International Review of Fuzzy Mathematics
Vol. 3 No. 1 (June, 2018)

Received: 23rd March 2017 Revised: 14th April 2017 Accepted: 10th September 2017

Yong Chan Kim & Jung Mi Ko

L-FUZZY TOPOLOGIES AND *L*-FUZZY QUASI-UNIFORM SPACES

ABSTRACT: We prove the L-fuzzy topology induced by the initial L-fuzzy (quasi-) uniform structure coincides with the initial L-fuzzy topology for each L-fuzzy topology induced by L-fuzzy (quasi-) uniform structures.

2000 Mathematics Subject Classification: *54A40, 54A05, 54A15, 54E15.*

Keywords: *L-fuzzy topological spaces, L-fuzzy (quasi-)uniform spaces, Initial L-fuzzy (quasi-)uniform structures, LF-(uniformly-) continuous.*

1. INTRODUCTION

Šostak [19] introduced the notion of L-fuzzy topological spaces as a generalization of [0, 1]-topological spaces [4]. Moreover, Samanta [18] introduced the concept of [0, 1]-fuzzy uniform spaces as an expansion of Hutton L-uniform spaces [11]. Kim [14] defined an L-fuzzy (resp. quasi-) uniform space in a somewhat different view of the definition of Samanta [18]. Ko and Kim [15] proved the existence of initial L-fuzzy (quasi-)uniform structures.

In this paper, we show that the L-fuzzy topology induced by the initial L-fuzzy (quasi-) uniform structure coincides with the initial L-fuzzy topology for each L-fuzzy topology induced by L-fuzzy (quasi-) uniform structures.

2. PRELIMINARIES

In this paper, let X be a nonempty set. Let $L = (L, \leq, \vee, \wedge',)$ be a completely distributive lattice with an order-reversing involution$'$, 0 and 1 denote the least and the greatest element in L. For $\alpha \in L$, $\underline{\alpha}(x) = \alpha$ for each $x \in X$ and $L_1 = L - \{1\}$. Notions and notations not described in this paper are standard and usual.

Definition 2.1 [19] A function $\mathcal{T}: L^X \to L$ is called an *L-fuzzy topology* on X if it satisfies the following conditions:

(O1) $\mathcal{T}(\underline{0}) = \mathcal{T}(\underline{1}) = 1$,

(O2) $\mathcal{T}(\mu_1 \wedge \mu_2) \geq \mathcal{T}(\mu_1) \wedge \mathcal{T}(\mu_2)$, for all $\mu_1, \mu_2 \in L^X$,

(O3) $\mathcal{T}(\vee_{i\in\Lambda} \mu_i) \geq \wedge_{i\in\Lambda} \mathcal{T}(\mu_i)$, for any $\{\mu_i\}_{i\in\Lambda} \subset L^X$.

The pair $(X, \mathcal{T})$ is called an *L-fuzzy topological space*.

Let $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ be L-fuzzy topological spaces. A function $\psi : (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ is called *LF-continuous* if $\mathcal{T}_2(\lambda) \leq \mathcal{T}_1(\psi^{\leftarrow}(\lambda))$, for all $\lambda \in L^Y$. Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be L-fuzzy topologies on X. We say that $\mathcal{T}_1$ is *finer* than $\mathcal{T}_2$ ($\mathcal{T}_2$ is *coarser* than $\mathcal{T}_1$) if $\mathcal{T}_2(\lambda) \leq \mathcal{T}_1(\lambda)$ for all $\lambda \in L^X$.

Definition 2.2 [13] Let $\underline{0} \notin \Theta$ be a subset of L^X. A function $\beta : \Theta \to L$ is called an *L-fuzzy basis* on X if it satisfies the following conditions:

(B1) $\beta(\underline{1}) = 1$,

(B2) $\beta(\mu_1 \wedge \mu_2) \geq \beta(\mu_1) \wedge \beta(\mu_2)$, for all $\mu_1, \mu_2 \in \Theta$.

Theorem 2.3 [13] *Let $\beta : \Theta \to L$ be an L-fuzzy basis on X. For each $\mu \in L^X$, we define the function $\mathcal{T}_\beta : L^X \to L$ as follows:*

$$\mathcal{T}_\beta(\mu) = \begin{cases} \vee\{\wedge_{j\in\Lambda} \beta(\mu_j)\}, & \text{if } \mu = \vee_{j\in\Lambda} \mu_j, \ \text{for } \{\mu_j\}_{j\in\Lambda} \subset \Theta \\ 1, & \text{if } \mu = \underline{0}, \\ 0, & \text{otherwise}. \end{cases}$$

Then

(1) *$(X, \mathcal{T}_\beta)$ is an L-fuzzy topological space.*

(2) *A function $\psi : (Y, T^*) \to (X, T_\beta)$ is LF-continuous iff $\mathcal{T}^* (\psi^{\leftarrow}(\mu)) \geq \beta(\mu)$ for each $\mu \in \Theta$.*

Theorem 2.4 [13] *Let $\{(X_i, \mathcal{T}_i)\}_{i\in\Gamma}$ be a family of L-fuzzy topological spaces, X a set and, for each $i \in \Gamma$, $\psi_i : X \to X_i$ a function. Let*

$$\Theta = \{\underline{0} \neq \mu = \wedge_{j=1}^{n} \psi_{k_j}^{\leftarrow}(\nu_{k_j}) \,|\, \mathcal{T}_{k_j}(\nu_{k_j}) > 0 \ \text{for all } k_j \in K\}$$

for every finite set $K = \{k_1, \ldots, k_n\} \subset \Gamma$. *Define the function* $\beta : \Theta \to L$ *on* X *by*

$$\beta(\mu) = \vee\{\wedge_{j=1}^{n} \mathcal{T}_{k_j}(\nu_{k_j}) \mid \mu = \wedge_{j=1}^{n} \psi_{k_j}^{\leftarrow}(\nu_{k_j})\}$$

where the $\vee$ *is taken over all finite index sets* $K = \{k_1, \ldots, k_n\} \subset \Gamma$. *Then:*

(1) β *is an L-fuzzy basis on X.*

(2) *The L-fuzzy topology* $\mathcal{T}_\beta$ *generated by* β *is the coarsest L-fuzzy topology on X for which all* ψ_i, $i \in \Gamma$, *are LF-continuous.*

(3) *A function* $\psi : (Y, T^*) \to (X, \mathcal{T}_\beta)$ *is LF-continuous iff for each* $i \in \Gamma$,

$\psi_i \circ \psi : (Y, \mathcal{T}^*) \to (X_i, \mathcal{T}_i)$ *is LF-continuous.*

Definition 2.5 [10, 13] Let **A** and **B** be categories. A functor $V : \mathbf{A} \to \mathbf{B}$ is called *topological* if every V-structured source $(\psi_i : B \to V(A_i, S_i))_{i\in I}$ has an unique V-initial lift $(\overline{\psi}_i : (A, S) \to A_i)_{i\in I}$ such that $V(A, S) = B$ and $V(\overline{\psi}_i) = \psi_i$. The structure S is called an *V-initial structure* on A with respect to $(B, \psi_i, (A_i, S_i), I)$.

The category of L-fuzzy topological spaces and LF-continuous maps is denoted by L-FTOP.

Theorem 2.6 [10, 13] *The forgetful functor* V: L-**FTOP** $\to$ **Set** *defined by* $V(X, \mathcal{T}) = X$ *and* $V(f) = f$ *is topological.*

Let Ω_X denote the family of all functions $f : L^X \to L^X$ with the following properties:

(1) $f(\underline{0}) = \underline{0}$, $\mu \le f(\mu)$, for every $\mu \in L^X$,

(2) $f(\vee\mu_i) = \vee f(\mu_i)$, for $\mu_i \in L^X$.

For $f, g \in \Omega_X$, we define, for all $\mu \in L^X$,

$$f^{-1}(\mu) = \wedge\{\rho \mid f(\underline{1} - \rho) \le \underline{1} - \mu\},$$

$$(f \sqcap g)(\mu) = \wedge\{f(\mu_1) \vee g(\mu_2) \mid \mu_1 \vee \mu_2 = \mu\},$$

$$f \circ g(\mu) = f(g(\mu)).$$

Then $f^{-1}, f \sqcap g, f \circ g \in \Omega_X$.

Lemma 2.7 [11, 14,17] *For every* $f, g, h, f_1, g_1 \in \Omega_X$, *the following properties hold:*

(1) *If* $f \le f_1$, $g \le g_1$, *then* $f \sqcap g \le f_1 \sqcap g_1$.

(2) $f \sqcap g \le f$, $f \sqcap g \le g$, $(f \sqcap g) \sqcap h = f \sqcap (g \sqcap h)$ *and* $f \sqcap f = f$.

(3) $(f^{-1})^{-1} = f$.

(4) $f \le g$ *iff* $f^{-1} \le g^{-1}$.

(5) $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

(6) $(f \sqcap g)^{-1} = f^{-1} \sqcap g^{-1}$.

(7) *Define* $f_\rho : L^X \to L^X$ *as follows:*

$$f_\rho(\lambda) = \begin{cases} \underline{0} & \text{if } \lambda = \underline{0}, \\ \rho & \text{if } \underline{0} \ne \lambda \le \rho, \\ \underline{1} & \text{otherwise.} \end{cases}$$

Then:

(a) $f_\rho \in \Omega_X$ *and* $f_\rho^{-1} = f_{\rho'}$.

(b) $f_\rho \circ f_\rho = f_\rho$, $(f_\rho \sqcap f_\mu) \circ (f_\rho \sqcap f_\mu) = f_\rho \sqcap f_\mu$ *and* $f \le f_{\underline{1}}$ *for all* $f \in \Omega_X$.

(c) $f_{\underline{1}} = f_{\underline{1}}^{-1}$, $f \sqcap f_{\underline{1}} = f$ *and* $f \le f_{\underline{1}}$ *for all* $f \in \Omega_X$.

Definition 2.8 [14] A function $\mathcal{U} : \Omega_X \to L$ is said to be an *L-fuzzy quasi-uniformity* on X if it satisfies the following conditions:

(FQU1) $\mathcal{U}(f_1 \sqcap f_2) \ge \mathcal{U}(f_1) \wedge \mathcal{U}(f_2)$, for $f_1, f_2 \in \Omega_X$.

(FQU2) For $f \in \Omega_X$, we have $\vee\{\mathcal{U}(f_1) \mid f_1 \circ f_1 \le f\} \ge \mathcal{U}(f)$.

(FQU3) If $f_1 \ge f$, then $\mathcal{U}(f_1) \ge \mathcal{U}(f)$.

(FQU4) There exists $f \in \Omega_X$ such that $\mathcal{U}(f) = 1$.

The pair $(X, \mathcal{U})$ is said to be an *L-fuzzy quasi-uniform space*.

An L-fuzzy quasi-uniform space $(X, \mathcal{U})$ is called an *L-fuzzy uniform space* if it satisfies:

(FU) for $f \in \Omega_X$, we have $\vee\{\mathcal{U}(f_1) \leq f^{-1}\} \geq \mathcal{U}(f)$.

Let $\mathcal{U}_1$ and $\mathcal{U}_2$ be L-fuzzy (resp. quasi-)uniformities on X. We say $\mathcal{U}_1$ is *finer* than $\mathcal{U}_2$ (or $\mathcal{U}_2$ is *coarser* than $\mathcal{U}_1$), denoted by $\mathcal{U}_2 \leq \mathcal{U}_1$, iff for any $f \in \Omega_X$, $\mathcal{U}_2(f) \leq \mathcal{U}_1(f)$.

Let $(X, \mathcal{U})$ be an L-fuzzy quasi-uniform space. We define for $f \in \Omega_X$, $\mathcal{U}^{-1}(f) = \mathcal{U}(f^{-1})$. From Lemma 2.7, we easily show that $\mathcal{U}^{-1}$ is an L-fuzzy quasi-uniformity on X.

Theorem 2.9 [5, 10] *Let $\mathcal{U}$ be an L-fuzzy quasi-uniformity on X. For each $r \in L_1$, $\lambda \in L^X$, we define*

$$\mathcal{C}_{\mathcal{U}}(\lambda, r) = \wedge \{f^{-1}(\lambda) \mid \mathcal{U}(f) > r\}.$$

Then it satisfies the followings:

(1) $\mathcal{C}_{\mathcal{U}}(\underline{0}, r) = \overline{0}$

(2) $\mathcal{C}_{\mathcal{U}}(\lambda, r) \geq \lambda$.

(3) *If $\lambda_1 \leq \lambda_2$, then $\mathcal{C}_{\mathcal{U}}(\lambda_1, r) \leq \mathcal{C}_{\mathcal{U}}(\lambda_2, r)$.*

(4) *If $r \leq s$, then $\mathcal{C}_{\mathcal{U}}(\lambda, r) \leq \mathcal{C}_{\mathcal{U}}(\lambda, s)$.*

(5) $\mathcal{C}_{\mathcal{U}}(\lambda_1 \vee \lambda_2, r) \leq \mathcal{C}_{\mathcal{U}}(\lambda_1, r) \vee \mathcal{C}_{\mathcal{U}}(\lambda_2, r)$.

(6) *If L is a chain, $\mathcal{C}_{\mathcal{U}}(\mathcal{C}_{\mathcal{U}}(\lambda, r), r) \leq \mathcal{C}_{\mathcal{U}}(\lambda, r)$.*

Theorem 2.10 [5, 10] *Let $(X, \mathcal{U})$ be an L-fuzzy quasi-uniform space. The function* $\mathcal{T}_{\mathcal{U}} : L^X \to L$ is defined by, for each $\lambda \in L^X$,

$$\mathcal{T}_{\mathcal{U}}(\lambda) = \vee\{r \in L \mid \mathcal{C}_{\mathcal{U}}(\lambda', r) \leq \lambda'\}.$$

Then $\mathcal{T}_{\mathcal{U}}$ is an L-fuzzy topology on X.

Lemma 2.11 [11, 14] *Let $\psi : X \to Y$ be a function. For each $f \in \Omega_Y$, a function* $\psi^{-1}(f) : L^X \to L^X$ is defined by, for all $\mu \in L^X$,

$$\psi^{-1}(f)(\mu) = (\psi^{\leftarrow} \circ f \circ \psi^{\rightarrow})(\mu) = \psi^{\leftarrow}(f(\psi^{\rightarrow}(\mu))).$$

For $f, f_1, f_2 \in \Omega_Y$, we have the following properties.

(1) $\psi^{-1}(f) \in \Omega_X$.

(2) *If* $f_1 \leq f_2$, *then* $\psi^{-1}(f_1) \leq \psi^{-1}(f_2)$.

(3) $\psi^{-1}(f_1) \circ \psi^{-1}(f_2) \leq \psi^{-1}(f_1 \circ f_2)$ *with equality if* ψ *is onto.*

(4) $(\psi^{-1}(f))^{-1} = \psi^{-1}(f^{-1})$.

(5) $\psi^{-1}(f_1) \sqcap \psi^{-1}(f_2) = \psi^{-1}(f_1 \sqcap f_2)$.

(6) $\psi^{\rightarrow}((\psi^{-1}(f))^{-1}(\lambda)) \leq f^{-1}(\psi^{\rightarrow}(\lambda))$, *for all* $\lambda \in L^X$.

(7) $\psi^{-1}(f_\rho) = f_{\psi^{-1}}(\rho)$.

Definition 2.12 [14] Let $(X, \mathcal{U})$ and $(Y, \mathcal{V})$ be L-fuzzy (quasi-)uniform spaces. A function $\psi : (X, \mathcal{U}) \rightarrow (Y, \mathcal{V})$ is *LF-uniformly continuous* if $\mathcal{V}(f) \leq \mathcal{U}(\psi^{-1}(f))$, for every $f \in \Omega_Y$.

Theorem 2.13 [14] *Let* $(X, \mathcal{U})$, $(Y, \mathcal{V})$ *and* $(Z, \mathcal{W})$ *be L-fuzzy (quasi-)uniform spaces. If* $\psi : (X, \mathcal{U}) \rightarrow (Y, \mathcal{V})$ *and* $\phi : (Y, \mathcal{V}) \rightarrow (Z, \mathcal{W})$ *are LF-uniformly continuous, then* $\phi \circ \psi : (X, \mathcal{U}) \rightarrow (Z, \mathcal{W})$ *is LF-uniformly continuous.*

Theorem 2.14 [14] *Let* $(X, \mathcal{U})$ *and* $(Y, \mathcal{V})$ *be L-fuzzy quasi-uniform spaces. Let* $\psi : (X, \mathcal{U}) \rightarrow (Y, \mathcal{V})$ *be LF-uniformly continuous. For each* $\lambda \in L^X$, $\nu \in L^Y$ *and* $r \in L_1$, *we have the following properties.*

(1) $\psi : (X, \mathcal{U}^{-1}) \rightarrow (Y, \mathcal{V}^{-1})$ *is LF-uniformly continuous.*

(2) $\mathcal{C}_{\mathcal{V}}(\psi^{\rightarrow}(\lambda), r)) \geq \psi^{\rightarrow}(\mathcal{C}_{\mathcal{U}}(\lambda, r))$, $\mathcal{C}_{\mathcal{V}^{-1}}(\psi^{\rightarrow}(\lambda), r)) \geq \psi^{\rightarrow}(\mathcal{C}_{\mathcal{U}^{-1}}(\lambda, r))$.

(3) $\mathcal{C}_{\mathcal{U}}(\psi^{\leftarrow}(\nu), r)) \leq \psi^{\leftarrow}(\mathcal{C}_{\mathcal{V}}(\nu, r))$.

(4) $\psi : (X, \mathcal{T}_{\mathcal{U}}) \rightarrow (Y, \mathcal{T}_{\mathcal{V}})$ *is LF-continuous.*

(5) $\psi : (X, \mathcal{T}_{\mathcal{U}^{-1}}) \rightarrow (Y, \mathcal{T}_{\mathcal{V}^{-1}})$ *is LF-continuous.*

Theorem 2.15 [15] *Let* $\{(X_k, \mathcal{V}_k) \mid k \in \Gamma\}$ *be a family of L-fuzzy (resp. quasi) uniform spaces, X a set and for each* $k \in \Gamma$, $\psi_k : X \rightarrow X_k$ *a function. We define a function* $\mathcal{U} : \Omega_X \rightarrow L$ *by*

$$U(f) = \vee\{\bigwedge_{i=1}^{n} \mathcal{V}_{k_i}(f_{k_i}) \mid \sqcap_{i=1}^{n} \psi_{k_i}{}^{-1}(f_{k_i}) \leq f\}$$

where the $\vee$ *is taken over every finite index* $K = \{k_1, \ldots, k_n\} \subset \Gamma$. *Then:*

(1) *The structure $\mathcal{U}$ is the coarsest L-fuzzy (resp. quasi-)uniformity on X for which each* ψ_k is LF-uniformly continuous.

(2) *A map* $f : (Z, \mathcal{W}) \to (X, \mathcal{U})$ *is LF-uniformly continuous iff for each* $k \in \Gamma$, $\psi_k \circ f : (Z, \mathcal{W}) \to (X_k, \mathcal{V}_k)$ *is LF-uniformly continuous.*

The category of *L*-fuzzy uniform spaces and *LF*-uniformly continuous maps is denoted by *L*-UNIF.

Theorem 2.16 [15] *The forgetful functor* $W : L\text{-UNIF} \to \text{Set}$ *defined by* $W(X, \mathcal{U}) = X$ *and* $W(\psi) = \psi$ *is topological.*

3. *L*-FUZZY TOPOLOGIES AND *L*-FUZZY QUASI-UNIFORMITY SPACES

We show that the *L*-fuzzy topology induced by the initial *L*-fuzzy (quasi-)uniform structure coincides with the initial *L*-fuzzy topology for each *L*-fuzzy topology induced by *L*-fuzzy (quasi-)uniform structures.

Theorem 3.1 *Let L be a chain,* $(X_i, \mathcal{U}_i)_{i\in\Gamma}$ *L-fuzzy (resp. quasi-)uniform spaces and* $\psi_i : X \to X_i$ *a function for each* $i \in \Gamma$. *Let the structure $\mathcal{U}$ be the initial L-fuzzy (resp. quasi-)uniformity on X with respect to* $(X, \psi_i, (X_i, \mathcal{U}_i), i \in \Gamma)$ *for which each* ψ_i *is LF-uniformly continuous. Then:*

(1) $\mathcal{C}_{\mathcal{U}}(\lambda, r) = \wedge\{ \sqcap_{i=1}^{n} \psi_{k_i}^{-1}(f_{k_i}^{-1})(\lambda) \mid \mathcal{V}_{k_i}(f_{k_i}) > r,\ \forall k_i \in K\}$

where the $\wedge$ *is taken over every finite index* $K = \{k_1, \ldots, k_n\} \subset \Gamma$,

(2) *The L-fuzzy topology* $\mathcal{T}_{\mathcal{U}}$ *induced by* $\mathcal{U}$ *coincides with the initial L-fuzzy topology* $\mathcal{T}_\beta$ *on X with respect to* $(X, \psi_i, (X_i, \mathcal{T}_{\mathcal{U}_i}), i \in \Gamma)$ *where all* $\psi_i : (X, \mathcal{T}_\beta) \to (X_i, \mathcal{T}_{\mathcal{U}_i})$ *are LF-continuous.*

Proof: (1) From Theorem 2.9, we only show that

$$\wedge\{f^{-1}(\lambda) \mid \mathcal{U}(f) > r\} = \wedge\{\sqcap_{i=1}^{n} \psi_{k_i}^{-1}(f_{k_i}^{-1})(\lambda) \mid \mathcal{V}_{k_i}(f_{k_i}) > r,\ \forall k_i \in K\}$$

where the $\wedge$ is taken over every finite index $K = \{k_1, ..., k_n\} \subset \Gamma$.

Since $\mathcal{U}(\psi_{k_i}{}^{-1}(f_{k_i})) \geq \mathcal{V}_{k_i}(f_{k_i}) > r$ and $(\psi_{k_i}{}^{-1}(f_{k_i}))^{-1} = \psi_{k_i}{}^{-1}(f_{k_i}^{-1})$ from Lemma 2.11(4), we have

$$\wedge\{f^{-1}(\lambda) | \mathcal{U}(f) > r\} \leq \wedge\{\sqcap_{i=1}^{n} \psi_{k_i}{}^{-1}(f_{k_i}^{-1})(\lambda) \mid \mathcal{V}_{k_i}(f_{k_i}) > r\}$$

Conversely, suppose that

$$\wedge\{f^{-1}(\lambda) | \mathcal{U}(f) > r\} \not\leq \wedge\{\sqcap_{i=1}^{n} \psi_{k_i}{}^{-1}(f_{k_i}^{-1})(\lambda) \mid \mathcal{V}_{k_i}(f_{k_i}) > r\}$$

Then there exists $f \in \Omega_X$ with $\mathcal{U}(f) > r$ such that

$$f^{-1}(\lambda) \not\leq \wedge\{\sqcap_{i=1}^{n} \psi_{k_i}{}^{-1}(f_{k_i}^{-1})(\lambda) \mid \mathcal{V}_{k_i}(f_{k_i}) > r\}$$

On the other hand, since L is a chain and $\mathcal{U}(f) > r$, there exists a finite index $K = \{k_1, \ldots, k_n\} \subset \Gamma$ such that

$$\mathcal{U}(f) \geq \bigwedge_{i=1}^{n} \mathcal{V}_{k_i}(f_{k_i}) > r, \ \sqcap_{i=1}^{n} \psi_{k_i}{}^{-1}(f_{k_i}) \leq f.$$

It implies

$$\begin{aligned}\wedge\{\sqcap_{i=1}^{n} \psi_{k_i}{}^{-1}(f_{k_i}^{-1})(\lambda) \mid \mathcal{V}_{k_i}(f_{k_i}) > r\} &\leq \sqcap_{i=1}^{n} \psi_{k_i}{}^{-1}(f_{k_i}^{-1})(\lambda) \\ &\leq f^{-1}(\lambda)\end{aligned}$$

It is a contradiction.

(2) Suppose there exists $\lambda \in L^X$ such that $\mathcal{T}_\beta(\lambda) \not\geq \mathcal{T}_{\mathcal{U}}(\lambda)$. By the definition of $\mathcal{T}_{\mathcal{U}}$ from Theorem 2.10, there exists $r_0 \in L$ such that $\mathcal{C}_{\mathcal{U}}(\lambda', r_0) = \lambda'$ and $\mathcal{T}_\beta(\lambda) \not\geq r_0$.

On the other hand, since $\mathcal{C}_{\mathcal{U}}(\lambda', r_0) = \lambda'$, we have

$$\begin{aligned}\lambda' &= \mathcal{C}_{\mathcal{U}}(\lambda', r_0) \\ &= \wedge \ \{f^{-1}(\lambda') \mid \mathcal{U}(f) > r_0\} \\ &= \wedge \ \{ \ \wedge\{\sqcap_{i=1}^{n} \psi_{k_i}{}^{-1}(f_{k_i}^{-1})(\lambda') \mid \mathcal{U}_{k_i}(f_{k_i}) > r_0\} \quad \text{(by (1))}\end{aligned}$$

where the $\wedge$ is taken over every finite index $K = \{k_1, ..., k_n\} \subset \Gamma$. Since $\mathcal{U}_{k_i}(f_{k_i}) > r_0$ for all $k_i \in \Gamma$, we have

$$\sqcap_{i=1}^{n} \psi_{k_i}{}^{-1}(f_{k_i}^{-1})(\lambda') = \wedge\{\vee_{i=1}^{n} \psi_{k_i}{}^{-1}(f_{k_i}^{-1}(\mu_i))\}$$

$$= \wedge\{\vee_{i=1}^{n} \psi_{k_i}^{\leftarrow}(f_{k_i}^{-1}(\psi_{k_i}^{\rightarrow}(\mu_i))\}$$

where $\wedge$ is taken over every finite family $\{\mu_i \mid i = 1, ..., n\}$ such that $\lambda' = \vee_{i=1}^{n} \mu_i$. It implies

$$\lambda' = \wedge\{\sqcap_{i=1}^{n} \psi_{k_i}{}^{-1}(f_{k_i}^{-1})(\lambda') \mid \mathcal{U}_{k_i}(f_{k_i}) > r_0\}$$

$$= \wedge\{\wedge\{\vee_{i=1}^{n} \psi_{k_i}^{\leftarrow}(f_{k_i}^{-1}(\psi_{k_i}^{\rightarrow}(\mu_i))\} \mid \mathcal{U}_{k_i}(f_{k_i}) > r_0\}$$

$$= \wedge\{\vee_{i=1}^{n} \psi_{k_i}^{\leftarrow}(\mathcal{C}_{\mathcal{U}_{k_i}}(\psi_{k_i}^{\rightarrow}(\mu_i), r_0))\}$$

Thus,

$$\lambda = \{\wedge\{\wedge\{\vee_{i=1}^{n} \psi_{k_i}^{\leftarrow}(\mathcal{C}_{\mathcal{U}_{k_i}}(\psi_{k_i}^{\rightarrow}(\mu_i), r_0))\}\}\}'$$

$$= \vee\{\vee\{\vee_{i=1}^{n} \psi_{k_i}^{-1}(\mathcal{C}_{\mathcal{U}_{k_i}}(\psi_{k_i}^{\rightarrow}(\mu_i), r_0))\}\}\}'\}$$

$$= \vee\{\vee\{\wedge_{i=1}^{n} \psi_{k_i}^{-1}(\mathcal{C}_{\mathcal{U}_{k_i}}(\psi_{k_i}^{\rightarrow}(\mu_i), r_0)))'\}\}$$

$$= \vee\{\vee\{\wedge_{i=1}^{n} \psi_{k_i}^{-1}(\mathcal{C}_{\mathcal{U}_{k_i}}(\psi_{k_i}^{\rightarrow}(\mu_i), r_0)'\}\}$$

where the first $\vee$ is taken over every finite index $K = \{k_1, \ldots, k_n\} \subset \Gamma$ and the second $\vee$ is taken over every finite family $\{\nu_i \mid i = 1, ..., n\}$ such that $\lambda' = \vee_{i=1}^{n} \mu_i$. Since

$\mathcal{C}_{\mathcal{U}k_i}(\psi_{k_i}^{\rightarrow}(\mu_i), r_0) = \mathcal{C}_{\mathcal{U}k_i}(\mathcal{C}_{\mathcal{U}k_i}(\psi_{k_i}^{\rightarrow}(\mu_i), r_0), r_0)$, from Theorem 2.9, we have

$$\mathcal{T}_{\mathcal{U}_{k_i}}(\mathcal{C}_{\mathcal{U}_{k_i}}(\psi_{k_i}^{\rightarrow}(\mu_i), r_0)') \geq r_0.$$

Put $\nu_i = f_{k_i}^{-1}(\mathcal{C}_{\mathcal{U}_{k_i}}(\psi_{k_i}^{\rightarrow}(\mu_{k_i}), r_0)')$. From Theorem 2.4, we have

$$\beta(\nu_i) \geq \mathcal{T}_{\mathcal{U}_{k_i}}(\mathcal{C}_{\mathcal{U}_{k_i}}(\psi_{k_i}^{\rightarrow}(\mu_{k_i}), r_0)') \geq r_0.$$

It implies $\beta(\wedge_{i=1}^{n} \nu_i) \geq r_0$. By the definition of $\mathcal{T}_\beta$ from Theorem 2.3, we have

$$\mathcal{T}_\beta(\lambda) \geq \wedge \beta(\bigwedge_{i=1}^{n} \nu_i) \geq r_0.$$

It is a contradiction. Therefore $\mathcal{T}_\beta(\mu) \geq \mathcal{T}_{\mathcal{U}}(\mu)$ for all $\mu \in L^X$.

We will show that $\mathcal{T}_\beta(\mu) \leq \mathcal{T}_{\mathcal{U}}(\mu)$ for every $\mu \in L^X$, equivalently, the identity function $id_X : (X, \mathcal{T}_{\mathcal{U}}) \to (X, \mathcal{T}_\beta)$ is LF-continuous. We only show that $\psi_i \circ id_X : (X, \mathcal{T}_{\mathcal{U}}) \to (X_i, \mathcal{T}_{\mathcal{U}_i})$ is LF-continuous from Theorem 2.4(3). It is obvious from Theorems 2.15 and 2.14.

Let $\Delta(X)$ denote the family of all L-fuzzy quasi-uniformities on X. Define $\mathcal{U}^0, \mathcal{U}^1 \in \Delta(X)$ as follows

$$\mathcal{U}^0(f) = \begin{cases} 1 & \text{if } f = f_{\underline{1}}, \\ 0 & \text{otherwise} \end{cases}$$

and

$$\mathcal{U}^1(f) = 1 \ \forall f \in \Omega_X.$$

Then $\mathcal{U}^1$ ($\mathcal{U}^0$) is the finest (the coarsest) quasi-uniformity on X.

For each $\mathcal{U}_1, \mathcal{U}_2 \in \Delta(X)$, we define $\mathcal{U}_1 \vee \mathcal{U}_2$ as follows:

$\mathcal{U}_1 \vee \mathcal{U}_2$ is the coarsest quasi-unformity finer than $\mathcal{U}_1$ and $\mathcal{U}_2$.

Example 3.2 Let $L = [0, 1]$ be the unit interval with an order reversing involution $x' = 1-x$ and $\mathcal{U}_1$ and $\mathcal{U}_2 \in \Delta(X)$ as follows:

$$\mathcal{U}_1(f) = \begin{cases} 1 & \text{if } f = f_{\underline{1}}, \\ \frac{1}{2} & \text{if } f_\rho \leq f < f_{\underline{1}}, \\ 0 & \text{otherwise} \end{cases}$$

and

$$\mathcal{U}_2(f) = \begin{cases} 1 & \text{if } f = f_{\underline{1}}, \\ \frac{1}{3} & \text{if } f_\mu \leq f < f_{\underline{1}}, \\ 0 & \text{otherwise} \end{cases}$$

Let $\mathcal{U} = \mathcal{U}_1 \vee \mathcal{U}_2$ be given. From Theorem 2,15, we have

$$\mathcal{U}(f) = \begin{cases} 1 & \text{if } f = f_{\underline{1}}, \\ \frac{1}{2} & \text{if } f_\rho \leq f < f_{\underline{1}}, \\ \frac{1}{3} & \text{if } f_\rho \sqcap f_\mu \leq f < f_{\underline{1}}, f \not\geq f_\rho, \\ 0 & \text{otherwise} \end{cases}$$

where

$$f_\rho \sqcap f_\mu(\lambda) = \begin{cases} \underline{0} & \text{if } \lambda = \underline{0}, \\ \rho \vee \mu & \text{if } \underline{0} \neq \lambda \leq \rho \wedge \mu, \\ \rho & \text{if } \lambda \leq \rho,\ \lambda \not\leq \mu, \\ \mu & \text{if } \lambda \leq \mu,\ \lambda \not\leq \rho, \\ \rho \vee \mu & \text{if } \lambda \leq \rho \vee \mu,\ \lambda \not\leq \rho,\ \lambda \not\leq \mu \\ \underline{1} & \text{otherwise.} \end{cases}$$

From Theorem 2.9 and Lemma 2.7, we have the following: for every $\epsilon > 0$,

$$\mathcal{C}_{\mathcal{U}}((\underline{1} - \rho) \vee (\underline{1} - \mu),\ \frac{1}{3} - \epsilon)$$

$$= \ \wedge\{f^{-1}((\underline{1} - \rho) \vee (\underline{1} - \mu) \mid \mathcal{U}(f) > \frac{1}{3} - \epsilon\}$$

$$= (f_\rho \sqcap f_\mu)^{-1}((\underline{1}-\rho)\vee(\underline{1}-\mu))$$

$$= (f_\rho^{-1} \sqcap f_\mu^{-1})((\underline{1}-\rho)\vee(\underline{1}-\mu))$$

$$= (f_{\underline{1}-\rho} \sqcap f_{\underline{1}-\mu})((\underline{1}-\rho)\vee(\underline{1}-\mu))$$

$$= (\underline{1}-\rho)\vee(\underline{1}-\mu),$$

where $f_\rho^{-1} = f_{\rho'} = f_{\underline{1}-\rho}$ and $f_\mu^{-1} = f_{\underline{1}-\mu}$ from Lemma 2.7(7).

Hence $\mathcal{T}_{\mathcal{U}}(\rho \vee \mu) = \frac{1}{3}$ from Theorem 2.10. Similarly, we have

$$\mathcal{T}_{\mathcal{U}}(\lambda) = \begin{cases} 1 & \textit{if } \lambda = \underline{1} \text{ or } \underline{0} \\ \frac{1}{2} & \textit{if } \lambda = \rho \\ \frac{1}{3} & \textit{if } \lambda = \mu \\ \frac{1}{3} & \textit{if } \lambda = \rho\vee\mu \\ \frac{1}{3} & \textit{if } \lambda = \rho\wedge\mu \\ 0 & \textit{otherwise} \end{cases}$$

On the other hand, from Theorem 2.10, we obtain $\mathcal{T}_{\mathcal{U}_1}$ and $\mathcal{T}_{\mathcal{U}_2}$ on X as follows:

$$\mathcal{T}_{\mathcal{U}_1}(\lambda) = \begin{cases} 1 & \textit{if } \lambda = \underline{1} \text{ or } \underline{0} \\ \frac{1}{2} & \textit{if } \lambda = \rho, \\ 0 & \textit{otherwise} \end{cases}$$

and

$$\mathcal{T}_{\mathcal{U}_2}(\lambda) = \begin{cases} 1 & \textit{if } \lambda = \underline{1} \text{ or } \underline{0} \\ \frac{1}{3} & \textit{if } \lambda = \mu, \\ 0 & \textit{otherwise} \end{cases}$$

From Theorem 2.4, there exists the coarsest [0, 1]-fuzzy topology $\mathcal{T}_{\mathcal{U}_1} \vee \mathcal{T}_{\mathcal{U}_2}$ finer than $\mathcal{T}_{\mathcal{U}_1}$ and $\mathcal{T}_{\mathcal{U}_2}$ as follows:

$$(\mathcal{T}_{\mathcal{U}_1} \vee \mathcal{T}_{\mathcal{U}_2})(\lambda) = \begin{cases} 1 & \text{if } \lambda = \underline{1} \text{ or } \underline{0}, \\ \frac{1}{2} & \text{if } \lambda = \rho, \\ \frac{1}{3} & \text{if } \lambda = \mu, \\ \frac{1}{3} & \text{if } \lambda = \rho \vee \mu, \\ \frac{1}{3} & \text{if } \lambda = \rho \wedge \mu, \\ 0 & \text{otherwise.} \end{cases}$$

Hence $\mathcal{T}_{\mathcal{U}} = \mathcal{T}_{\mathcal{U}_1} \vee \mathcal{T}_{\mathcal{U}_2}$.

Theorem 3.3 *Let $\mathcal{V}$ and $\mathcal{V}^{-1}$ be L-fuzzy quasi-uniformities on X and $\mathcal{U} = \mathcal{V} \vee \mathcal{V}^{-1}$. Then $\mathcal{U}$ is the coarsest L-fuzzy uniformity on X finer than $\mathcal{V}$ and $\mathcal{V}^{-1}$.*

Proof: From Theorem 2.15, $\mathcal{U}$ is the coarsest L-fuzzy quasi-uniformity on X finer than $\mathcal{V}$ and $\mathcal{V}^{-1}$. We only show that $\mathcal{U}$ satisfies the condition (FU).

Suppose there exist $f \in \Omega_X$ such that

$$\vee\{\mathcal{U}(h) \mid h \leq f^{-1}\} \not\geq \mathcal{U}(f).$$

By the definition of $\mathcal{U}$, there exists $f_1, f_2 \in \Omega_X$ such that $f_1 \sqcap f_2 \leq f$ and

$$\vee\{\mathcal{U}(h) \mid h \leq f^{-1}\} \not\geq \mathcal{V}(f_1) \wedge \mathcal{V}^{-1}(f_2).$$

On the other hand, since $f_1 \sqcap f_2 \leq f$ implies $f_1^{-1} \sqcap f_2^{-1} \leq f^{-1}$ from Lemma 2.7, we have

$$\vee\{\mathcal{U}(h) \mid h \leq f^{-1}\} \geq \mathcal{V}(f_2^{-1}) \wedge \mathcal{V}^{-1}(f_1^{-1})$$

$$= \mathcal{V}(f_1) \wedge V^{-1}(f_2).$$

It is a contradiction. Hence $\vee\{\mathcal{U}(h) \mid h \leq f^{-1}\} \geq \mathcal{U}(f)$, for all $f \in \Omega_X$.

Theorem 3.4 *Let L be a chain, $\mathcal{V}$ and $\mathcal{V}^{-1}$ be L-fuzzy quasi-uniformities on X. Let $\mathcal{U} = V \otimes \mathcal{V}^{-1}$ on $X \times X$ be the L-fuzzy quasi-uniformity induced by projections*

$\pi_1 : X \times X \to (X, \mathcal{V})$ and $\pi_2 : X \times X \to (X, \mathcal{V}^{-1})$. Then:

$$\mathcal{T}_{\mathcal{U}} = \mathcal{T}_{\mathcal{V}} \otimes \mathcal{T}_{\mathcal{V}^{-1}},\ \mathcal{T}_{\mathcal{U}^{-1}} = \mathcal{T}_{\mathcal{V}^{-1}} \otimes \mathcal{T}_{\mathcal{V}}.$$

Proof: From Theorem 2.15, we only prove it from the followings: for each $f \in \Omega_{X\times X}$,

$$\begin{aligned}
\mathcal{U}^{-1}(f) = \mathcal{U}(f^{-1}) &= \vee\{\mathcal{V}(f_1^{-1}) \wedge \mathcal{V}^{-1}(f_2^{-1}) \mid \pi_1^{-1}(f_1^{-1}) \sqcap \pi_2^{-1}(f_2^{-1}) \leq f^{-1}\} \\
&= \vee\{\mathcal{V}(f_1^{-1}) \wedge \mathcal{V}^{-1}(f_2^{-1}) \mid \pi_1^{-1}(f_1) \sqcap \pi_2^{-1}(f_2) \leq f\} \\
&= \vee\{\mathcal{V}^{-1}(f_1) \wedge \mathcal{V}(f_2) \mid \pi_1^{-1}(f_1) \sqcap \pi_2^{-1}(f_2) \leq f\} \\
&= \mathcal{V}^{-1} \otimes \mathcal{V}(f).
\end{aligned}$$

Example 3.5 Let $L = [0; 1]$ be the unit interval with an order reversing involution $x' = 1 - x$. Let $\mathcal{V}$ be an L-fuzzy quasi-uniformity on X as follows:

$$\mathcal{V}(f) = \begin{cases} 1 & if\ f = f_{\underline{1}}, \\ \frac{1}{2} & if\ f_\rho \leq f < f_{\underline{1}}, \\ 0 & otherwise. \end{cases}$$

We obtain

$$\mathcal{V}^{-1}(f) = \begin{cases} 1 & if\ f = f_{\underline{1}}, \\ \frac{1}{2} & if\ f_\rho^{-1} \leq f < f_{\underline{1}}, \\ 0 & otherwise. \end{cases}$$

From Lemma 2.11(6) and Theorem 2.15,

$$\mathcal{V} \otimes \mathcal{V}^{-1}(f) = \begin{cases} 1 & if\ f = f_{\underline{1}}, \\ \frac{1}{2} & if\ f_{\pi_1^{-1}(\rho)} \sqcap f_{\underline{1} - \pi_2^{-1}(\rho)} \leq f < f_{\underline{1}}, \\ 0 & otherwise. \end{cases}$$

For every $\epsilon > 0$, we have the followings:

$$\mathcal{C}_{\mathcal{U}}((\underline{1}-\pi_1^{-1}(\rho))\vee\pi_2^{-1}(\rho),\tfrac{1}{2}-\epsilon)$$

$$=\bigwedge\{f^{-1}((\underline{1}-\pi_1^{-1}(\rho))\vee\pi_2^{-1}(\rho))\mid\mathcal{U}(f)>\tfrac{1}{2}-\epsilon\}$$

$$=(f_{\pi_1^{-1}(\rho)}\sqcap f_{\underline{1}-\pi_2^{-1}(\rho)})^{-1}((\underline{1}-\pi_1^{-1}(\rho))\vee\pi_2^{-1}(\rho))$$

$$=(f_{\pi_1^{-1}(\rho)})^{-1}\sqcap(f_{\underline{1}-\pi_2^{-1}(\rho)})^{-1}((\underline{1}-\pi_1^{-1}(\rho))\vee\pi_2^{-1}(\rho))$$

$$=f_{\underline{1}-\pi_1^{-1}(\rho)}\sqcap f_{\pi_2^{-1}(\rho)}((\underline{1}-\pi_1^{-1}(\rho))\vee\pi_2^{-1}(\rho))$$

$$=(\underline{1}-\pi_1^{-1}(\rho))\vee(\pi_2^{-1}(\rho))$$

and

$$\mathcal{C}_{\mathcal{U}}((\underline{1}-\pi_1^{-1}(\rho))\wedge\pi_2^{-1}(\rho),\tfrac{1}{2}-\epsilon)$$

$$=(f_{\pi_1^{-1}(\rho)}\sqcap f_{\underline{1}-\pi_2^{-1}(\rho)})^{-1}((\underline{1}-\pi_1^{-1}(\rho))\wedge\pi_2^{-1}(\rho))$$

$$=(\underline{1}-\pi_1^{-1}(\rho))\wedge(\pi_2^{-1}(\rho)).$$

Hence $\mathcal{T}_{\mathcal{V}\otimes\mathcal{V}^{-1}}\ (\pi_1^{-1}(\rho)\wedge(\underline{1}-\pi_2^{-1}(\rho))=\dfrac{1}{2}$ and $\mathcal{T}_{\mathcal{V}\otimes\mathcal{V}^{-1}}\ (\pi_1^{-1}(\rho)\vee(\underline{1}-\pi_2^{-1}(\rho))=\dfrac{1}{2}$.

Similarly, from Theorem 2.15, we have

$$\mathcal{T}_{\mathcal{V}\otimes\mathcal{V}^{-1}}(\lambda)=\begin{cases}1 & if\ \lambda=\underline{0},\lambda=\underline{1},\\ \frac{1}{2} & if\ \lambda=\underline{1}-\pi_2^{-1}(\rho),\ \lambda=\pi_1^{-1}(\rho),\\ \frac{1}{2} & if\ \lambda=(\underline{1}-\pi_2^{-1}(\rho))\wedge\pi_1^{-1}(\rho),\\ \frac{1}{2} & if\ \lambda=(\underline{1}-\pi_2^{-1}(\rho)\vee\pi_1^{-1}(\rho),\\ 0 & otherwise.\end{cases}$$

On the other hand, we have

$$\mathcal{T}_{\mathcal{V}}(\lambda) = \begin{cases} 1 & if\ \lambda = \underline{0}, \lambda = \underline{1}, \\ \frac{1}{2} & if\ \lambda = \rho, \\ 0 & otherwise \end{cases}$$

and

$$\mathcal{T}_{\mathcal{V}^{-1}}(\lambda) = \begin{cases} 1 & if\ \lambda = \underline{0}, \lambda = \underline{1}, \\ \frac{1}{2} & if\ \lambda = \underline{1} - \rho, \\ 0 & otherwise \end{cases}$$

By Theorem 2.4, we have

$$\Theta = \{\underline{1}, \pi_1^{-1}(\rho), \underline{1} - \pi_2^{-1}(\rho), \pi_1^{-1}(\rho) \wedge (\underline{1} - \pi_2^{-1}(\rho))\}$$

and

$$\mathcal{T}_{\mathcal{V}} \otimes \mathcal{T}_{\mathcal{V}^{-1}}(\lambda) = \begin{cases} 1 & if\ \lambda = \underline{0},\ \lambda = \underline{1}, \\ \frac{1}{2} & if\ \lambda = \underline{1} - \pi_2^{-1}(\rho),\ \lambda = \pi_1^{-1}(\rho), \\ \frac{1}{2} & if\ \lambda = (\underline{1} - \pi_2^{-1}(\rho)) \wedge \pi_1^{-1}(\rho) \\ \frac{1}{2} & if\ \lambda = (\underline{1} - \pi_2^{-1}(\rho)) \vee \pi_1^{-1}(\rho), \\ 0 & otherwise. \end{cases}$$

Hence $\mathcal{T}_{\mathcal{V} \otimes \mathcal{V}^{-1}} = \mathcal{T}_{\mathcal{V}} \otimes \mathcal{T}_{\mathcal{V}^{-1}}$

REFERENCES

[1] G. Artico, R. Moresco, Fuzzy Proximities and totally Bounded Fuzzy Uniformities, J. Math. Anal. Appl., **99** (1984), 320–337.

[2] G. Artico, R. Moresco, Fuzzy Proximities According with Lowen Fuzzy Uniformities, Fuzzy Sets and Systems **21** (1987), 85–98.

[3] M. H. Burton, The Relationship between a Fuzzy Uniformity and its Family of α-level uniformities, Fuzzy Sets and Systems, **54** (1993), 311–316.

[4] C. L. Chang, Fuzzy Topological Spaces, J. Math. Anal. Appl. **24** (1968), 182–190.

[5] K. C. Chattopadhyay, S. K. Samanta, Fuzzy Topology: Fuzzy Closure Operator, Fuzzy Compactness and Fuzzy Connectedness, Fuzzy Sets and Systems **54** (1993), 207–212.

[6] M.H. Ghanim, O.A. Tantawy, F.M. Selim, Gradations of Uniformity and Gradations of Proximity, Fuzzy Sets and Systems **79** (1996), 373–382.

[7] U. Höhle, Probabilistic topologies induced by L-fuzzy uniformities, Manuscripta Math. **38** (1982), 289–323.

[8] U. Höhle, Many Valued Topology and its Applications, Kluwer Academic Publisher, Boston, 2001.

[9] U. Höhle, S.E. Rodabaugh, Mathematics of Fuzzy Sets, Logic, Topology and Measure Theory, The Handbooks of Fuzzy Sets Series, Volume 3, Kluwer Academic Publishers, Dordrecht , 1999.

[10] U. Höhle, A. Šostak, Axiomatic Foundations of Fixed-Basis fuzzy topology, The Handbooks of Fuzzy Sets Series, Volume 3, Kluwer Academic Publishers, Dordrecht, 123-272, 1999.

[11] B. Hutton, Uniformities on fuzzy topological spaces, J. Math. Anal. Appl. **58** (1977), 559–571.

[12] A. K. Katsaras, Fuzzy quasi-proximities and fuzzy quasi-uniformities, Fuzzy Sets and Systems **27** (1988), 335–343.

[13] Y.C. Kim, Initial *L*-fuzzy closure spaces, Fuzzy Sets and Systems **133** (2003), 277–297.

[14] Y.C. Kim, *L*-fuzzy quasi-uniformizable spaces, Indian J. pure and appl. Math. **35** (2004), vol 5, 599–619.

[15] J.M. Ko, Y.C. Kim, Initial *L*-fuzzy quasi-uniform spaces, (press in) Int. J. Fuzzy Logic and Intelligent Systems.

[16] W. Kotzé, Uniform spaces, The Handbooks of Fuzzy Sets Series (Chapter 8), Volume 3, Kluwer Academic Publishers, Dordrecht, 1999.

[17] Liu Ying-Ming and Luo Mao-Kang , Fuzzy topology, World Scientific Publishing Co., Singapore, 1997.

[18] S. K. Samanta, Fuzzy proximities and fuzzy uniformities, Fuzzy Sets and Systems 70 (1995), 97–105.

[19] A. P. Šostak, On a fuzzy topological structure, Suppl. Rend. Circ. Matem. Palermo2 Ser II 11 (1985), 89–103.

Yong Chan Kim and Jung Mi Ko
Department of Mathematics
Kangnung, National University Gangneung
Gangwondo, 210-702, Korea