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COMMON FIXED POINT THEOREMS FOR NON-COMPATIBLE MAPPINGS AND MEIR-KEELER TYPE CONTRACTIVE CONDITION IN FUZZY METRIC SPACES

***ABSTRACT**: We define a new commutativity condition in fuzzy metric spaces. We also define an analogue of the (ϵ, δ) contractive condition in the settings of fuzzy metric spaces. As applications we prove some fixed point theorems in fuzzy metric spaces. We also show that completeness of the whole space can be replaced by a weaker condition and continuity of any mapping is not necessary for the existence of a common fixed point.*

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1. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [33] in 1965. Deng [3], Erceg [4], Kaleva and Seikkala [12], Kramosil and Michalek [14], George and Veeramani [6] have introduced the concept of fuzzy metric spaces in different ways. Many authors have studied the fixed point theory in these fuzzy metric spaces ([1], [2], [5], [6], [8], [9], [15], [25]-[32]).

Grabiec [7] followed Kramosil and Michalek [14] and obtained fuzzy version of Banach's fixed point theorem. In 1976, Jungck [10] established common fixed point theorem for commuting maps generalizing the Banach's fixed point theorem. Sessa [23] defined a generalization of commutativity. Further Jungck [11] introduced more generalized commutativity so-called compatibility. Mishra et. al [15] introduced the concept of compatibility in fuzzy metric spaces.

Most of the fixed point theorems existing in the mathematical literature deal with compatible and continuous mappings so it would be natural question: what about the mappings which are not compatible and continuous?

Banach fixed point theorem has many application but suffers from one draw back, the definition requires continuity of the function. It has been known from the paper of Kanan [13] that there exists maps that have a discontinuity in the domain but which has a fixed point. Moreover the maps involved in every case were continuous at the fixed point.

These observations motivated several authors of the field to prove fixed point theorems for non-compatible, discontinuous mappings. Pant [17]-[20] initiated the study of non-compatible maps and introduced pointwise R- weak commutativity of mappings in [17]. He also showed that pointwise R-weak commutativity is a necessary, hence minimal condition for the existence of a common fixed point of contractive type maps [17].

Vasuki [32] defined R-weak commutativity in fuzzy metric spaces. Pathak, Cho and Kang [21] introduced the concept of R-weakly commuting mappings of type (A) in metric spaces and showed that this type of mappings are non-compatible. He also showed that R-weakly commuting mappings are not necesssarily R-weakly commuting of type (A).

In this paper, we introduce the concept of R-weak commuting mappings of type (A) in the setting of fuzzy metric spaces and define (DS)-weak commutativity in fuzzy metric spaces. As an application of this concept we prove a common fixed point theorem [Theorem 1] for two mappings in fuzzy metric spaces. In our Theorem 1, we show that completeness of the whole space can be replaced by a weaker condition. We also show that continuity of any mapping is not necessary for existence of common fixed point.

In fixed point theory, the class of (ϵ, δ) contraction maps is much wider than the class of Banach contraction and the class of ϕ – contractions. Meir-Keeler [16], Pant [17], [20], Pathak, Cho and Kang [21] proved fixed point theorems for (ϵ, δ) contractions in metric spaces. However, so far (ϵ, δ) contraction maps have not been used in fuzzy metric spaces to prove fixed point theorems.

In this paper, we introduce an (ϵ, δ) contractive condition in the settings of fuzzy metric spaces and we apply it with our newly defined concept of (DS)-weak commutativity to prove fixed point theorems in fuzzy metric spaces. We also show that continuity of any mapping and completeness of the whole space are not necessary to prove fixed point theorem for (ϵ, δ) contractive condition. We improve, extend and generalize the results of Meir-Keeler [16] and Pant [20]. We improve and generalize the result of Pant [17] and Pathak Cho and Kang [21].

2. PRELIMINARIES

Definition 1. [22] A binary operation $* : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous t-norm if $\{[0, 1], *\}$ is an abelian topological monoid with unit 1 such that $a*b \leq c*d$ whenever $a \leq c$ and $b \leq d$, a, b, c, d $\in [0, 1]$.

Examples of t-norm are $a*b = \min\{a, b\}$.

Definition 2. [6] The 3-tuple (X, M, ∗) is called a fuzzy metric space if X is an arbitrary set, ∗ is a continuous t-norm and M is a fuzzy set in X^2 *x* $[0, \infty)$ satisfying the following conditions for all x, y, z $\in$ X and t, s > 0,

(i) M(x, y, 0) 0,

(ii) M(x, y, t) = 1 for all t > 0 if and only if x = y,

(iii)M(x, y, t) = M(y, x, t),

(iv)M(x, y, t) ∗ M(y, z, s) ≤ M(x, z, t + s),

(v) M(x, y, •)): $[0, \infty]$ [0, 1] is continuous.

In this paper (X, M, ∗) will denote a fuzzy metric space in the sense of the above definition with the following condition:

(vi) $\lim_{t\to\infty} M(x, y, t) = 1$ for all x, y in X.

Note that M(x, y, t) can be thought as the degree of nearness between x and y with respect to t. We identify x = y with M(x, y, t) =1 for all t > 0 and M(x, y, t) = 0 with ∞ and we can find some topological properties and examples of fuzzy metric spaces in [6].

In the following example, we know that every metric induces a fuzzy metric.

Example 1: [6] Let (X, d) be a metric space. Define a∗b = ab or a∗b = min{a, b} and for all x, y $\in$ X, t > 0,

$$M(x, y, t) = \frac{t}{t + d(x, y)} \tag{i}$$

Then (X, M, ∗) is a fuzzy metric space. We call this fuzzy metric M induced by the metric d the standard fuzzy metric.

Definition 3. [7] A sequence $\{x_n\}$ in a fuzzy metric space (X, M, ∗) is called Cauchy if $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ for every $t > 0$ and each $p > 0$. (X, M, ∗) is complete if every Cauchy sequence in X converges in X. A sequence $\{x_n\}$ in X converges to $x \in X$ if $\lim_{n\to\infty} M(x_n, x, t) = 1$ for every $t > 0$.

Definition 4. [6] A sequence $\{x_n\}$ in a fuzzy metric space (X, M, ∗) is a Cauchy sequence if for each $\in > 0$, $t > 0$, there exists $n_0 \in N$ such that $M(x_m, x_n, t) > 1 - \in$ for all $m, n \geq n_0$, where N is the set of natural numbers.

Song [24] defined the Cauchy sequence in the following manner.

Definition 5. [24] A sequence $\{x_n\}$ in a fuzzy metric space (X, M, ∗) is defined to be a Cauchy sequence if $M(x_{n+p}, x_n, t) \to 1$ (for all $t > 0$) as $n \to \infty$ uniform on $p \in N$, N being the set of natural numbers.

If t- norm satisfies the condition $a*b = \min\{a, b\}$ or $a*a \geq a$ for all $a, b \in [0, 1]$ then Song [24] has shown that a Cauchy sequence in the sense of Grabiec [7] or George and Veerami [6] is a Cauchy sequence in the sense of Song [24].

Definition 6. [15] Let (X, M, ∗) be a fuzzy metric space and let f, g be self mappings of X. The mappings f and g are said to be compatible if

$$\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1,$$

for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\alpha} gx_n = z \text{ for some } z \in X.$$

Definition 7. [32] Let (X, M, ∗) be a fuzzy metric space and let f, g be self mappings of X. The mappings f and g are said to be R-weakly commuting if there exists a positive real number R such that

$$M(fgx, gfx, t) \geq M(fx, gx, \frac{t}{R}) \text{ for all x in X.}$$

Definition 8. [21] Let (X, d) be a metric space and let f, g be self-mappings of X. The mappings f and g are said to be R-weakly commuting of type (Af) if there exists a positive real number R such that

$$d(fgx, ggx) \le Rd(fx, gx) \quad \text{for all } x \in X.$$

Definition 9. [21] Let (X, d) be a metric space and let f, g be self mappings of X. The mappings f and g are said to be R-weakly commuting of type (A_g) if there exists a positive real number R such that

$$d(gfx, ffx) \le Rd(fx, gx) \quad \text{for all } x \in X.$$

Remark 1. [21] R-weakly commuting mappings are not necessarily R-weakly commuting of type (A_f) or R-weakly commuting of type (A_g).

Definition 10. Let (X, M, ∗) be a fuzzy metric space and let f, g be self-mappings of X. The mappings f and g are said to be (DS_f)-weakly commuting at x ∈ X, if there exists a positive real number R such that

$$M(fgx, ggx, t) \ge M(fx, gx, \frac{t}{R})$$

Here f and g are (DS_f)-weakly commuting on X if the above inequality holds for all x ∈ X.

Definition 11. Let (X, M, ∗) be a fuzzy metric space and let f, g be self mappings of X. The mappings f and g are said to be (DS_g)-weakly commuting at x ∈ X if there exists a positive real number R such that

$$M(gfx, ffx, t) \ge M(fx, gx, \frac{t}{R})$$

Here f and g are (DS_g)-weakly commuting on X if the above inequality holds for all x ∈ X.

Example 2. Let X = [0, 2] with the metric d defined by d(x, y) = |x - y|. For each $t \in (0, \infty)$ define

$$M(x, y, t) = \frac{t}{t + d(x, y)} x, y \in X,$$

$$M(x, y, 0) = 0 \; x, y \in X.$$

Clearly M(x, y, ∗) is a fuzzy metric space on X where $_*$ is defined by $a_* b = ab$ or $a*b = \min\{a, b\}$.

Define f, g: X → X by

$fx = x$ if $x \in [0, \frac{1}{4})$,

$fx = \frac{1}{4}$ if $x \geq \frac{1}{4}$,

$gx = \frac{x}{1+x}$ for all $x \in [0, 2]$.

Consider the sequence $\{x_n = \frac{1}{3} + \frac{1}{n} : n \geq 1\}$ in X.

Then $\lim_{n\to\infty} fx_n = \frac{1}{4}$, $\lim_{n\to\infty} gx_n = \frac{1}{4}$ but $\lim_{n\to\infty} (fgx_n, gfx_n, t) = \frac{t}{t + |\frac{1}{4} - \frac{1}{5}|} \neq 1$.

Thus f and g are noncompatible. If we take t = 1 and $x = \frac{1}{5}$ then M(fgx, ggx, t)

$= M(fg\left(\frac{1}{5}\right), gg\left(\frac{1}{5}\right), t) = \frac{1}{1 + |\frac{1}{6} - \frac{1}{7}|} = \frac{42}{43}$ and $M(fx, gx, \frac{1}{R}) = \frac{30}{30+R}$. For $R \geq \frac{5}{7}$,

f and g are (DS_f)- weakly commuting at $x = \frac{1}{5}$.

Example 3: Let X = [1, 10] with the metric d defined by d(x, y) = |x - y|. For each $t \in (0, \infty)$ define

$M(x, y, t) = \frac{t}{t + d(x, y)} \; x, y \in X,$

$M(x, y, 0) = 0$ $x, y \in X$.

Clearly $M(x, y, *)$ is a fuzzy metric space on X where $*$ is defined by $a_* b = ab$ or $a*b = \min\{a, b\}$.

Define f, g: X – X by

$fx = x$ if $1 \le x \le 5$,

$fx = \dfrac{x+3}{4}$ if $x > 5$,

$gx = 2$ if $1 \le t \le 5$,

$gx = \dfrac{x+1}{3}$ if $x > 5$.

Consider the sequence $\{x_n = 5 + \frac{1}{n}, n \ge 1\}$ in X

Then $\lim_{n\to\infty} fx_n = 2$, $\lim_{n\to\infty} gx_n = 2$ but

$$\lim_{n\to\infty} M(fgx_n, gfx_n, t) = \lim_{n\to\infty} \frac{t}{t + |2 - \frac{1}{3t}} = 1.$$

Then f and g are compatible. If we take $t = 1$, then it is easy to see that for $R \ge 4$, f and g are (DS_f)-weakly commuting at $x = 8$.

3. MAIN RESULTS

Theorem 1: Let $(X, M, *)$ be a fuzzy metric space with $t * t \ge t$ for all $t \in [0, 1]$ and let f, g be mappings from X into itself such that f and g are (DS_f)-weakly commuting mappings or (DS_g)-weakly commuating mappings at coincidence points and satisfying the following condition:

$$M(fx, fy, t) \ge r(M(gx, gy, t)) \tag{1.1}$$

for all $x, y \in X$, where $r : [0, 1] \to [0, 1]$ is a continuous function such that $r(t) > t$ for each $0 < t < 1$.

If $f(X) \subseteq g(X)$ and if either $f(X)$ or $g(X)$ is a complete subspace of X, then f and g have a unique common fixed point.

Proof. Let x_0 be an arbitrary point in X. Since $f(X) \subseteq g(X)$ choose a point x_1 in X such that $fx_0 = gx_1$. In general having chosen x_n, we can choose x_{n+1} such that $fx_n = gx_{n+1}$, for n = 0, 1, 2,... Then for t > 0.

$$\begin{aligned} M(fx_n, fx_{n+1}, t) &\geq r(M(gx_n, gx_{n+1}, t) \\ &= r(M(fx_{n-1}, fx_n, t)) \\ &\geq M(fx_{n-1}, fx_{n,t}) \text{ since } r(t) > t \text{ for } 0 < t < 1. \end{aligned} \tag{1.2}$$

Thus $\{M(fx_n, fx_{n+1}, t), n \geq 0\}$ is an increasing sequence of positive real numbers in [0, 1] and, therefore, tends to a limit $L \leq 1$. We claim that L = 1. For if L < 1 on making $n \to \infty$ in (1.2), we get $L \geq r(L) > L$, a contradiction. Hence L = 1. Now for any positive integer p,

$$M(fx_n, fx_{n+p}, t \geq M(fx_n, fx_{n+1}, \frac{t}{p}) * \ldots * M(fx_{n+p-1}\ (x_{n-p}, \frac{t}{p}).$$

Since by the above argument $\lim_{n\to\infty}$. $M(fx_n, fx_{n+1}, t) = 1$ for t > 0, it follows that

$$\lim_{n\to\infty} M(fx_n, fx_{n+p,}\ t) \geq 1 * 1 * \ldots * 1 \geq 1.$$

Thus $\{fx_n\} = \{gx_{n+1}\}$ is Cauchy sequence. Now suppose g(X) is complete. Note that the sequence $\{gx_{n+1}\}$ is contained in g(X) and has a limit in g(X) call it z. Let $u \in g^{-1}z$. Then gu = z. By (1.1), we have

$$M(fx_n, fu, t) \geq r(M(gx_n, gu, t)).$$

Proceeding limit as $n \to \infty$, we have

$M(z, fu, t) \geq 1$ since r(t) = 1 for t = 1, which implies that fu = z i.e. fu = gu = z. If f(X) is complete, then $z \in f(X) \subseteq g(X)$ and we have fu = gu = z that is u is coincidence point of f and g. If f and g are (DS_f)-weakly commuting mappings at coincidence point then for a positive real number R, we have

$$M(fgu, ggu, t) \geq M(fu, gu\ \frac{t}{R}) = 1,$$

which implies fgu = ggu that is fz = gz. Similarly, if f and g are (DS_g)-weakly commuting at coincidence point, we get fz = gz. By (1.1), we have

$$M(fx_n, fz, t) \geq r(M(gx_n, gz, t), M(gx_n, gz, t).$$

Proceeding limit as $n \to \infty$, we get

$M(z, fz, t)\ M(z, fz, t)$,

which is a contradiction. Thus $fz = z = gz$. Thus z is a common fixed point of f and g. Now to prove the uniqueness, let if possible $z_1 \neq z$ be another common fixed point of f and g. Then there exists $t > 0$ such that

$$\begin{aligned} M(z, z_1, t) &= M(fz, fz_1, t), \\ &\geq r(M(gz, gz_1, t)), \\ &\geq r(M(z, z_1, t)), \\ &> M(z, z_1, t) \text{ since } r(t)\ t \text{ for } 0 < t < 1, \end{aligned}$$

which is a contradiction. Therefore $z = z_1$ that is z is a unique common fixed point of f and g. This completes the proof.

Remark 2. Theorem 1, improve and generalize the results of Pant [17] and Vasuki [32].

Theorem 2. Let (X, M, $*$) be a fuzzy metric space with $t*t \geq t$ for all $t \in [0, 1]$. Let f and g be mappings from X into itself such that f and g are (DS_f)-weakly commuting self mappings or (DS_g)-weakly commuting self mappings at coincidence points and satisfying the following conditions:

(2.1) Given $\epsilon \in (0, 1)$ there exists $\delta \in (0, \epsilon)$ such that

$$\epsilon \geq M(gx, gy, t) > \epsilon - \delta \Rightarrow M(fx, fy, t) > \epsilon,$$

(2.2) $fx = fy$, whenever $gx = gy$.

If $f(X) \subseteq g(X)$ and if either g(X) or f(X) is complete subspace of X, then f and g have a unique common fixed point in X.

Proof. As defined in Theorem 1, we can define a sequence $\{x_n\}$ in X such that $fx_n = gx_{n+1}$, for n = 1, 2, ... By (2.1), for all x, y in X with $gx \neq gy$, we have

$M(fx, fy, t)\ M(gx, gy, t)$.

Thus we have

$$M(fx_n, fx_{n+1}, t) > M(gx_n, gx_{n+1}, t)$$
$$= M(fx_{n-1}, fx_n, t) \quad (2.3)$$

Thus $\{M(fx_n, fx_{n+1}, t): n \geq 1\}$ is an increasing sequence of positive real numbers and so it tends to the limit $L \leq 1$, we claim that $L = 1$. For if $L < 1$ for given $\delta > 0$ however small δ may be there exists a positive real number N such that for all $m \geq N$

$$L \geq M(fx_m, fx_{m+1}, t) = M(gx_{m+1}, gx_{m+2}, t) > L - \delta \quad (2.4)$$

Select δ in (2.4) in accordance with (2.1), for each $m \geq N$, we then obtain

$$M(fx_{m+1}, fx_{m+2}, t) > L,$$

which contradicts (2.4). Therefore, we have

$$\lim_{n\to\infty} M(fx_m, fx_{m+1}, t) = \lim_{n\to\infty} M(gx_{m+1}, gx_{m+2}, t)$$

Thus by the same argument, as in proof of Theorem 1, $\{fx_n\} = \{gx_{n+1}\}$ is cauchy sequence.

Now suppose g(X) is complete. Note that the sequence $\{gx_{n+1}\}$ is contained in g(X) and has a limit in g(X) call it z. Let $u \in g^{-1} z$. Then gu = z. By (2.1), we have

$$M(fx_n, fu, t) \geq M(gx_n, gu, t).$$

Proceeding limit as $n \to \infty$, we have

$$M(z, fu, t) \geq 1,$$

which implies that fu = z that is fu = gu = z. If f(X) is complete then $z \in f(X) \subseteq g(X)$ and we have fu = gu = z that u is coincidence point of f and g. Since f and g are (DS_f)-weakly commuting at coincidence points then for a positive real number R, we have

$$M(fgu, ggu, t) \geq M(fu, gu, \frac{t}{R}),$$
$$= 1,$$

which implies that fgu = ggu that is fz = gz. Similarly if f and g are (DS_g)-weakly commuting at coincidence points we get fz = gz.

By (2.1), we have

$$M(fx_n, fz, t) > M(gx_n, gz, t).$$

Taking limit as $n \to \infty$, we have

$M(z, fz, t) > M(z, fz, t)$,

which is a contradiction. Thus $fz = z = gz$. Thus z is a common fixed point of f and g. The uniqueness of z follows by using an argument similar to that used in the corresponding part of Theorem 1. This completes the proof.

Remark 2: Theorem 2, improves, extends and generalizes the results of Meir-Keeler [16] and Pant [20]. It improves and generalizes the result of Pant [17] and Pathak, Cho and Kang [21].

If we put $g = I_X$ (the identity mapping on X) in Theorem 2, we have the following:

Corollary 3: Let $(X, M, *)$ be a fuzzy metric space with $t * t \geq t$ for all $t \in [0, 1]$. Let f be mapping from X into itself satisfying the following condition:

(3.1) Given $\epsilon \in (0, 1)$ there exists $\delta \in (0, \epsilon)$ such that

$$\epsilon \geq M(x, y, t) > \epsilon - \delta \Rightarrow M(fx, fy, t) > \epsilon,$$

Then f has a unique fixed point.

Remark 3. Corollary 3 is fuzzy analogue of the well-known Meir-Keeler fixed point Theorem [16].

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