x\in I_\alpha} \alpha \geq \beta$ and so $x \in \mu_\beta$. Hence, $I_\beta \subseteq \mu_\beta$ Now, let $x \in \mu_\beta$. Then

$\bigvee_{x\in I_\alpha} \alpha = \mu(x) \geq \beta$ and so for $M = \{\alpha \in [0, 1] : x \in I_\alpha\}$ we have

$$x \in \bigcap_{\alpha\in M} I_\alpha = I_{\bigwedge_{\alpha\in M}\alpha} \subseteq I_B \quad \text{by Remark 2.2}$$

This implies that $x \in I_\beta$ and so $\mu_\beta \subseteq I_\beta$. Therefore, $\mu_\beta = I_\beta$, for all $\beta \in [0, 1]$.

Conversely, let there exists fuzzy subset μ of *H* such that for all $\alpha \in [0, 1]$, $\mu_\alpha = I_\alpha$, $M \subseteq [0, 1]$ and $x \in I_{\bigvee_{\alpha\in M}\alpha}$. Then $x \in \mu_{\bigvee_{\alpha\in M}\alpha}$ and so $\mu(x) \geq \bigvee_{\alpha\in M} \alpha \geq \alpha$, for all $\alpha \in M$. This implies that $x \in \mu_\alpha = I_\alpha$, for all $\alpha \in M$ and so $x \in \bigcap_{\alpha\in M} I_\alpha$. Hence,

$$I_{\underset{\alpha\in M}{\vee}\alpha} \subseteq \bigcap_{\alpha\in M} I_\alpha .$$

Similarly, we can show that

$$\bigcap_{\alpha\in M} I_\alpha \subseteq I_{\underset{\alpha\in M}{\vee}\alpha} .$$

Therefore,

$$I_{\underset{\alpha\in M}{\vee}\alpha} = \bigcap_{\alpha\in M} I_\alpha .$$

Example 3.2. Let $H = [0, \infty)$. Then H together with the following hyperoperation is a hyper BCK-algebra (for details see [3]):

$$x \circ y = \begin{cases} [0, x] & x \leq y, \\ (0, y] & x > y \neq 0, \\ \{x\} & y = 0 \end{cases}$$

for all $x, y \in H$. Also, for all $a \in H$, $I_a = \{0\} \cup [a, \infty)$ is a weak hyper BCK-ideal of H. Now, let $I_\alpha = \{0\} \cup [\alpha, \infty)$, for all $\alpha \in [0, 1]$. We observe that,

$$I_\alpha \subseteq I_\beta \text{ if and only if } \alpha \geq \beta, \text{ for all } \alpha, \beta \in [0, 1] \quad (3)$$

Now, let $M \subseteq [0, 1]$. By relation (3), it is easy to check that $\bigcap_{\alpha\in M} I_\alpha = I_\beta$, where $\beta = \underset{\alpha\in M}{\vee}\alpha$ and so we have $I_{\underset{\alpha\in M}{\vee}\alpha} = \bigcap_{\alpha\in M} I_\alpha .$ Now, if we define fuzzy subset μ of H by $\mu(x) = \underset{\alpha\in I_\alpha}{\vee}\alpha$, for all $x \in H$, then we can see that μ is a fuzzy weak hyper BCK-ideal of H and satisfies the condition $\mu_\alpha = I_\alpha$, for all $\alpha \in [0, 1]$.

Theorem 3.3. Let $\{I_\alpha\}_{\alpha\in[0,1]}$ be a class of hyper BCK-ideals of hyper BCK-algebra H. The necessary and sufficient condition for there exists a fuzzy hyper BCK-ideal μ of H such that for all $\alpha \in [0, 1]$ we have $\mu_\alpha = I_\alpha$ is that

$$I_{\underset{\alpha\in M}{\vee}\alpha} = \bigcap_{\alpha\in M} I_\alpha .$$

for all M $\subseteq$ [0, 1].

Proof. Let fuzzy subset μ of H is defined as (2) in Theorem 3.1. We must show that μ is a fuzzy hyper *BCK*-ideal of H. Let $x << y$, for $x, y \in H$. Now, there exists $\alpha \in [0, 1]$ such that $y \in I_\alpha$. It must be noted that such α exists, because, $\{I_\alpha\}_{\alpha\in[0,1]}$ satisfies the condition (1) of Theorem 2.1 and so by Remark 2.2, for $\alpha = 0$ we have $I_0 = H$ which implies that $y \in I_0$. Since $x << y \in I_\alpha$ and I_α is a hyper *BCK*-ideal of H, then $x \in I_\alpha$. This implies that

$$\{\alpha \in [0, 1] : y \in I_\alpha\} \subseteq \{\alpha \in [0, 1] : x \in I_\alpha\}$$

and so

$$\mu(y) = \underset{y\in I_\alpha}{\vee} \alpha \le \underset{x\in I_\alpha}{\vee} \alpha = \mu(x)$$

The rest of the proof is similar to the proof of Theorem 3.1.

Example 3.4. Let H = {0, 1, 2, 3}. Then the following table shows a hyper BCK-algebra structure on *H*:

o	0	1	2	3
0	{0}	{0}	{0}	{0}
1	{1}	{0}	{0}	{0}
2	{2}	{1, 2}	{0, 1, 2}	{0, 1, 2}
3	{3}	{1, 2, 3}	{1, 2, 3}	{0, 2, 3}

It is easy to check that {0}, {0, 1}, {0, 1, 2} and *H* are hyper *BCK*-ideals of *H*. Now, let $0 \le \alpha_1 < \alpha_2 < \alpha_3 \le 1$ and

$$\begin{array}{lll} I_\alpha = H & \text{for} & 0 \le \alpha \le \alpha_1, \\ I_\alpha = \{0, 1, 2\} & \text{for} & \alpha_1 < \alpha \le \alpha_2, \\ I_\alpha = \{0, 1\} & \text{for} & \alpha_2 < \alpha \le \alpha_3, \\ I_\alpha = \{0\} & \text{for} & \alpha_3 < \alpha \le 1. \end{array}$$

It is easy to check that $\{I_\alpha\}_{\alpha\in[0,1]}$ satisfies the condition (1) that is for any subset $M \subseteq [0, 1]$, $I_{\underset{\alpha\in M}{\vee}\alpha} = \bigcap_{\alpha\in M} I_\alpha.$ Now, let fuzzy subset μ of H is defined as follows:

$$\mu(x) = \begin{cases} 1 \quad , & x = 0, \\ \alpha_3 \ , & x = 1, \\ \alpha_2 \ , & x = 2, \\ \alpha_1 \ , & x = 3. \end{cases}$$

for all $x \in H$. The proof of this fact that μ is a fuzzy hyper *BCK*-ideal is easy. Now, we show that $\mu_\alpha = I_\alpha$, for all $\alpha \in [0, 1]$.

Let $\alpha \in [0, \alpha_1]$. Then

$$\mu_\alpha = \{x \in H : \mu(x) \geq \alpha\} = \{0, 1, 2, 3\} = I_\alpha.$$

Also, for $\alpha \in (\alpha_1, \alpha_2]$, $\mu_\alpha = \{0, 1, 2\} = I_\alpha$. Similarly, for all $\alpha \in (\alpha_2, \alpha_3]$ and $\alpha \in (\alpha_3, 1]$, we have $\mu_\alpha = I_\alpha$.

Theorem 3.5. Let $\{S_\alpha\}_{\alpha\in[0,1]}$ be a class of hyper subalgebras of hyper *BCK*-algebra *H*. The necessary and sufficient condition for there exists a fuzzy hyper subalgebra μ of *H* such that for all $\alpha \in [0, 1]$ we have $\mu_\alpha = S_\alpha$ is that

$$S_{\underset{\alpha\in M}{\vee}\alpha} = \bigcap_{\alpha\in M} S_\alpha$$

for all $M \subseteq [0, 1]$.

Proof. Let fuzzy subset μ of H is defined as (2) in Theorem 3.1. We must show that μ is a fuzzy hyper subalgebra. Let $\mu(x) \wedge \mu(y) = t$, for $x, y \in H$. Thus, $\underset{x\in S_\alpha}{\vee}\alpha = \mu(x) \geq t$ and similarly $\underset{y\in S_\alpha}{\vee}\alpha \geq t$ and so for $M = \{\alpha \in [0, 1] : x \in S_\alpha\}$ we have

$$x \in \bigcap_{\alpha\in M} S_\alpha = S_{\underset{\alpha\in M}{\vee}\alpha} \subseteq S_t \text{ by Remark 2.2}$$

Similarly, $y \in S_t$. Since S_t is a hyper subalgebra of H, then $x \circ y \subseteq S_t$ and so for all $a \in x \circ y$, $\mu(a) = \bigvee_{\alpha \in S_\alpha} \alpha \geq t = \mu(x) \wedge \mu(y)$. Hence,

$$\bigwedge_{\alpha \in xoy} \mu(a) \geq \mu(x) \wedge \mu(y)$$

This implies that μ is a fuzzy hyper subalgebra of H. The rest of the proof is similar to the proof of Theorem 3.1.

Example 3.6. Let $H = \{0, 1, 2, 3\}$. Then the following table shows a hyper BCK-algebra structure on H:

o	0	1	2	3
0	{0}	{0}	{0}	{0}
1	{1}	{0}	{0}	{0}
2	{2}	{3}	{0, 3}	{1, 3}
3	{3}	{3}	{0, 3}	{0, 3}

It is easy to check that {0, 3} and {0, 2, 3} are hyper subalgebras of H. Now, let

$$0 \leq \alpha_1 < \alpha_2 < \alpha_3 \leq 1$$

and

$$\begin{array}{lll} S_\alpha = H & \text{for} & 0 \leq \alpha \leq \alpha_1, \\ S_\alpha = \{0, 2, 3\} & \text{for} & \alpha_1 < \alpha \leq \alpha_2, \\ S_\alpha = \{0, 3\} & \text{for} & \alpha_2 < \alpha \leq \alpha_3, \\ S_\alpha = \{0\} & \text{for} & \alpha_3 < \alpha \leq 1. \end{array}$$

It is easy to check that $\{S_\alpha\}_{\alpha \in [0,1]}$ satisfies the condition (1). Now, let fuzzy subset μ of H is defined as follows: for all $x \in H$,

$$\mu(x) = \begin{cases} 1 & , \quad x = 0, \\ \alpha_3 & , \quad x = 3, \\ \alpha_2 & , \quad x = 2, \\ \alpha_1 & , \quad x = 1. \end{cases}$$

The proof of this fact that μ is a fuzzy hypersubalgebra is easy. Now, we show that $\mu_\alpha = S_\alpha$, for all $\alpha \in [0, 1]$. Let $\alpha \in [0, \alpha_1]$. Then

$$\mu_\alpha = \{x \in H : \mu(x) \geq \alpha\} = \{0, 1, 2, 3\} = S_\alpha.$$

Also, for $\alpha \in (\alpha_1, \alpha_2]$, $\mu_\alpha = \{0, 2, 3\} = S$. Similarly, for all $\alpha \in (\alpha_2, \alpha_3]$ and $\alpha \in (\alpha_3, 1]$, we have $\mu_\alpha = S_\alpha$.

3.2. Representation Theorem for Fuzzy Subhypergroups of a Hypergroup

Definition 3.7. A hypergroup H is said to be invertible on the left if for all $a \in H$, $x \in a \circ y$ implies that $y \in a \circ x$, for every $x, y \in H$. Similarly, invertibility on the right is defined. A hypergroup which is invertible on the left and right is said to be invertible.

Definition 3.8. [1] A fuzzy subset μ of hypergroup H is said to be a fuzzy subhypergroup if satisfies the following conditions:

(i) $\bigwedge_{a \in x \circ y} \mu(a) \geq \mu(x) \wedge \mu(y)$, for all $x, y \in H$,

(ii) for all $x, a \in H$ there exist $y, z \in H$ such that $x \in a \circ y \cap z \circ a$ and

$$\mu(y) \wedge \mu(z)) \geq \mu(a) \wedge \mu(x).$$

Theorem 3.9. Let $\{S_\alpha\}_{\alpha \in [0,1]}$ be a class of subhypergroups of invertible hypergroup H. The necessary and sufficient condition for there exists a fuzzy subhypergroup μ of H such that for all $\alpha \in [0, 1]$ we have $\mu_\alpha = S_\alpha$ is that

$$S_{\underset{a \in M}{\wedge} \alpha} = \bigcap_{\alpha \in M} S_\alpha$$

for all $M \subseteq [0, 1]$.

Proof. Let $\{S_\alpha\}_{\alpha \in [0,1]}$ be a family of subhypergroups of H such that

$$S_{\underset{a \in M}{\vee} \alpha} = \bigcap_{\alpha \in M} S_\alpha$$

for all $M \subseteq [0, 1]$. We define fuzzy subset μ of H as (2) in the Theorem 3.1. We first show that μ is a fuzzy subhypergroup of H. For this let $t = \mu(x) \wedge \mu(y)$. Thus

$$\bigvee_{x \in S_\alpha} \alpha = \mu(x) \geq t \quad \text{and} \quad \bigvee_{y \in S_\alpha} \alpha = \mu(y) \geq t$$

and so for $M = \{\alpha \in [0, 1] : x \in S_\alpha\}$ we have

$$\mathrm{x} \in \bigcap_{\alpha \in M} S_\alpha = S_{\bigvee_{\alpha \in M} \alpha} \subseteq S_t.$$

Similarly, we can show that $y \in S_t$. Since S_t is a subhypergroup of H, then $x \text{ o } y \subseteq S_t$ and so for all $a \in x \text{ o } y$ we have $\mu(a) = \bigvee_{a \in S_\alpha} \alpha \geq t$. Thus

$$\bigwedge_{a \in x \circ y} \mu(a) \geq t = \mu(x) \wedge \mu(y).$$

Now, let $x, a \in H$. Since H is a hypergroup, then $x \in H = a \text{ o } H = H \text{ o } a$ and so there exist $y, z \in H$ such that $x \in a \text{ o } y \cap z \text{ o } a$. Since H is invertible, then $y \in a \text{ o } x$ and $z \in x \text{ o } a$ and so

$$\mu(y) \geq \bigwedge_{\mu \in a \circ x} \mu(u) \geq \mu(x) \wedge \mu(a)$$

Similarly, we can show that $\mu(z) \geq \mu(x) \wedge \mu(a)$ and so

$$\mu(y) \wedge \mu(z) \geq \mu(x) \wedge \mu(a)$$

Hence, μ is a fuzzy subhypergroup of H. It is easy to check that $\mu_\alpha = S_\alpha$, for all $\alpha \in [0, 1]$ (See the proof of Theorem 3.1.)

3.3. Representation Theorem for fuzzy subhyperrings and hyperideals

Theorem 3.10. Let $\{I_\alpha\}_{\alpha \in [0,1]}$ be a class of hyperideals of hyperring R. The necessary and sufficient condition for there exists a fuzzy hyperideal μ of R such that for all $\alpha \in [0, 1]$ we have $\mu_\alpha = \mathrm{I}_\alpha$ is that

$$I_{\bigvee_{\alpha \in M} \alpha} = \bigcap_{\alpha \in M} I_\alpha$$

for all $M \subseteq [0, 1]$.

Proof. Let $\{I_\alpha\}_{\alpha\in[0,1]}$ be a class of hyperideals of hyperring R such that

$$I_{\bigvee_{\alpha\in M}\alpha} = \bigcap_{\alpha\in M} I_\alpha$$

for all M $\subseteq$ [0, 1] and fuzzy subset μ of R is defined as (2) in Theorem 3.1. Let $\mu(x) \wedge \mu(y) = t$, for $x, y \in R$. Thus $\bigvee_{x\in I_\alpha} \alpha = \mu(x) \geq t$. Now, let $M = \{\alpha \in [0, 1] : x \in I_\alpha\}$. Then

$$x \in \bigcap_{\alpha\in M} I_\alpha = I_{\bigvee_{\alpha\in M}\alpha} \subseteq I_t \quad \text{by Remark 2.2}$$

Hence x $\in I_t$. Similarly, we can show that $y \in I_t$. Since I_t is a hyperideal of R, then $x + y \subseteq I_t$ and so for all $z \in x + y$,

$$\mu(z) = \bigvee_{z\in I_\alpha} \alpha \geq t = \mu(x) \wedge \mu(y).$$

So,

$$\bigwedge_{z\in x+y} \mu(z) \geq \mu(x) \wedge \mu(y).$$

Now, let $\mu(x) = t$. Thus $\bigvee_{x\in I_\alpha} \alpha = t$. This implies that

$$x \in \bigcap_{\alpha\in M} I_\alpha = I_{\bigvee_{\alpha\in M}\alpha} = I_t.$$

Since I_t is a hyperideal of R, then $-x \in I_t$ and so

$$\mu(-x) = \bigvee_{-x\in I_\alpha} \alpha \geq t = \mu(x).$$

Now, let $\mu(x) \vee \mu(y) = t$, for $x, y \in R$. Thus $\mu(x) = t$ or $\mu(y) = t$ and so $\bigvee_{x\in I_\alpha} \alpha = \mu(x) \geq t$ or $\bigvee_{y\in I_\alpha} \alpha = \mu(y) \geq t$ Hence,

$$x \in \bigcap_{\alpha \in M} I_\alpha = I_{\underset{\alpha\in M}{\vee}\alpha} \subseteq I_t \quad \text{by Remark 2.2}$$

or y $\in I_t$. Since I_t is a hyperideal of R, then $xy \in I_t$ and so

$$\mu(xy) = \underset{xy\in I_\alpha}{\vee} \alpha \geq t = \mu(x) \vee \mu(y).$$

Therefore, μ is a fuzzy hyperideal of R. The proof of $\mu_\alpha = I_\alpha$ is similar to the proof of Theorem 3.1.

Theorem 3.11. Let $\{S_\alpha\}_{\alpha\in[0,1]}$ be a class of subhyperrings of hyperring R. The necessary and sufficient condition for there exists a fuzzy subhyperring μ of R such that for all $\alpha \in [0, 1]$ we have $\mu_\alpha = S_\alpha$ is that

$$S_{\underset{\alpha\in M}{\vee}\alpha} = \bigcap_{\alpha\in M} S_\alpha$$

for all $M \subseteq [0, 1]$.

Proof. Since every fuzzy hyperideal of R is a fuzzy subhyperring, then the properties (i) and (ii) of Definition 2.5 hold, by Theorem 3.10. Now, we prove the property (iii) of Definition 2.5. Let $\mu(x) \wedge \mu(y) = t$. Then $\underset{x\in S_\alpha}{\vee} \alpha = \mu(x) \geq t$. If $M = \{\alpha \in [0, 1] : x \in I_\alpha\}$, then

$$x \in \bigcap_{\alpha\in M} S_\alpha = S_{\underset{\alpha\in M}{\vee}\alpha} \subseteq S_t \quad \text{by Remark 2.2}$$

Similarly, we can show that y $\in S_t$ and so $xy \in S_t$. Thus

$$\mu(xy) = \underset{xy\in S_\alpha}{\vee} \alpha \geq t = \mu(x) \wedge \mu(y).$$

The proof of $\mu_\alpha = S_\alpha$ is similar to the proof of Theorem 3.1.

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