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WEAKLY θ-OPEN FUNCTIONS BETWEEN FUZZY TOPOLOGICAL SPACES

ABSTRACT: In this paper, we introduce and characterize fuzzy weakly θ-*open functions between fuzzy topological spaces as natural dual to the fuzzy weakly* θ-*continuous functions and also study these functions in relation to some other types of already known functions.*

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1. INTRODUCTION AND PRELIMINARIES

The concept of fuzzy sets was introduced by Prof. L.A. Zadeh in his classical paper [14]. After the discovery of the fuzzy subsets, much attention has been paid to generalize the basic concepts of classical topology in fuzzy setting and thus a modern theory of fuzzy topology is developed. The notion of fuzzy subsets naturally plays a very significant role in the study of fuzzy topology which was introduced by C.L. Chang [4] in 1968. In 1980, Ming and Ming [6], introduced the concepts of quasi-coincidence and q-neighbourhoods by which the extensions of functions in fuzzy setting can very interestingly and effectively be carried out. In 1985, D.A. Rose [13] defined weakly open functions in topological spaces. In 1997 J.H. Park, Y.B. Park and J.S. Park [9] introduced the notion of weakly open functions in between fuzzy topological spaces. In [12] Z. Petricevic has introduced and studied the concepts of fuzzy θ-continuous and fuzzy weakly θ-continuous functions. In this paper we introduce and discuss the concepts of fuzzy θ-open and fuzzy weakly θ-open functions and we obtain several characterizations and properties of these functions and also study these functions comparing with other types of already known functions.

Throughout this paper by (X, τ) or simply by X we mean a fuzzy topological space (fts, shorty) due to Chang [4]. A point fuzzy in X with support $x \in X$ and value p $(0 < p \leq 1)$ is denoted by x_p. Two fuzzy sets λ and β are said to be quasi-coincident (q-coincident, shorty) denoted by $\lambda q \beta$, if there exists $x \in X$ such that $\lambda(x) + \beta(x) > 1$ [6] and by $\overline{q}$ we denote "is not" q-coincident. It is known [6] that $\lambda \leq \beta$ if and only if $\lambda \overline{q}$ $(1-\beta)$. A fuzzy set λ is said to be q-neighbourhood (q-nbd) of x_p if there is a fuzzy open set μ such that $x_p q \mu$ and $\mu \leq \lambda$.

The interior, closure and the complement of a fuzzy set $\lambda \in X$ are denoted by $Int(\lambda)$, $Cl(\lambda)$ and $1 - \lambda$ respectively. For definitions and results not explained in this paper, the reader is referred to [1,4,5,7,11,13,14] assuming them to be well known.

DEFINITIONS 1.1. A fuzzy set λ in a fts X is called,

(1) Fuzzy preopen [3] if $\lambda \leq Int(Cl(\lambda))$.

(2) Fuzzy regular open [1] if $\lambda = Int(Cl(\lambda))$.

(3) Fuzzy α-open [3] if $\lambda \leq Int(Cl(Int(\lambda)))$.

(4) Fuzzy β-open [5] if $\lambda \leq Cl(Int(Cl(\lambda)))$.

DEFINITIONS 1.2. [7]. A fuzzy point x_p in a fts X is said to be a fuzzy θ-cluster point of a fuzzy set λ if and only if for every fuzzy open q-nbd μ of x_p, $Cl(\mu)$ is q-coincident with λ. The set of all fuzzy θ-cluster points of λ is called the fuzzy θ-closure of λ and is denoted by $Cl_\theta(\lambda)$. A fuzzy set λ is fuzzy θ-closed if and only if $\lambda = Cl_\theta(\lambda)$. The complement of a fuzzy θ-closed set is called of fuzzy θ-open and the θ-interior of λ denoted by $Int_\theta(\lambda)$ is defined as:

$Int_\theta(\lambda) = \{x_p :$ for some fuzzy open q-nbd β of x_p, $Cl(\beta) \leq \lambda\}$.

LEMMA 1.3. [2]. Let λ be a fuzzy set in a fts X, then:

(1) λ is a fuzzy θ-open if and only if $\lambda = Int_\theta(\lambda)$.

(2) $1 - Int_\theta(\lambda) = Cl_\theta(1 - \lambda)$ and $Int_\theta(1 - \lambda) = 1 - Cl_\theta(\lambda)$.

(3) $Cl_\theta(\lambda)$ (resp. $Int_\theta(\lambda)$) is a fuzzy closed set (resp. fuzzy open set) but not necessarily is a fuzzy θ-closed set (resp. fuzzy θ-open set).

RESULT. 1.4. (i) It is easy to see that $Cl(\lambda) \leq Cl_\theta(\lambda)$ and $Int_\theta(\lambda) \leq Int(\lambda)$ for any fuzzy set λ in a fts X:

(ii) For a fuzzy open (resp. fuzzy closed) set λ in a fts X, $Cl(\lambda) = Cl_\theta(\lambda)$ (resp. $Int_\theta(\lambda) = Int(\lambda)$).

DEFINITION 1.5. Let $f : (X, \tau) \to (Y, \sigma)$ be a function from a fts (X, τ) into a fts (Y, σ). The function f is called:

(i) fuzzy weakly open [10] if $f(\lambda) \leq Int\,(f(Cl(\lambda)))$ for each fuzzy open set λ in X.

(ii) fuzzy almost open (written as f.a.o.N) [8] if $f(\lambda)$ is a fuzzy open set of Y for each fuzzy regular open set λ in X.

(iii) fuzzy β-open [5] if $f(\lambda)$ is a fuzzy β-open set of Y for each fuzzy open set λ of X.

(iv) fuzzy θ-continuous [12] (resp. fuzzy weakly θ-continuous [12]) if for each fuzzy point x_p and each open nbd λ of of $f(x_p)$, there is a fuzzy open nbd μ of x_p such that $f(Cl(\mu)) \leq Cl(\lambda)$ (resp. $f(Int(Cl(\mu))) \leq Cl(\lambda)$).

2. FUZZY WEAKLY θ-OPEN FUNCTIONS

Since fuzzy θ-continuity [12] is dual to fuzzy θ-openness (It might be new one), we define in this paper the concept of fuzzy weak θ-openness as natural dual to the fuzzy weak θ-continuity [12].

DEFINITION 2.1. A function $f : (X, \tau_1) \to (Y, \tau_2)$ is said to be fuzzy weakly θ-open if $f(\lambda) \leq Int_\theta(f(Cl(\lambda)))$ for each fuzzy open set λ of X.

DEFINITION 2.2. A function $f : (X, \tau_1) \to (Y, \tau_2)$ is said to be fuzzy θ-open if $f(\lambda)$ is a fuzzy θ-open set of Y for each fuzzy open set λ of X:

REMARK. 2.3. Clearly, every fuzzy weakly θ-open function is fuzzy weakly open and every fuzzy θ-open function is fuzzy weakly θ-open.

EXAMPLE 2.4. Let $X = \{a, b\}$ and $Y = \{x, y\}$. Fuzzy sets A, B, E and H be defined as:

A(a) = 0.2 , A(b) = 0.3;

B(a) = 0.8 , B(b) = 0.9;

E(x) = 0.5 , E(y) = 0.7;

H(x) = 0.4 , H(y) = 0.3.

Let $\tau = \{0, A, 1_X\}$, $\sigma = \{0, B, 1_X\}$ and $\gamma = \{0, E, H, 1_Y\}$. Then the mapping $f : (X, \tau) \to (Y, \gamma)$ defined by $f(a) = x$ and $f(b) = y$ is fuzzy weakly θ-open which is not fuzzy θ-open and the mapping $g : (X, \sigma) \to (Y, \gamma)$ defined by $g(a) = x$ and $g(b) = y$ is fuzzy weakly open but not fuzzy weakly θ-open.

THEOREM 2.5. For a function $f : (X, \tau_1) \to (Y, \tau_2)$, the following conditions are equivalent :

(i) f is fuzzy weakly θ-open,

(ii) $f(Int_\theta(\lambda)) \leq Int_\theta(f(\lambda))$ for every fuzzy subset λ of X,

(iii) $Int_\theta(f^{-1}(\beta)) \leq f^{-1}(Int_\theta(\beta))$ for every fuzzy subset β of Y,

(iv) $f^{-1}(Cl_\theta(\beta)) \leq Cl_\theta(f^{-1}(\beta))$ for every fuzzy subset β of Y.

PROOF. $(i) \to (ii)$: Let λ be any fuzzy subset of X and x_p a fuzzy point in $Int_\theta(\lambda)$. Then, there exists a fuzzy open q-nbd γ of x_p such that $\gamma \leq Cl(\gamma) \leq \lambda$. Then, $f(\gamma) \leq f(Cl(\gamma)) \leq f(\lambda)$. Since f is fuzzy weakly θ-open, $f(\gamma) \leq Int_\theta(f(Cl(\gamma))) \leq Int_\theta(f(\lambda))$. It implies that $f(x_p)$ is a point in $Int_\theta(f(\lambda))$. This shows that $x_p \in f^{-1}(Int_\theta(f(\lambda)))$. Thus $Int_\theta(\lambda) \leq f^{-1}(Int_\theta(f(\lambda)))$, and so $f(Int_\theta(\lambda)) \leq Int_\theta(f(\lambda))$.

$(ii) \leq (i)$: Let μ be a fuzzy open set in X. As $\mu \leq Int_\theta(Cl(\mu))$ implies, $f(\mu) \leq f(Int_\theta(Cl(^1))) \leq Int_\theta(f(Cl(\mu)))$. Hence f is fuzzy weakly θ-open.

$(ii) \to (iii)$: Let β be any fuzzy subset of Y. Then by (ii), $f(Int_\theta(f^{-1}(\beta))) \leq Int_\theta(\beta)$. Therefore $Int_\theta(f^{-1}(\beta)) \leq f^{-1}(Int_\theta(\beta))$.

$(iii) \to (ii)$: This is obvious.

$(iii) \to (iv)$: Let β be any fuzzy subset of Y. Using (iii), we have

$$1 - Cl_\theta(f^{-1}(\beta)) = Int_\theta(1 - f^{-1}(\beta)) = Int_\theta(f^{-1}(1 - \beta)) \leq f^{-1}(Int_\theta(1 - \beta))$$

$= f^{-1}(1 - Cl_\theta(\beta)) = 1 - (f^{-1}(Cl_\theta(\beta))$. Therefore, we obtain $f^{-1}(Cl_\theta(\beta)) \leq Cl_\theta(f^{-1}(\beta))$.

$(iv) \to (iii)$: Similarly we obtain, $1 - f^{-1}(Int_\theta(\beta)) \le 1 - Int_\theta(f^{-1}(\beta))$, for every fuzzy subset β of Y, i.e., $Int_\theta(f^{-1}(\beta)) \le f^{-1}(Int_\theta(\beta))$.

THEOREM 2.6. If X is a fuzzy regular space, then for a function $f : (X, \tau_1) \to (Y, \tau_2)$, the following conditions are equivalent:

(i) f is fuzzy weakly θ-open,

(ii) For each fuzzy θ-open set λ in X, $f(\lambda)$ is fuzzy θ-open in Y,

(iii) For any fuzzy set β of Y and any fuzzy θ-closed set λ in X containing $f^{-1}(\beta)$, there exists a fuzzy θ-closed set δ in Y containing β such that $f^{-1}(\delta) \le \lambda$.

PROOF. $(i) \to (ii)$: Let λ be a fuzzy θ-open set in X. Then $1 - f(\lambda)$ is a fuzzy set in Y and by (i) and Theorem 2.5 (iv), $f^{-1}(Cl_\theta(1 - f(\lambda))) \le Cl_\theta(f^{-1}(1 - f(\lambda)))$. Therefore, $1 - f^{-1}(Int_\theta(f(\lambda))) \le Cl_\theta(1 - \lambda) = 1$. Then, we have $\lambda \le f^{-1}(Int_\theta(f(\lambda)))$ which implies $f(\lambda) \le Int_\theta(f(\lambda))$. Hence $f(\lambda)$ is fuzzy θ-open in Y.

$(ii) \to (iii)$: Let β be any fuzzy set in Y and λ be a fuzzy θ-closed set in X such that $f^{-1}(\beta) \le \lambda$. Since $1 - \lambda$ is fuzzy θ-open in X, by (ii), $f(1- \lambda)$ is fuzzy θ-open in Y. Let $\delta = 1 - f(1 - \lambda)$. Then δ is fuzzy θ-closed and $\beta \le \delta$. Now, $f^{-1}(\delta) = f^{-1}(1 - f(1 - \lambda)) = 1 - f^{-1}(f(\lambda)) \le \lambda$.

$(iii) \to (i)$: Let β be any fuzzy set in Y. Then by Corollary 3.6 of [7] $\lambda = Cl_\theta(f^{-1}(\beta))$ is fuzzy θ-closed set in X and $f^{-1}(\beta) \le \lambda$. Then there exists a fuzzy θ-closed set δ in Y containing β such that $f^{-1}(\delta) \le \lambda$. Since δ is fuzzy θ-closed $f^{-1}(Cl_\theta(\beta)) \le f^{-1}(\delta) \le Cl_\theta(f^{-1}(\beta))$. Therefore by Theorem 2.5, f is a weakly θ-open function.

Furthermore, we can prove the following,

THEOREM 2.7. If $f : (X, \tau_1) \to (Y, \tau_2)$ is fuzzy weakly θ-open, then for each x_p fuzzy point in X and each fuzzy open set μ of X containing x_p, there exists a fuzzy open set δ in Y containing $f(x_p)$ such that $\delta \le f(Cl(\mu))$.

PROOF. Let $x_p \in X$ and μ be a fuzzy open set in X containing x_p. Since f is fuzzy weakly θ-open. $f(\mu) \le Int_\theta(f(Cl(\mu)))$. Let $\delta = Int_\theta(f(Cl(\mu)))$. Hence $\delta \le f(Cl(\mu))$, with δ containing $f(x_p)$.

The reverse in the theorem above is true if f is a fuzzy closed function.

COROLLARY 2.8. Let $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ be a closed function. Then the statement following are equivalent:

(i) f is fuzzy weakly θ-open,

(ii) For each x_p fuzzy point in X and each fuzzy open set μ of X containing x_p, there exists a fuzzy open set δ containing $f(x_p)$ such that $\delta \leq f(Cl(\mu))$.

PROOF. $(i) \rightarrow (ii)$: Theorem 2.7.

$(ii) \rightarrow (i)$: Let μ be a fuzzy open set in X and let $y_p \in f(\mu)$. It follows from (ii) $\delta \leq f(Cl(\mu))$ for some δ fuzzy open in Y containing y_p. Hence as f is a closed function we have, $y_p \in \delta \leq Int_\theta(f(Cl(\mu)))$ by Result 1.4(ii) above. This shows that $f(\mu) \leq Int_\theta(f(Cl(\mu)))$, i.e., f is a fuzzy weakly θ-open function.

THEOREM 2.9. Let $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ be a bijective function. Then the following statements are equivalent:

(i) f is fuzzy weakly θ-open,

(ii) $Cl_\theta(f(\lambda)) \leq f(Cl(\lambda))$ for each λ fuzzy open of X,

(iii) $Cl_\theta(f(Int(\beta)) \leq f(\beta)$ for each β fuzzy closed of X.

PROOF. $(i) \rightarrow (iii)$: Let β be a fuzzy closed set in X. Then we have $f(1-\beta) = 1-f(\beta) \leq Int_\theta(f(Cl(1-\beta)))$ and so $1-f(\beta) \leq 1-Cl_\theta(f(Int(\beta))$. Hence $Cl_\theta(f(Int(\beta))) \leq f(\beta)$.

$(iii) \rightarrow (ii)$: Let λ be a fuzzy open set in X. Since $Cl(\lambda)$ is a fuzzy closed set and $\lambda \leq Int(Cl(\lambda))$ by (iii) we have $Cl_\theta(f(\lambda)) \leq Cl_\theta(f(Int(Cl(\lambda))) \leq f(Cl(\lambda))$.

$(ii) \rightarrow (iii)$: Similar to $(iii) \rightarrow (ii)$.

$(iii) \rightarrow (i)$: Clear.

The following theorem the proof is mostly straightforward and is omitted.

THEOREM 2.10. For a function $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ the following conditions are equivalent:

(i) f is fuzzy weakly θ-open,

(ii) For each fuzzy closed subset β of X, $f(Int(\beta)) \leq Int_\theta(f(\beta))$,

(iii) For each fuzzy open subset λ of X, $f(Int(Cl(\lambda))) \leq Int_\theta(f(Cl(\lambda)))$,

(iv) For every fuzzy preopen subset λ of X, $f(\lambda) \leq Int_{\theta}(f(Cl(\lambda)))$,

(v) For every fuzzy α-open subset λ of X, $f(\lambda) \leq Int_{\theta}(f(Cl(\lambda)))$.

Now, we give a fuzzy strong definition of continuity define that when combined with fuzzy weak θ-openness imply fuzzy θ-openness.

DEFINITION 2.11. A function $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ is said to be fuzzy strongly continuous if for every fuzzy subset λ of X, $f(Cl(\lambda)) \leq f(\lambda)$.

LEMMA 2.12. If $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ is fuzzy strongly continuous, then $Int_{\theta}(f(Cl(\lambda))) \leq f(\lambda)$ but the converse does not hold as is shown by the following example.

EXAMPLE 2.13. Let $X = \{a, b, c\}$ and $Y = \{x, y, z\}$. Fuzzy sets A and B be defined as :

$$A(a) = 0 , A(b) = 0.2 , A(c) = 0.8;$$

$$B(x) = 0 , B(y) = 0.7 , B(z) = 0.4.$$

Let $\tau = \{0, A, 1_X\}$ and $\sigma = \{0, B, 1_Y\}$. Then the mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ and $f(c) = y$ satisfies the condition $Int_{\theta}(f(Cl(\lambda))) \leq f(\lambda)$ but not fuzzy strongly continuous.

THEOREM 2.14. If $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ is fuzzy weakly θ-open and fuzzy strongly continuous, then f is fuzzy θ-open.

PROOF. Let λ be an fuzzy open subset of X. Since f is fuzzy weakly θ-open $f(\lambda) \leq Int_{\theta}(f(Cl(\lambda)))$. However, because f is fuzzy strongly continuous, $f(\lambda) \leq Int_{\theta}(f(\lambda))$ and therefore $f(\lambda)$ is fuzzy θ-open.

The following example shows that neither of this fuzzy strongly continuity yield a decomposition of fuzzy θ-openness.

EXAMPLE 2.15. Let $X = \{a, b\}$ and $Y = \{x, y\}$. Fuzzy sets A and B defined as:

$$A(a) = 0.4 , A(b) = 0.8;$$
$$B(x) = 0.4 , B(y) = 0.3 .$$

Let $\tau = \{0, A, 1_X\}$ and $\sigma = \{0, B, 1_X\}$. Then the mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a) = x$ and $f(b) = y$ satisfies fuzzy θ-openness but not fuzzy strongly continuity.

A function $f : (X, \tau_1) \to (Y, \tau_2)$ is said to be fuzzy contra θ-closed if $f(\lambda)$ is a fuzzy θ-open set of Y, for each fuzzy closed set λ in X.

THEOREM 2.16. If $f : (X, \tau_1) \to (Y, \tau_2)$ is fuzzy contra θ-closed, then f is a fuzzy weakly θ-open function.

PROOF. Let λ be an fuzzy open subset of X. Then, we have $f(\lambda) \leq f(Cl(\lambda))) = Int_\theta(f(Cl(\lambda)))$.

The converse of Theorem 2.16 does not hold.

EXAMPLE. 2.17. Let $X = \{a, b, c\}$ and $Y = \{x, y, z\}$.

Define fuzzy sets A, B and H as :

$$A(a) = A(b) = 1 \;,\; A(c)= 0;$$

$$B(a) = 0 \;,\; B(b) = B(c) = 1;$$

$$H(a) = 1 \;,\; H(b) = H(c) = 0.$$

Let $\tau = \{0, A, 1_X\}$ and $\sigma = \{0, B, H, 1_X\}$. Then the mapping $f : (X, \tau) \to (X, \sigma)$ defined as : $f(a) = f(c) = c$ and $f(b) = b$ is fuzzy weakly θ-open but not fuzzy contra θ-closed.

THEOREM 2.18. Let X be a fuzzy regular space. Then $f : (X, \tau_1) \to (Y, \tau_2)$ is fuzzy weakly θ-open if and only if f is fuzzy θ-open.

PROOF. The sufficiency is clear. Necessity. Let λ be a non-null fuzzy open subset of X. For each x_p fuzzy point in λ, let μ_{x_p} be an fuzzy open set such that $x_p \in \mu_{x_p} \leq Cl{=}(\mu_{x_p}) \leq \lambda$. Hence we obtain that $\lambda = \cup\{\mu_{x_p} : x_p \in \lambda\} = \cup\{Cl(\mu_{x_p}) : x_p \in \lambda\}$ and, $f(\lambda) = \cup\{f(\mu_{x_p}) : x_p \in \lambda\} \leq \cup\{Int_\theta(f(Cl(\mu_{x_p}))) : x_p \in \lambda\} \leq Int_\theta(f(\cup\{Cl(\mu_{x_p}) : x_p \in \lambda\}) = Int_\theta(f(\lambda))$. Thus f is fuzzy θ-open.

THEOREM 2.19. If $f : (X, \tau_1) \to (Y, \tau_2)$ is a f.a.o.N function and a fuzzy closed function, then it is a fuzzy weakly θ-open function.

PROOF. Let λ be a fuzzy open set in X. Since f is f.a.o.N and $Int(Cl(\lambda))$ is fuzzy regular open, $f(Int(Cl(\lambda)))$ is fuzzy open in Y and hence $f(\lambda) \leq f(Int(Cl(\lambda)) \leq Int(f(Cl(\lambda))) = Int_\theta(f(Cl(\lambda)))$. This shows that f is fuzzy weakly θ-open.

It is obvious that converse of Theorem 2.19 is not true in general.

LEMMA 2.20 [6]. If $f : (X, \tau_1) \to (Y, \tau_2)$ is a fuzzy continuous function, then for any fuzzy subset λ of X, $f(Cl(\lambda)) \leq Cl(f(\lambda))$.

THEOREM 2.21. If $f : (X, \tau_1) \to (Y, \tau_2)$ is a fuzzy weakly θ-open and fuzzy continuous function, then f is a fuzzy β-open function.

PROOF. Let λ be a fuzzy open set in X. Then by fuzzy weak θ-openness of f, $f(\lambda) \leq Int_\theta(f(Cl(\lambda)))$. Since f is fuzzy continuous $f(Cl(\lambda)) \leq Cl(f(\lambda))$. Hence we obtain that, $f(\lambda) \leq Int_\theta(f(Cl(\lambda))) \leq Int_\theta(Cl(f(\lambda))) \leq Cl(Int(Cl(f(\lambda))))$. Therefore, $f(\lambda) \leq Cl(Int(Cl(f(\lambda)))$ which shows that $f(\lambda$ is a fuzzy β-open set in Y. Thus f is a fuzzy β-open function.

Since every fuzzy strongly continuous function is fuzzy continuous we have the following corollary.

COROLLARY 2.22. If $f : (X, \tau_1) \to (Y, \tau_2)$ is a fuzzy weakly θ-open and fuzzy strongly continuous function. Then f is a fuzzy β-open function.

Recall that, two non-empty fuzzy sets λ and β in a fuzzy topological spaces X (i.e., neither λ nor β is 0_X) are said to be fuzzy θ-separated [9] if $\lambda \overline{q} Cl_\theta(\beta)$ and $\beta \overline{q} C_\theta l(\lambda)$ or equivalently if there exist two fuzzy θ-open sets μ and ν such that $\lambda \leq \mu$, $\beta \leq \nu$, $\lambda \overline{q} \nu$ and $\beta \overline{q} \mu$.

A fuzzy topological space X which can not be expressed as the union of two fuzzy θ-separated sets is said to be a fuzzy θ-connected space [10].

THEOREM 2.23. If $f : (X, \tau_1) \to (Y, \tau_2)$ is a fuzzy weakly θ-open from a space X onto a fuzzy θ-connected space Y; then X is fuzzy connected.

PROOF. If possible, let X be not connected. Then there exist fuzzy separated sets β and γ in X such that $X = \beta \cup \gamma$. Since β and γ are fuzzy separated, there exist two fuzzy open sets μ and ν such that $\beta \leq \mu$, $\gamma \leq \nu$, $\beta \overline{q} \nu$ and $\gamma \overline{q} \mu$. Hence we have $f(\beta) \leq f(\mu)$, $f(\gamma) \leq f(\nu)$, $f(\beta)\, \overline{q}\, f(\nu)$ and $f(\gamma)\, \overline{q}\, f(\mu)$. Since f is fuzzy weakly θ-open, we have $f(\mu) \leq Int_\theta(f(Cl(\mu)))$ and $f(\nu) \leq Int_\theta(f(Cl(\nu)))$ and since μ and ν are fuzzy open and also fuzzy closed, we have $f(Cl(\mu)) = f(\mu)$, $f(Cl(\nu)) = f(\nu)$. Hence $f(\mu)$ and $f(\nu)$ are fuzzy θ-open in Y. Therefore, $f(\beta)$ and $f(\gamma)$ are fuzzy θ-separated sets in Y and Y

$= f(X) = f(\beta \cup \gamma) = f(\beta) \cup f(\gamma)$. Hence this contrary to the fact that Y is fuzzy θ-connected. Thus X is fuzzy connected.

DEFINITION 2.24. A space X is said to be fuzzy hyperconnected if every non-empty fuzzy open subset of X is fuzzy dense in X.

THEOREM 2.25. If X is a fuzzy hyperconnected space, then a function $f : (X, \tau_1) \to (Y, \tau_2)$ is fuzzy weakly θ-open if and only if $f(X)$ is fuzzy θ-open in Y.

PROOF. The su±ciency is clear. For the necessity observe that for any fuzzy open subset λ of X, $f(\lambda) \leq f(X) = Int_\theta(f(X)) = Int_\theta(f(Cl(\lambda)))$.

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