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ANALYTICAL INVESTIGATION OF NATURAL CONVECTION IN A HEATED CYLINDER USING HOMOTOPY PERTURBATION METHOD

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Abstract: *In this paper, homotopy perturbation Method (HPM) has been applied to solve a nonlinear heat transfer problem. Natural convection around an isothermal horizontal cylinder was studied. Heat transfer coefficient and specific heat coefficient was assumed to be dependent on temperature. Outcomes were compared with solution of heat transfer equation with constant properties. Solutions of HPM were compared with numerical results for different cases, Also variation of Nusselt number obtained and investigated.*

Keywords: *Homotopy Perturbation Method (*HPM*); Nonlinear Heat Transfer; Numerical Runge-Kutta Method (*NM*); Natural Convection; Nusselt number.*

NOMENCLATURE

A	Surface	NM	Numerical Runge-Kutta Method
pr	Prandtl number	Nu	Nusselt
c	Specific heat coefficient	p	Small parameter
Ra_d	Rayleigh number	**Greek Symbol**	
d	Diameter	α	Thermal diffusivity
T	Temperature	β	Thermal Expansion coefficient
h	Heat transfer coefficient	ξ	Constant parameter
T_∞	Ambient temperature	θ	Temperature difference
HPM	Homotopy Perturbation Method	ν	Kinematic Viscosity
k	Thermal conductivity	ρ	Density

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1. INTRODUCTION

Many familiar heat transfer application involve natural convection as the primary mechanism of heat transfer. Some examples are cooling of electronic equipments such as power transistors, TVs and VCRs; heat transfer in electronic baseboard heaters or steam radiators; heat transfer phenomena in the refrigeration coils and power transmission lines.

The fluid velocities associated with natural convection are low; typically less than 1 m/s therefore, the heat transfer coefficient encountered in natural convection are usually much lower than those encountered in forced convection [1-3].

So, studying natural convection that rises in applicable engineering problems, help to manipulate better natural convection as one of the heat transfer mechanism. Cooling cylindrical fin with natural convection is a good application of this heat transfer mechanism.

The boundary layer over a hot horizontal cylinder start to develop at the bottom, increasing in thickness along the circumference, and forming a rising plum at the top, as shown in Fig. (1). Therefore, the local Nusselt number is highest at the bottom and lowest at the top of cylinder when the boundary layer flow remains laminar [4].

Most of problems arising in heat transfer area are nonlinear and through the majority of them only a limited numbers of them have exact analytical solution so these nonlinear equations should be solved using other methods. Other methods include numerical and semi exact methods, scientists believe that the combination of these two methods can be more cost effective method and also lead to useful results.

One of the semi-exact methods is the homotopy perturbation method (HPM), which is established by He in 1999 [5]. This method has been applied by many authors to solve a wide variety of scientific and engineering problems. Ganji [6-8] use this method and other semi exact methods to solve nonlinear heat transfer problems. Ghasemi [9] solve a nonlinear and inhomogeneous two-dimensial wave equation problem by HPM. It was shown by many authors such as Ganji and He that this method provides improvements over existing numerical techniques [10-11].

In this paper, the mathematical model of this method is introduced and then its application in natural convection flow over a horizontal hot cylinder is studied.

The aim of this study is to consider the variation of temperature with the time in an isothermal horizontal cylinder that has been cooled with the natural convection of airflow. In recent years, much attention has been devoted to the newly developed methods to construct an analytic solution of some heat transfer equation; such methods include the HPM [6-8].

Therefore in the present work we study the influence of heat transfer coefficient, h, and specific heat coefficient, c, when they are variable with temperature or they are constant on isothermal cylinder, and how long it takes to be cooled. We find solution for these kinds of problems by HPM and compare it with numerical method (NM).

1.1. Analysis of the Homotopy Perturbation Method

The Homotopy perturbation method is a combination of the classical perturbation technique and Homotopy technique. To explain the basic idea of the HPM for solving nonlinear differential equations we consider the following nonlinear differential equation:

$$A(u) - f(r) = 0, \qquad r \in \Omega . \tag{1}$$

Subject to boundary condition

$$B(u, \partial u/\partial n) = 0, \qquad r \in \Gamma . \tag{2}$$

where A is a general differential operator, B a boundary operator, $f(r)$ is a known analytical function, Γ is the boundary of domain Ω and $\partial u/\partial n$ denotes differentiation along the normal drawn outwards from Ω. The operator A can, generally speaking, be divided into two parts: a linear part L and a nonlinear part N. Eq. (1) therefore can be rewritten as follows:

$$L(u) + N(u) - f(r) = 0 . \tag{3}$$

In case that the nonlinear Eq. (1) has no "small parameter", we can construct the following Homotopy:

$$H(v, p) = L(v) - L(u_0) + pL(u_0) + p(N(v) - f(r)) = 0 , \tag{4}$$

where

$$\nu(r, p) : \Omega \times [0, 1] \to R . \tag{5}$$

In Eq. (7), $p \in [0, 1]$ is an embedding parameter and u_0 is the first approximation that satisfies the boundary condition. We can assume that the solution of Eq. (4) can be written as a power series in p, as following:

$$\nu = \nu_0 + p\nu_1 + p^2\nu_2 + \ldots . \tag{6}$$

And the best approximation for solution is:

$$u = \lim_{p \to 1} \nu = \nu_0 + \nu_1 + \nu_2 + \ldots . \tag{7}$$

When, Eq. (4) correspond to Eq. (1) and Eq. (7) becomes the approximate solution of Eq. (1). Some interesting results have been attained using this method. Convergence and stability of this method is shown in [9].

2. DESCRIPTION OF THE PROBLEM

The aim of this study is to consider the temperature variation of a small hot isothermal horizontal cylinder in Fig. 1. with diameter and length of 1 cm that is being cooled with natural convection of air flow.

In this article, 3 cases have been investigated which are presented in Table. (1)

Table 1
An Isothermal Horizontal Cylinder

Case	*h*	*c*
1	variable	variable
2	constant	variable
3	constant	constant

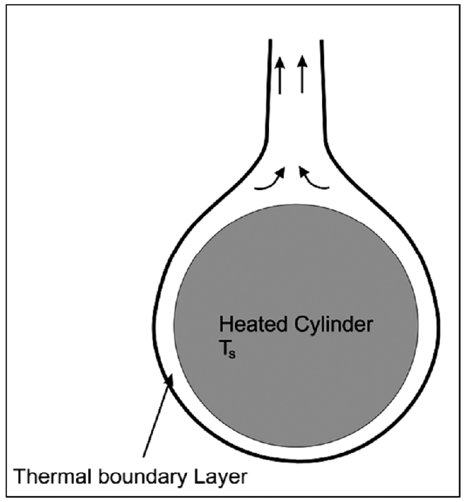

Figure 1: An Isothermal Horizontal Cylinder

In each case parameters and the following equations have been introduced:

3. APPLICATIONS

3.1. Case 1

In this case heat transfer coefficient, *h*, and specific heat coefficient, *c*, are variable with temperature. Eq. (8) represents the heat equation of a lump system [14]:

$$\rho V c\left(\frac{d}{dt}T(t)\right)+hA(T(t)-T_\infty)=0\,. \tag{8}$$

Which *c* is the quality of temperature dependency of specific heat on temperature.

$$c=c_0(1+\xi(T-T_\infty))\,. \tag{9}$$

The average Nusselt number over the entire surface can be determined from [15] for an isothermal horizontal cylinder:

$$Nu=\left(0.6+\frac{0.387\,Ra_d^{1/6}}{\left(1+\left(\frac{0.559}{\Pr}\right)^{9/16}\right)^{8/27}}\right)^2. \tag{10}$$

Where Rayleigh number and heat transfer coefficient are as follow:

$$Ra_d=\frac{g\beta d^3(T-T_\infty)}{\nu\alpha}, \tag{11}$$

$$h=\frac{k}{d}Nu\,. \tag{12}$$

From Eq. (13), the variation of h against θ could be found as follow:

$$h=\frac{k}{d}\left(0.6+\frac{0.387\left(\frac{g\beta d^3}{\nu\alpha}\right)^{\frac{1}{6}}}{\left(1+\left(\frac{0.559}{pr}\right)^{\frac{9}{16}}\right)^{\frac{8}{27}}}\theta(t)^{\frac{1}{6}}\right)^2. \tag{13}$$

Substituting Esq. (13) and (9) in (8), we have:

$$b\left(\frac{d}{dt}\theta(t)\right)+b\beta\theta(t)\left(\frac{d}{dt}\theta(t)\right)+0.36e\theta(t)+1.2a_0e\theta(t)^{7/6}+ea_0^2\theta(t)^{4/3}=0, \tag{14}$$

where:

$$b=\rho Vc_0 \tag{15}$$

$$e=\frac{Ak}{d} \tag{16}$$

$$a_0=\frac{0.387\left(\frac{g\beta d^3}{\nu\alpha}\right)^{1/6}}{\left(1+\left(\frac{0.559}{\mathrm{Pr}}\right)^{9/16}\right)^{8/27}} \tag{17}$$

Solving this equation for a real condition with the air as a cooling flow so we reach to the following coefficients and equation:

$$b=1.5 \tag{18}$$

$$e=0.000208 \tag{19}$$

$$a_0=0.0192 \tag{20}$$

$$1.57\left(\frac{d}{dt}\theta(t)\right)+0.0051810\theta(t)\left(\frac{d}{dt}\theta(t)\right)+0.000074880\theta(t)$$
$$+4.79232\times10^{-6}+7.667712\times10^{-8}\theta(t)^{4/3}=0. \tag{21}$$

3.2. Case 2

Consider heat transfer in a lumped system, Eq. (8), with constant h and variable c. The specific heat coefficient varies linearly with temperature as shown in Eq. (9).

Substituting Eq. (9) in Eq. (8), we have:

$$b\left(\frac{d}{dt}\theta(t)\right)+b\beta\theta(t)\left(\frac{d}{dt}\theta(t)\right)+f\theta(t)=0. \tag{22}$$

Where:

$$b = \rho V c_0 \tag{23}$$

$$f = hA \tag{24}$$

For a real condition with the air as a cooling flow, we reached the following coefficients:

$$b = 1.57 \tag{25}$$

$$f = 1.13097 \times 10^{-4} \tag{26}$$

$$\beta = 0.0033 \tag{27}$$

So we have Eq. (35):

$$1.57\left(\frac{d}{dt}\theta(t)\right) + 5.181\times10^{-3}\theta(t)\left(\frac{d}{dt}\theta(t)\right) + 1.13097\times10^{-4}\theta(t) = 0\,. \tag{28}$$

3.3. Case 3

In this case both h and c are constant.

We solve this equation for a real condition with the air as a cooling flow. So, we reach the following coefficients and equation:

$$\rho V c = 1.57, \tag{29}$$

$$hA = 1.13097 \times 10^{-4}, \tag{30}$$

$$1.57\left(\frac{d}{dt}\theta(t)\right) + 1.13097\times10^{-4}\theta(t) = 0\,. \tag{31}$$

4. SOLUTION USING HOMOTOPY PERTURBATION METHOD

4.1. Case 1

In this section, we will apply the HPM to nonlinear ordinary differential Eq. (21). According to the HPM, we can construct a homotopy of Eq. (21) as follows:

$$\begin{aligned} H(\theta, p) = (1-p)&\left(1.57\left(\frac{d}{dt}\theta(t)\right) + 7.488\times10^{-5}\theta(t)\right) \\ &+ p\left(1.57\left(\frac{d}{dt}\theta(t)\right) + 0.005181\,\theta(t)\left(\frac{d}{dt}\theta(t)\right) + 0.00007488\,\theta(t)\right. \\ &\left. + 4.79232\times10^{-6} + 7.667712\times10^{-8}\,\theta(t)^{4/3}\right). \end{aligned} \tag{32}$$

$$\theta(t) = \theta_0(t) + p\cdot\theta_1(t) + p^2\cdot\theta_2(t)\,. \tag{33}$$

Substituting Eq. (33) into Eq. (32) and collect $H(\theta, p)$ and then put the coefficients of p equal to zero, we have:

$$p^0: \quad 1.57\left(\frac{d}{dt}\theta_0(t)\right) + 7.488\times10^{-5}\theta_0(t) = 0 \qquad \theta_0(0) = 100\,. \tag{34}$$

$$p^1: \quad 0.005181\,\theta_0(t)\left(\frac{d}{dt}\theta_0(t)\right)+1.57\left(\frac{d}{dt}\theta_1(t)\right)$$
$$+\,7.667712\times10^{-8}\theta_0(t)^{4/3}+4.79232\times10^{-6}\theta_0(t)^{7/6}=0 \qquad \theta_1(0)=0\,. \tag{35}$$

$$p^2: \quad 7.488\times10^{-5}\theta_2(t)+5.181\times10^{-3}\theta_0(t)\left(\frac{d}{dt}\theta_1(t)\right)+1.57\left(\frac{d}{dt}\theta_2(t)\right)$$
$$+\,5.181\times10^{-3}\theta_1(t)\left(\frac{d}{dt}\theta_0(t)\right)+7.667712\times10^{-6}\theta_1(t)^{4/3}=0 \quad \theta_2(0)=0\,. \tag{36}$$

Solving Esq. (34-36) with initial conditions, we have:

$$\theta_0(t)=100\,e^{-\frac{117}{2453125}t} \tag{37}$$

$$\theta_1(t)=-\frac{33}{2}e^{-\frac{234}{2453125}t}+\frac{48}{625}\times10^{\frac{2}{3}}\left(e^{-\frac{117}{2453125}t}\right)^{\frac{4}{3}}+\frac{192}{35}\times10^{\frac{1}{3}}\left(e^{-\frac{117}{2453125}t}\right)^{\frac{7}{6}}$$
$$+\frac{33}{2}-\frac{48}{625}\times10^{\frac{1}{3}}-\frac{192}{35}\times10^{\frac{1}{3}}. \tag{38}$$

$$\theta_2(t)=0. \tag{39}$$

So:

$$\theta(t)=\theta_0(t)+\theta_1(t)+\theta_2(t). \tag{40}$$

$$\theta(t)=-\frac{33}{2}e^{-\frac{234}{2453125}t}+\frac{48}{625}\times10^{\frac{2}{3}}\left(e^{-\frac{117}{2453125}t}\right)^{\frac{4}{3}}+\frac{192}{35}\times10^{\frac{1}{3}}\left(e^{-\frac{117}{2453125}t}\right)^{\frac{7}{6}}$$
$$+\frac{33}{2}-\frac{48}{625}\times10^{\frac{1}{3}}-\frac{192}{35}\times10^{\frac{1}{3}}+100\,e^{-\frac{117}{2453125}t}. \tag{41}$$

4.1. Case 2

By applying the HPM to nonlinear ordinary differential Eq. (28) According to the HPM, we can construct a homotopy of Eq. (28) as follows:

$$H(\theta,p)=(1-p)\left(1.57\left(\frac{d}{dt}\theta(t)\right)+11.309724\times10^{-4}\theta(t)\right)$$
$$+\,p\left(1.57\left(\frac{d}{dt}\theta(t)\right)+5.181\times10^{-3}\theta(t)\left(\frac{d}{dt}\theta(t)\right)+11.309724\times10^{-4}\theta(t)\right. \tag{42}$$

$$\theta(t)=\theta_0(t)+p\cdot\theta_1(t)+p^2\cdot\theta_2(t) \tag{43}$$

Substituting Eq. (43) into Eq. (42) and collect $H(\theta, p)$ and then put the coefficients of p equal zero, we have:

$$p^0: \quad 1.57\left(\frac{d}{dt}\theta_0(t)\right)+11.309724\times 10^{-4}\theta_0(t)=0 \qquad \theta_0(0)=100\,. \tag{44}$$

$$p^1: \quad 1.57\left(\frac{d}{dt}\theta_1(t)\right)+11.309724\times 10^{-4}\theta_1(t) + 5.181\times 10^{-3}\left(\frac{d}{dt}\theta_0(t)\right)=0 \qquad \theta_1(0)=0\cdot \tag{45}$$

$$\begin{aligned} p^2: \quad & 1.57\left(\frac{d}{dt}\theta_2(t)\right)+11.309724\times 10^{-4}\theta_2(t)-1.309724\times 10^{-4}\theta_1(t) \\ & +5.181\times 10^{-3}\theta_0(t)\left(\frac{d}{dt}\theta_1(t)\right)+1.57\left(\frac{d}{dt}\theta_2(t)\right) \\ & +5.181\times 10^{-3}\theta_0(t)\left(\frac{d}{dt}\theta_1(t)\right) \\ & +5.181\times 10^{-3}\theta_1(t)\left(\frac{d}{dt}\theta_0(t)\right)=0 \qquad \theta_2(0)=0 \end{aligned} \quad . \tag{46}$$

Solving Eqs. (44-46) with initial conditions, we have:

$$\theta_0(t)=100\, e^{-\frac{2827431}{39250000000}t}. \tag{47}$$

$$\theta_1(t)=\left(-\frac{93305223}{2827432}e^{-\frac{353429}{4906250000}t}+\frac{93305223}{2827432}\right)e^{-\frac{282743}{3925000000}t}. \tag{48}$$

$$\begin{aligned}\theta_2(t)=\Bigg(&-\frac{6158143629}{282743200}e^{-\frac{282743}{39250000000}t}+\frac{26381407997213}{7994371714624}e^{-\frac{353429}{4906250000}t} \\ &+\frac{26381407997213}{110976706000000000}t+\frac{261175939172391 87}{1598874060181600}e^{-\frac{5654863}{39250000000}t} \\ &-\frac{6228400385156744314971}{2260353840863690825 60}\Bigg)e^{-\frac{282743}{3925000000}t}.\end{aligned} \tag{49}$$

So:

$$\theta(t)=\theta_0(t)+\theta_1(t)+\theta_2(t). \tag{50}$$

$$\theta(t) = 100\, e^{-\frac{2827431}{39250000000}t} + \left(-\frac{93305223}{2827432} e^{-\frac{353429}{4906250000}t} + \frac{93305223}{2827432}\right) e^{-\frac{282743}{3925000000}t}$$
$$+ \left(-\frac{6158143629}{282743200} e^{-\frac{282743}{39250000000}t} + \frac{26381407997213}{7994371714624} e^{-\frac{353429}{4906250000}t}\right.$$
$$+ \frac{26381407997213}{110976706000000000}t + \frac{26117593917239187}{1598874060181600} e^{-\frac{5654863}{39250000000}t}$$
$$\left. - \frac{622840038515674431 4971}{226035384086369082560}\right) e^{-\frac{282743}{3925000000}t}. \quad (51)$$

4.3. Case 3

$$H(\theta, p) = (1-p)\left(1.57\left(\frac{d}{dt}\theta(t)\right) + 11.309724\times10^{-4}\theta(t)\right)$$
$$+ p\left(1.57\left(\frac{d}{dt}\theta(t)\right) + 11.309724\times10^{-4}\theta(t).\right. \quad (52)$$

$$\theta(t) = \theta_0(t) + p\cdot\theta_1(t) + p^2\cdot\theta_2(t). \quad (53)$$

Substituting Eq. (53) into Eq. (52) and collect $H(\theta, p)$ and then put the coefficients of p equal zero, we have:

$$p^0: \quad 1.57\left(\frac{d}{dt}\theta_0(t)\right) + 7.488\times10^{-5}\theta_0(t) = 0 \qquad \theta_0(0) = 100. \quad (54)$$

$$p^1: \quad 1.57\left(\frac{d}{dt}\theta_1(t)\right) + 7.488\times10^{-5}\theta_1(t) = 0 \qquad \theta_1(0) = 0. \quad (55)$$

$$p^2: \quad 1.57\left(\frac{d}{dt}\theta_2(t)\right) + 7.488\times10^{-5}\theta_2(t) = 0 \qquad \theta_0(0) = 0. \quad (56)$$

Solving Esq. (634-66) with initial conditions, we have:

$$\theta_0(t) = 100\, e^{-\frac{2827431}{39250000000}t}. \quad (57)$$

$$\theta_1(t) = 0. \quad (58)$$

$$\theta_2(t) = 0. \quad (59)$$

So we have:

$$\theta(t) = \theta_0(t) + \theta_1(t) + \theta_2(t). \quad (60)$$

$$\theta(t) = 100\, e^{-\frac{2827431}{39250000000}t}. \quad (61)$$

5. RESULT AND DISCUSSION

In Fig. 1. The temperature distribution is compared with numerical solution. The heat transfer coefficient and heat specific is taken variable. It can be seen that there is good agreement between them**.**

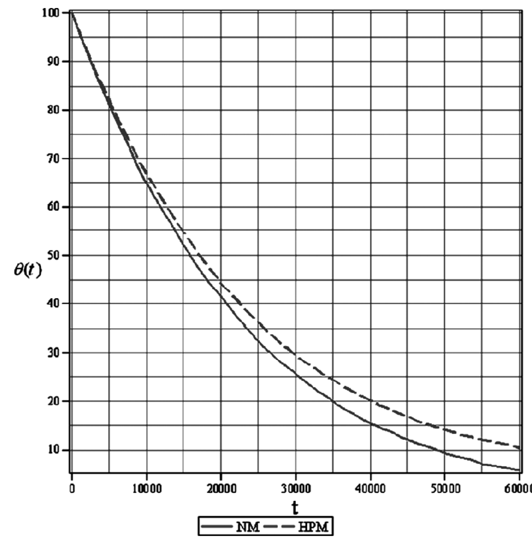

Figure 2: $\theta(t) - t,\ h \neq cte,\ c \neq cte$

In Fig. 2, the temperature gradient is depicted. It is increased with increasing time.

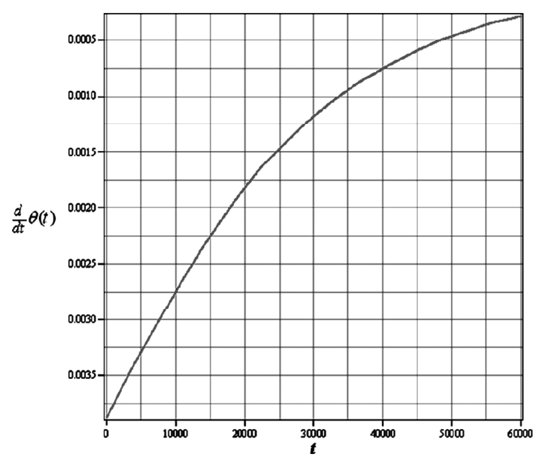

Figure 3: $\frac{d}{dt}\theta(t) - t,\ h \neq cte,\ c \neq cte$

In Fig. 3, the temperature distribution for case of constant heat transfer coefficient and variable specific heat is drawn.

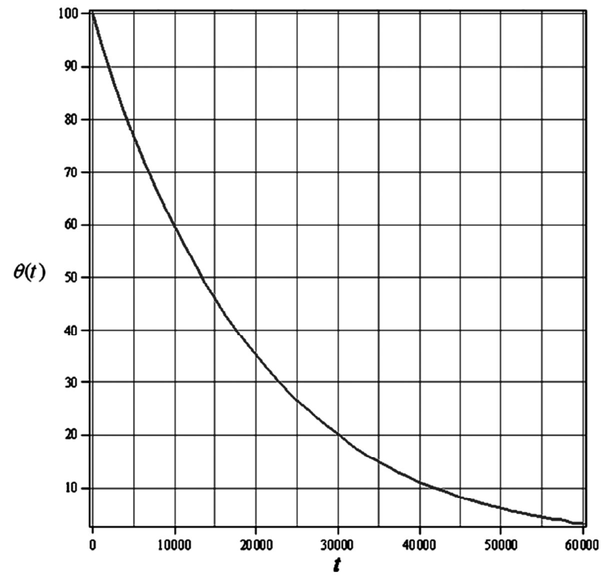

Figure 4: $\theta(t) - t,\ h \neq cte,\ c \neq cte,$ Solved by HPM

For this case the temperature gradient is depicted also. It is depicted in Fig. 4.

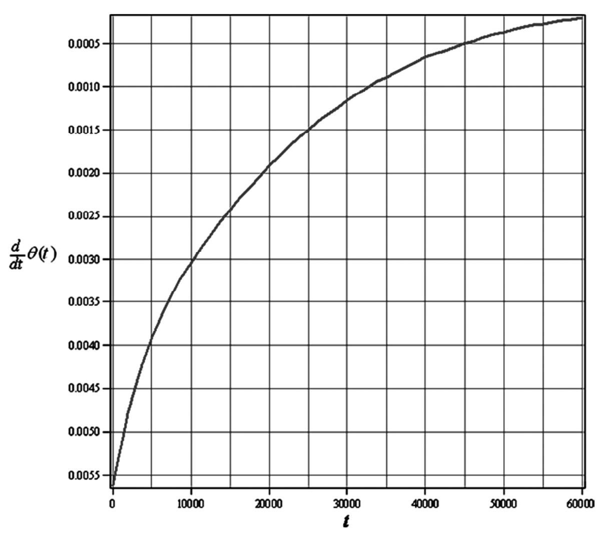

Figure 5: $\frac{d}{dt}\theta(t) - t,\ h = cte,\ c \neq cte,$ Solved by HPM

In Fig. 5, the temperature distribution for the case of variable heat transfer coefficient and constant specific heat with both Numerical method and HPM is depicted. With growth of time the temperature of cylinder closes to ambient temperature.

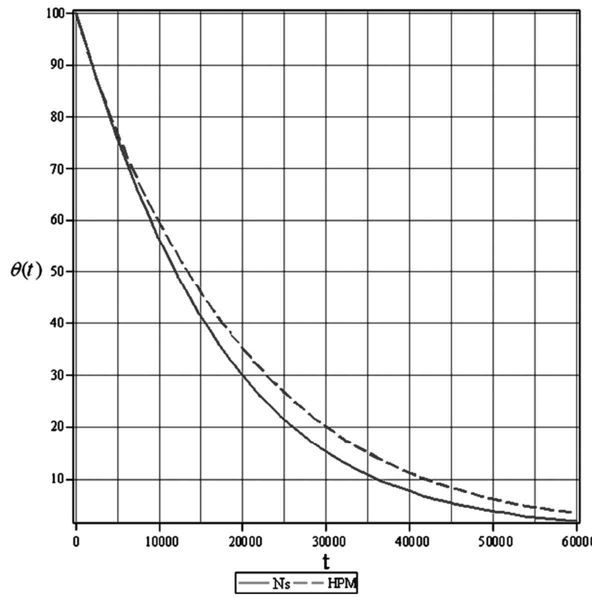

Figure 6: $\theta(t) - t,\ h \neq cte,\ c = cte$

For this case the temperature gradient is depicted also in Fig. 6.

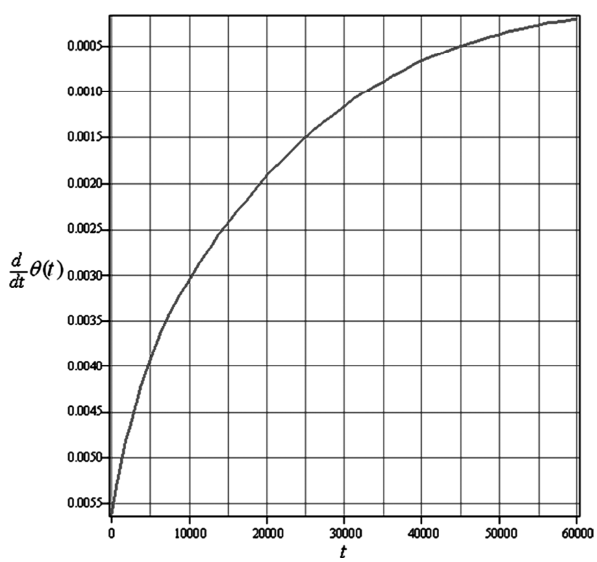

Figure 7: $\frac{d}{dt}\theta(t) - t,\ h \neq cte,\ c = cte$

For case of constant thermal properties the temperature distribution and gradient of is depicted in Figs. 7 and 8.

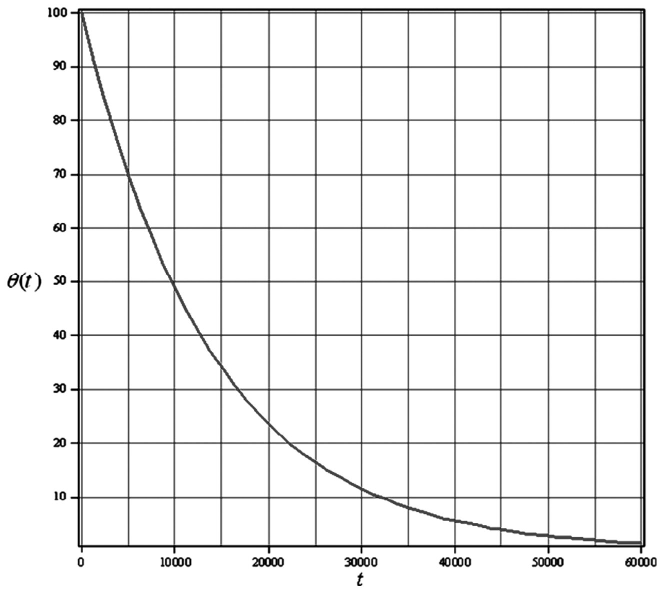

Figure 8: $\theta(t) - t,\ h = cte,\ c = cte$

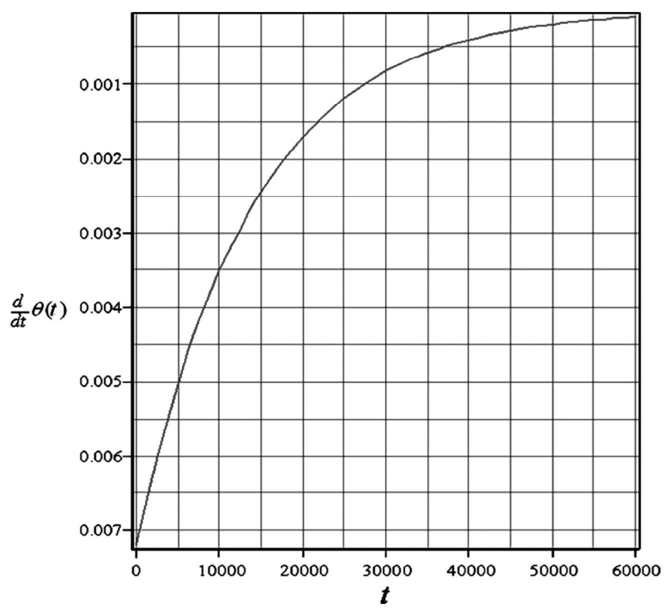

Figure 9: $\frac{d}{dt}\theta(t) - t,\ h = cte,\ c = cte$

In Figs. 9 and 10. The result of constant properties and variable properties is compared .it can be seen in the case of variable properties cylinder reach the ambient temperature at a shorter time and temperature gradient tend to zero faster.

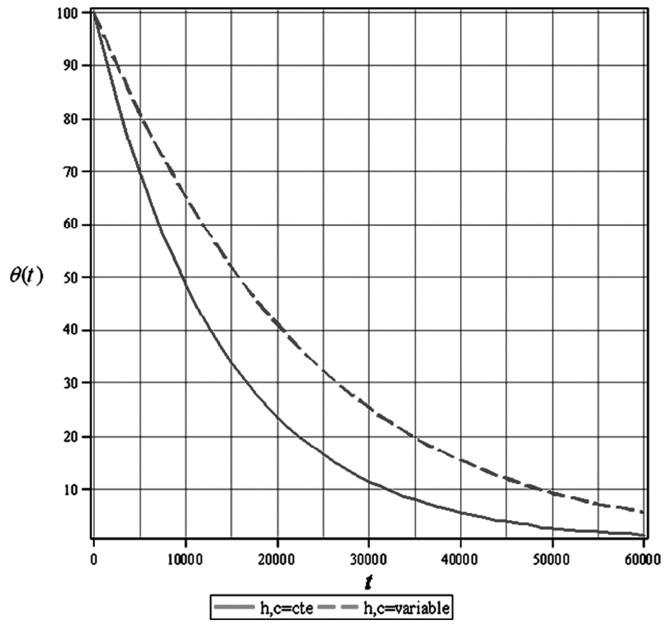

Figure 10: Compare $\theta(t) - t,\ h,\ c \neq cte,\ c = cte$ and $h,\ c = cte$

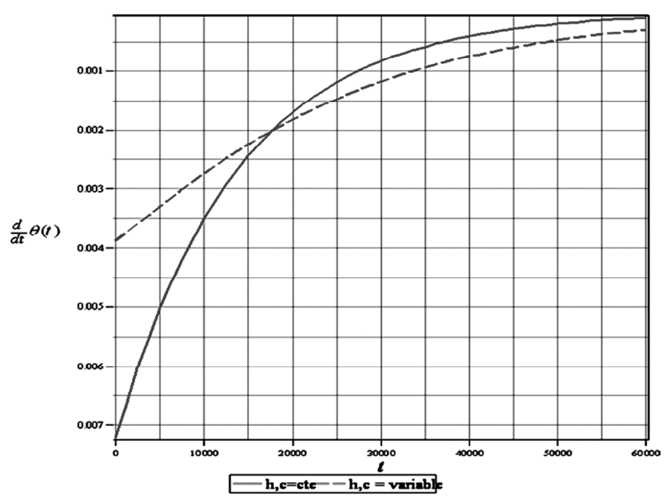

Figure 11: Compare $\frac{d}{dt}\theta(t) - t,\ h, c = cte,$ and $h, c = cte$

In Figs. 11 and 12. The result of variable properties and case of constant heat transfer coefficient and variable specific heat are compared . It can be seen in the case of variable properties cylinder reach the ambient temperature at a shorter time and temperature gradient tend to zero faster.

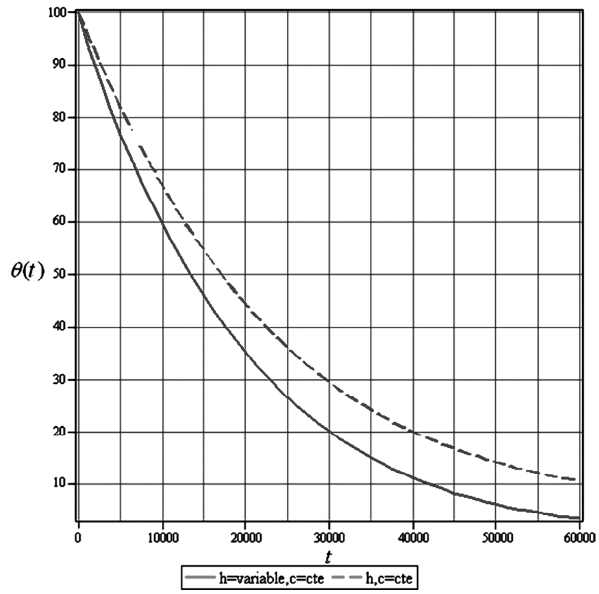

Figure 12: Compare $\theta(t)-t,\ h,c\neq cte$ and $h=cte, c\neq cte$

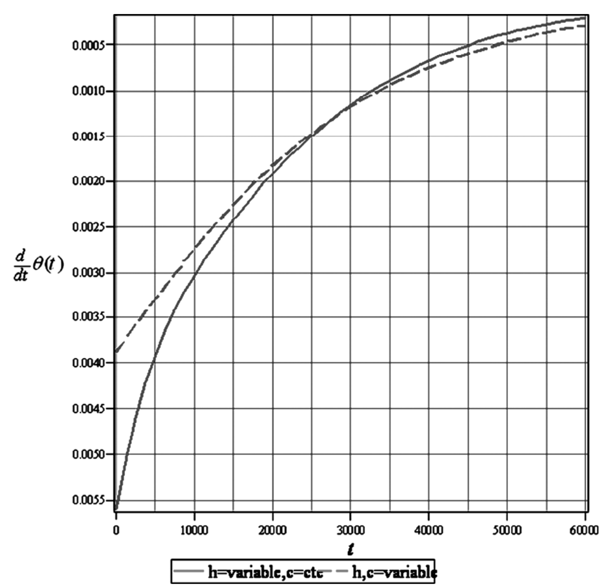

Figure 13: Compare $\frac{d}{dt}\theta(t)-t,\ h,c\neq cte,$ and $h=cte,\ c\neq cte$

In Fig. 13. Average Nusselt number in each time is shown. At the beginning time because of high gradient temperature Nusselt number is great. With growth of time Nusselt number tend to zero, because of zero temperature gradient.

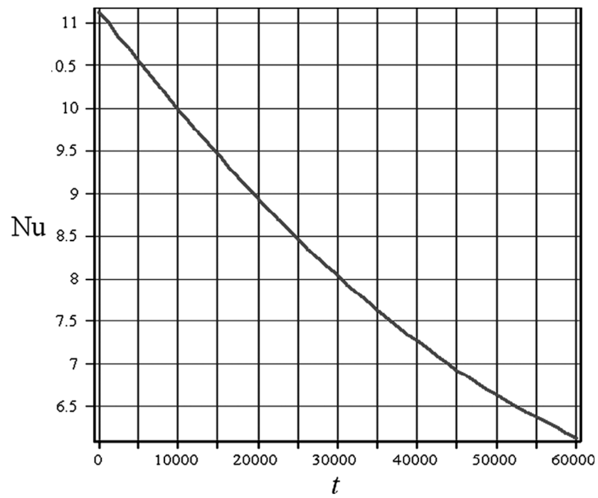

Figure 14: $Nu,\ h,\ c \neq cte$

For case of constant heat transfer properties the Nusselt number is zero because it related to heat transfer coefficient and thermal conductivity and they are constant.

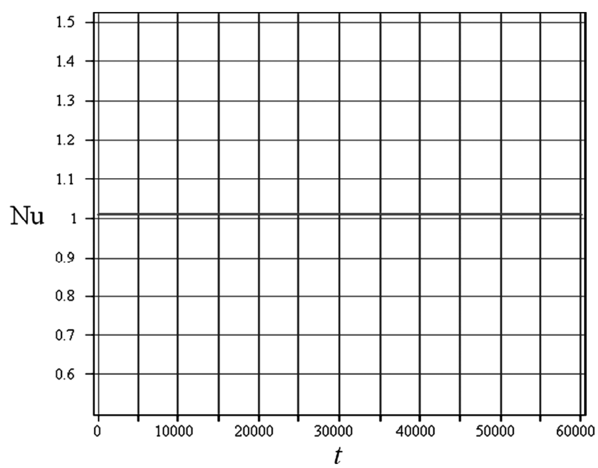

Figure 15: $Nu,\ h,\ c = cte,\ h \neq cte,\ c \neq cte$

6. CONCLUSIONS

In the present work, we have analyzed natural convection flow over a hot isothermal horizontal cylinder. We study the influence of variable h and *c*, and solved the nonlinear equation that is extracted by He's Homotopy Perturbation Method (*HPM*). These considered equations are easily solved by mentioned analytical method. Consequently, these equations are solved by the numerical method (Runge-Kutta fourth-order) using the software Maple 12® and the results of the *HPM* and *NM* are compared in Figs. 2, 6, 10, 11, 12, 13. Then effects of *h* and *c* when they are variable or constant are shown in Fig.3, 4, 5, 7, 8 and 8. Also the variations of Nusselt number are shown in Figs.14 and 15. So the following results are obtained:

(i) The effect of c is stronger than h when temperature is decreasing.

(ii) With increasing h and decreasing c, the time of cooling approach will be decreased.

(iii) The natural convection is not appropriate in industry that time is an important parameter.

(iv) Obtained results from case 1, 2 and 3 are approximately similar.

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