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THERMAL PERFORMANCE ASSESSMENT OF A CONVECTIVE POROUS FIN WITH VARIABLE CROSS SECTION BY MEANS OF OHAM

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Abstract: *In this study, thermal performance of a convective porous fin with variable cross section has been investigated using a simulation method called Optimal Homotopy Asymptotic Method (OHAM). The concept of OHAM is briefly introduced, and then employed it to derive solutions of governing nonlinear equation. The obtained results from this method are compared with those from the numerical solution to verify the accuracy of the proposed method. It is found that the OHAM can achieve suitable results in predicting the solution of such problems. Also the effects of some physical applicable parameters in this problem on temperature distribution and fin efficiency have been analyzed. The results show that increasing porous parameter, convective parameter or fin profile parameter lead to decreasing both the temperature variation and fin efficiency which indicated significantly of these parameters.*

Keywords: *Porous fin, Thermal performance, Variable cross section, Optimal homotopy asymptotic method (OHAM).*

1. INTRODUCTION

Enhancement the rate of heat transfer has been concerned in many thermal engineering applications especially in cooling systems for electronic equipment, chemical processes, energy systems equipment and high performance heat exchangers. Extended surfaces or fins are practical and efficient means of enhancing heat transfer between a primary surface and its environment where increasing the heat transfer coefficient is not an option. Fins are frequently used in engineering to enhance the rate of heat transfer on a solid surface. For the cases of constant heat transfer coefficient or constant cross section, the analytical solutions of temperature distribution as well as heat transfer rate can be easily obtained. However, in some situations such as fins in with variable cross section the heat transfer coefficient is no longer uniform and varies with the temperature difference between the surface and the adjacent fluid in a nonlinear manner. Consequently, the equation for temperature becomes highly nonlinear and is difficult to obtain an analytical solution. Also these extended surfaces are widely applied in various industrial applications. Due to this fact, fins have been the topic of many studies and extensive

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researches have been done in this area and many references are available (see e.g., Refs. [1-7]). In the present paper, the governing nonlinear differential equation is solved by Optimal Homotopy Asymptotic Method (OHAM) to assess thermal performance of a convective porous fin. The influence of various parameters namely, porous parameter (S_h), Convective parameter (m) and fin profile parameter (λ) upon temperature variations as well as fin efficiency are investigated.

Most scientific problems demonstrate themselves in the mathematical relations that are modeled principally by ordinary or partial differential equations but there are few phenomena in different fields of science occurring linearly. Most of problems and scientific phenomenon such as heat transfer ones function nonlinearly.

Analytical solution would not able to solve these equations generally because they are innately of nonlinearity; therefore, special techniques should be applied to solve them. In most cases, the solution can be obtained either by numerical techniques [8-9] or by method of perturbation [10-11]. In the case of numerical methods, stability and convergency should be considered due to avoid inappropriate results. On the other hand, in perturbation method, the small parameter should be exerted on the equation. Thus, finding the small parameter is deficiency of this method. For this reason, some different techniques have recently introduced to eliminate the small parameter including Homotopy Perturbation Method [12-14], Differential Transformation Method [15-17], Homotopy Analysis Method [18-21] and so forth.

NOMENCLATURE

b	Thickness of the fin at the base
C_p	Specific heat
D_a	Darcy number, K/t^2
g	gravity constant
Gr	Grashof number
k	Thermal conductivity
k_r	Thermal conductivity ratio, $\frac{k_{eff}}{k_f}$
K	Permeability of porous fin
L	Length
m	Convective parameter
Pr	Prandtl number, $\frac{\upsilon}{\alpha}$
Ra	Rayleigh number, $Gr \times \mathrm{Pr}$
S_h	Porous parameter
T	Temperature
V_w	Velocity of fluid passing through the fin
W	Width of the fin
x	Axial coordinate
X	Dimensionless axial coordinate, $\frac{x}{L}$

Greek symbols

α	Thermal diffusivity
β	Coefficient of volumetric thermal expansion
θ	Dimensionless temperature, $\frac{T(x)-T_\infty}{T_b-T_\infty}$
υ	Kinematic viscosity

Subscripts

b	base condition of fine
eff	Porous properties
f	Fluid properties
s	Solid properties
∞	ambient condition

One of the semi-exact methods which does not require the small parameter is OHAM which is powerful method for solving nonlinear problems. This method has already been applied successfully to solve many engineering problems by some authors [22-24].

In this research, thermal performance of a convective porous fin with variable cross section has been investigated using a simulation method called Optimal Homotopy Asymptotic Method (OHAM). Numerous studies have devoted to the analysis of fin performance of this type of problems due to its important application in engineering and heat transfer applications. Mueller and Abu-Mulaweh [25] studied the efficiency of horizontal single pin fin subjected to free convection and radiation heat transfer. Mokheimer [26] investigated locally variable heat transfer coefficient on the performance of extended surfaces subject to natural convection. Kang and Look [27] presented optimum designs of a thermally asymmetric convecting and radiating rectangular annular fin. Razelos and Kakatsios [28] determined the optimum dimensions of convecting–radiating heat transfer fins. Yu and Chen [29] performed a study on optimization of circular fins with variable thermal parameters.

The objective of this study is to apply the Optimal Homotopy Asymptotic Method to investigate a convective porous fin with variable cross section based on the OHAM solution; while in previous studies [30-31], the simplest case, rectangular fin profile has been investigated. Moreover, we have made a comparison with the numerical solution via well-known fourth order Runge–Kutta method to check the validity of this method. The results showed that the method has many merits including fast convergence and high accuracy.

2. DESCRIPTION OF THE PROBLEM

We consider a longitudinal porous fin of exponential function profile as shown in Fig. 1, which extends into a fluid of temperature T_∞ and the base is maintained at constant temperature T_b. Let the fin length be L, width W and its thicknesses at the base b. This fin is porous to allow the flow of infiltrate through it. With the assumption of one-dimensional heat conduction along the fin, steady-state operation and also considering the fact that the porous medium is isotropic and saturated with single-phase fluid; an energy balance applied to a differential element according to Fig. 2 yields:

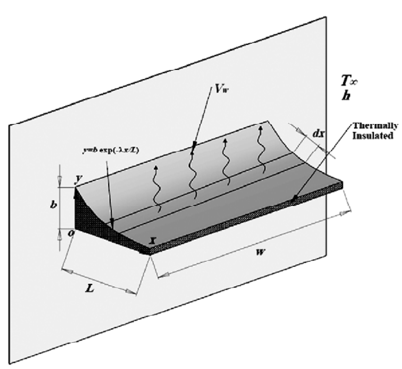

Figure 1: Schematic Diagram of Porous Fin Under Investigation

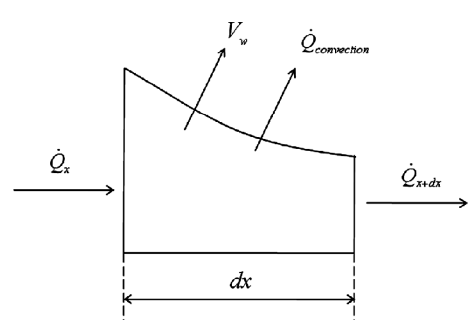

Figure 2: Control Volume for Thermal Analysis

$$\dot{Q}_x = \dot{Q}_{x+dx} + \dot{Q}_{\text{convection}} + \dot{m}C_p(T - T_\infty)\,. \tag{1}$$

Where, $\dot{m}$ accounts for mass flow rate of the fluid passing through the porous material can be written as,

$$\dot{m} = \rho W V_w\, dx\,. \tag{2}$$

From the Darcy's model we have,

$$V_w = \frac{gK\beta}{\upsilon}(T - T_\infty)\,. \tag{3}$$

With the use of standard expressions for conduction, convection the energy balance which can be written as,

$$\frac{d}{dx}\left(k_{eff} W b e^{-\lambda \frac{x}{L}} \frac{dT}{dx}\right) - hW(T - T_\infty) - \frac{\rho g K\beta C_p W}{\upsilon}(T - T_\infty)^2 = 0\,. \tag{4}$$

By introducing $X = \frac{x}{L}$, $\theta = \frac{T - T_\infty}{T_b - T_\infty}$ and some manipulating we have,

$$\frac{d^2\theta}{dx^2} - \lambda\frac{d\theta}{dx} - e^{\lambda X}(S_h\theta^2 + m^2\theta) = 0\,. \tag{5}$$

Where, $S_h = \frac{D_a x Ra}{k_r}(\frac{L}{b})^2$ is porous parameter and $m = (\frac{hL^2}{k_{eff}b})^{1/2}$ is convective parameter of the fin. Note that the temperature at the base of the fin is uniform T_b and also there is no heat transfer from the tip of the fin; boundary conditions for Eq. (5) can be written as:

$$\theta\big|_{X=0} = 1 \tag{6}$$

and

$$\left.\frac{d\theta}{dX}\right|_{X=1} = 0\,. \tag{7}$$

The efficiency of the fin η defined as the ratio of actual heat transfer to the other side while whole fin surface is at the same temperature which can be expressed as:

$$\eta = \frac{Q_{\text{actual}}}{Q_{\text{ideal}}} = \frac{\int_0^L hW(T - T_\infty)\,dx}{hWL(T_b - T_\infty)} = \int_0^1 \theta dX\,. \tag{8}$$

3. BASIC CONCEPTS OF OHAM

We apply the OHAM to the following differential equation:

$$L(u(\tau)) + g(\tau) + N(u(\tau)) = 0\,, \quad B(u) = 0\,. \tag{9}$$

Where L is a linear operator, τ denotes independent variable, $u(\tau)$ is an unknown function, $g(\tau)$ is a known function, $N(u(\tau))$ is a nonlinear operator and B is a boundary operator. Through OHAM one first constructs a family of equation:

$$(1-p)[L(\phi(\tau, p))+g(\tau)]=H(p)[L(\phi(\tau, p))+g(\tau)+N(\phi(\tau, p))] \quad B(\phi(\tau, p))=0\,. \tag{10}$$

Where $p \in [0,1]$ is an embedding parameter, $H(p)$ is a nonzero auxiliary function for $p \neq 0$ and $H(0) = 0$, $\phi(\tau, p)$ is an unknown function, respectively. Clearly, when $p = 0$ and $p = 1$, it holds that:

$$\phi(\tau, 0) = u_0(\tau), \qquad \phi(\tau, 1) = u(\tau). \tag{11}$$

Hence, when p increases from 0 to 1, the solution $\phi(\tau, p)$ varies from $u_0(\tau)$ to the solution $u(\tau)$, where $u_0(\tau)$ is obtained from Eq. (10) for $p = 0$:

$$L(u_0(\tau)) + g(\tau) = 0, \qquad B(u_0) = 0. \tag{12}$$

We choose the auxiliary function $H(p)$ in the form

$$H(p) = pC_1 + p^2C_2 + ... \tag{13}$$

Where $C_1, C_2, \ldots$ are constants which can be determined later. Expanding $\phi(\tau, p)$ in a series with respect to p, one has

$$\phi(\tau, p, C_i) = u_0(\tau) + \sum_{k\geq 1} u_k(\tau, C_i)\, p^k, \qquad i = 1, 2, ... \tag{14}$$

Substituting Eq. (14) into Eq. (10), collecting the same powers of p, and equating each coefficient of p to zero, we obtain set of differential equation with boundary conditions. Solving differential equations by boundary conditions, $u_0(\tau)$, $u_1(\tau, C_1)$, and $u_2(\tau, C_2)$, … are obtained. Generally, the solution of Eq. (9) can be determined approximately in the form:

$$\tilde{u}^{(m)} = u_0(\tau) + \sum_{k=1}^{m} u_k(\tau, C_i)\,. \tag{15}$$

Considering that the last coefficient C_m can be function of τ. Substituting Eq. (15) into Eq. (9), there results the following residual:

$$R(\tau, C_i) = L(\tilde{u}^{(m)}(\tau, C_i)) + g(\tau) + N(\tilde{u}^{(m)}((\tau, C_i))\,. \tag{16}$$

If $R(\tau, C_i) = 0$ then $\tilde{u}^{(m)}(\tau, C_i)$ would be the exact solution. As a whole, such a case will not arise for nonlinear problems, but we can minimize the functional:

$$J(C_1, C_2, ..., C_n) = \int_a^b R^2(\tau, C_1, C_2, ..., C_m)\, d\tau\,. \tag{17}$$

Where a and b are two values, depending on the given problem. The unknown constants C_i ($i = 1, 2, \ldots, m$) can be identified from the conditions

$$\frac{\partial J}{\partial C_1} = \frac{\partial J}{\partial C_2} = ... = 0\,. \tag{18}$$

With these constants, the approximate solution (of order m) (Eq. (15)) is well obtained.

4. SOLUTION WITH OHAM

In this section, OHAM is applied to nonlinear ordinary differential Eq. (5). According to the OHAM, applying Eq. (10) to Eq. (5):

$$(1-p)\,\theta'' - H(p)\,(\theta'' - \lambda\theta' - e^{\lambda X}\,(S_h\theta^2 + m^2\theta)) = 0 \tag{19}$$

where primes denote differentiation with respect to X.

We consider $H(p)$ as,

$$H(p) = C_1 p + C_2 p^2. \tag{20}$$

Substitution Eq. (20) into Eq. (19) and some simplification and rearranging based on powers of p-terms, we have:

$$\begin{aligned} &p^0 : \theta_0'' = 0, \\ &\theta_0(0) = 1, \quad \theta_0'(1) = 0. \end{aligned} \tag{21}$$

$$\begin{aligned} &p^1 : C_1\lambda\theta_0' + C_1 S_h e^{\lambda X}\theta_0^2 - \theta_0'' + C_1 m^2 e^{\lambda X}\theta_0 + \theta_1'' - C_1\theta_0'' = 0, \\ &\theta_1(0) = 0, \quad \theta_1'(1) = 0. \end{aligned} \tag{22}$$

$$\begin{aligned} &p^2 : -C_2\,\theta_0'' + \theta_2'' + C_1\lambda\theta_1' + C_2 m^2 e^{\lambda X}\theta_0 + C_2\lambda\theta_0' + 2C_1 S_h e^{\lambda X}\theta_0\theta_1 - \theta_1'' \\ &\qquad + C_1 m^2 e^{\lambda X}\theta_1 + C_2 S_h e^{\lambda X}\theta_0^2 - C_1\theta_1'' = 0, \\ &\theta_2(0) = 0, \quad \theta_2'(1) = 0. \end{aligned} \tag{23}$$

Since then, final expression for $\theta(X)$ is:

$$\theta(X) = \theta_0(X) + \theta_1(X) + \theta_2(X) + \ldots \tag{24}$$

From Eq. (16) by substituting $\theta(X)$ into Eq. (5), $R_1(X, C_1, C_2)$ and $R_2(X, C_1, C_2)$ are obtained and J_1 and J_2 can be attainable as follows:

$$\begin{aligned} J_1(C_1, C_2) &= \int_0^1 R_1^2(X, C_1, C_2)\,dX, \\ J_2(C_1, C_2) &= \int_0^1 R_2^2(X, C_1, C_2)\,dX. \end{aligned} \tag{25}$$

In the case of $S_h = 1$, $\lambda = 1$ and $m = 0.5$, the constants C_1 and C_2 are obtained from Eq. (18) as:

$$C_1 = -\,0.2630553214, \qquad C_2 = -\,0.03804816506 \tag{26}$$

By substituting Eq. (26) into Eq. (24), an expression for $\theta(X)$ is obtained.

5. RESULTS AND DISCUSSION

For various values of porous parameter (S_h), Convective parameter (m) and fin profile parameter (λ) results of the present analysis are compared with numerical solutions obtained by fourth-order Runge–Kutta in Fig. 3 and Table. 1. In these cases, a very good agreement between results is observed, which confirms the validity of the OHAM. This investigation is completed by depicting the effects of some important parameters to evaluate how these parameters influence the temperature variations along axial distance and fin efficiency as well.

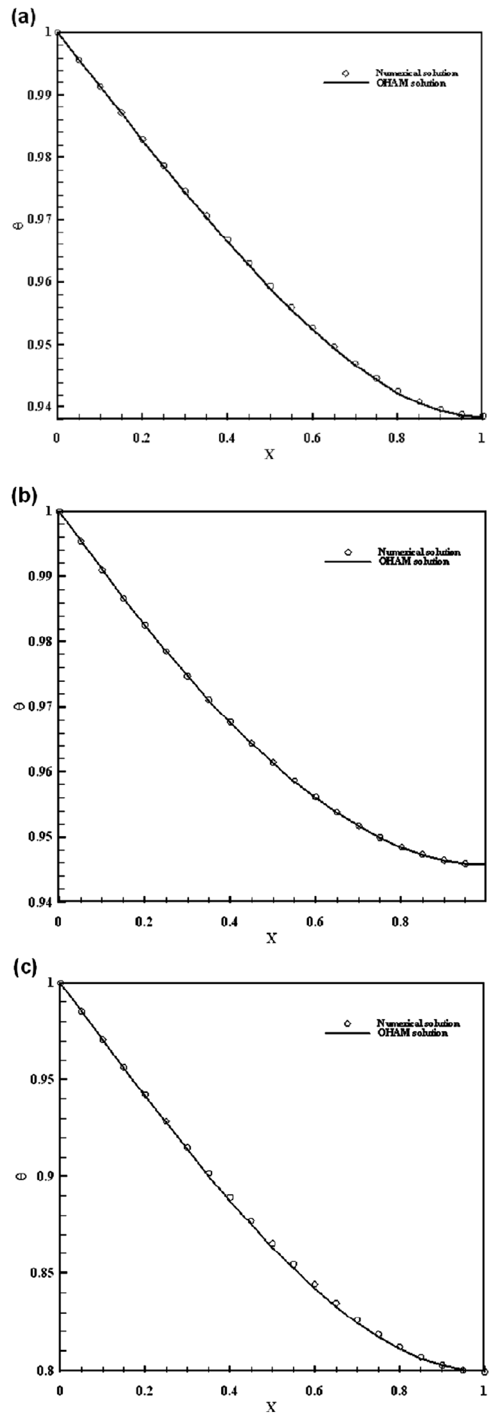

Figure 3: Comparison Between the Solutions via OHAM and Numerical Solution for $\theta(X)$ when (a) $S_h = 0$, $\lambda = 1$, $m = 0.3$ (b) $S_h = 0.1$, $\lambda = 0.5$, $m = 0$ (c) $S_h = 0.1$, $\lambda = 1$, $m = 0.5$

Table 1
The Values of OHAM and Numerical Solution for $\theta(X)$

	$S_h = 0, \lambda = 1, m = 0.3$			$S_h = 0.1, \lambda = 0.5, m = 0$			$S_h = 0.1, \lambda = 1, m = 0.5$		
X	*OHAM solution*	*Numerical solution*	*Relative error*	*OHAM solution*	*Numerical solution*	*Relative error*	*OHAM solution*	*Numerical solution*	*Relative error*
0.00	1.000000	1.000000	0.000000	1.000000	1.000000	0.000000	1.000000	1.000000	0.000000
0.05	0.995670	0.995674	0.000004	0.995407	0.995408	0.000002	0.985223	0.985256	0.000033
0.10	0.991340	0.991369	0.000028	0.990950	0.990958	0.000008	0.970530	0.970682	0.000152
0.15	0.987029	0.987095	0.000067	0.986637	0.986656	0.000019	0.955968	0.956315	0.000347
0.20	0.982753	0.982867	0.000114	0.982478	0.982511	0.000032	0.941594	0.942196	0.000601
0.25	0.978533	0.978699	0.000166	0.978482	0.978531	0.000049	0.927474	0.928368	0.000894
0.30	0.974388	0.974607	0.000219	0.974659	0.974725	0.000066	0.913677	0.914879	0.001201
0.35	0.970340	0.970609	0.000269	0.971019	0.971102	0.000082	0.900280	0.901779	0.001500
0.40	0.966411	0.966723	0.000312	0.967574	0.967671	0.000098	0.887360	0.889124	0.001764
0.45	0.962625	0.962972	0.000347	0.964334	0.964444	0.000110	0.874998	0.876972	0.001974
0.50	0.959006	0.959376	0.000370	0.961311	0.961429	0.000118	0.863279	0.865388	0.002109
0.55	0.955581	0.955961	0.000380	0.958517	0.958639	0.000122	0.852287	0.854441	0.002155
0.60	0.952375	0.952753	0.000377	0.955964	0.956084	0.000120	0.842105	0.844206	0.002102
0.65	0.949420	0.949781	0.000361	0.953665	0.953778	0.000113	0.832818	0.834766	0.001948
0.70	0.946744	0.947076	0.000333	0.951631	0.951731	0.000100	0.824510	0.826208	0.001698
0.75	0.944379	0.944673	0.000294	0.949875	0.949958	0.000083	0.817264	0.818631	0.001367
0.80	0.942360	0.942608	0.000249	0.948410	0.948473	0.000062	0.811161	0.812139	0.000978
0.85	0.940721	0.940922	0.000201	0.947249	0.947289	0.000041	0.806281	0.806849	0.000568
0.90	0.939500	0.939657	0.000156	0.946403	0.946423	0.000020	0.802703	0.802889	0.000186
0.95	0.938737	0.938860	0.000123	0.945887	0.945891	0.000005	0.800502	0.800398	0.000104
1.00	0.938473	0.938582	0.000108	0.945712	0.945710	0.000002	0.799752	0.799531	0.000221

Fig. 4 shows the variation of dimensionless temperature along the fin while porous parameter (S_h) varies from 0.01 to 10. Note that we have chosen m = 0.5 and λ = 1 here. As seen, when the value of S_h increases, there is rapid decrease in the fin temperature at a given axial location. Therefore, as the values of S_h increases, the fin cools down faster and tends to reach the surrounding temperature accordingly.

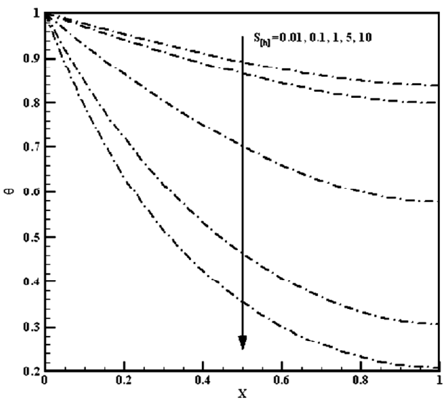

Figure 4: Temperature Distribution Along the Porous Fin with Variable Porous Parameter (S_h) when m = 0.5 and λ = 1

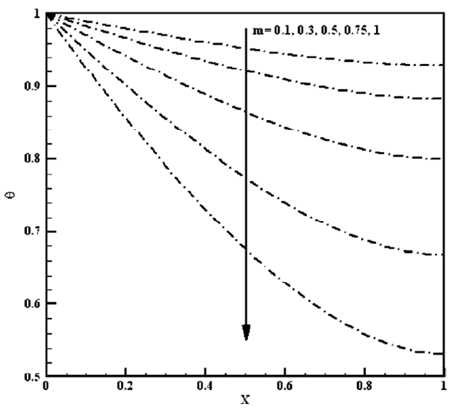

Figure 5: Temperature Distribution Along the Porous Fin with Variable Convective Parameter (m) when S_h = 1 and λ = 1

Fig. 5 indicates the effect of convective parameter (m) on temperature distribution along the fin. It should be noted that values of S_h and λ are constant here as S_h = 1 and λ = 1. As shown in Fig. 5, increase of m from 0.1 to 1 leads to decrease temperature along the fin. The influence of different value of λ on temperature distribution is illustrated in Fig. 6. Here we set S_h = 1 and m = 0.5. It is observed that temperature is decreasing when λ varies from 0.5 to 2.5; so this matter brings about the fin cools down faster. In addition, it is seen that these profiles satisfy the boundary condition at insulated tip where $\theta(1) = 0$ asymptotically, which support the obtained results by means of OHAM anyway.

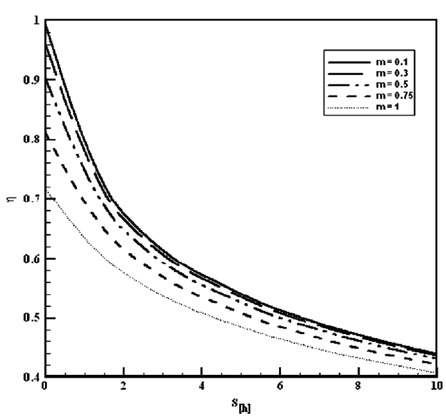

Figure 6: Variation of the Fin Efficiency with the Porous Parameter (S_h) for Different Values of the Convective Parameter (m) when λ = 1

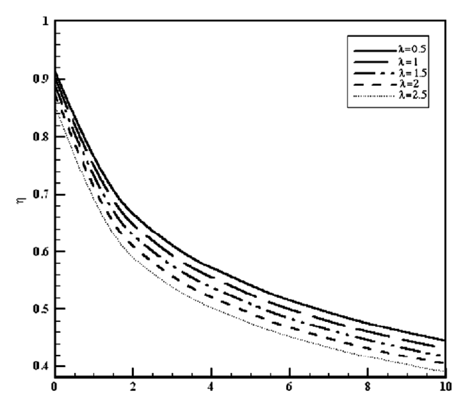

Figure 7: Variation of the Fin Efficiency with the Porous Parameter (S_h) for Different Values of the Fin Profile Parameter (λ) when m = 0.5

Figs. 6-7 represent the variation of the fin efficiency with S_h for different magnitudes of m and λ. From both figs, one may realize that increasing the value of S_h leads to deterioration fin efficiency. Furthermore, it is found that as the value of m decreases from 1 to 0.1, the fin efficiency improves at a given value of S_h. Similarly, the fin efficiency would increase with decreasing the value of λ.

6. CONCLUSION

In this paper, thermal performance of a convective porous fin with variable cross section has been investigated. A second order non-linear ordinary differential equation has been derived as the governing equation, and then solved using the OHAM. To check the validity of OHAM results, numerical solutions via fourth grade order Runge–Kutta was employed and a very excellent agreement between the solutions obtained from OHAM was observed.

The effects of different parameters namely, porous parameter S_h, convective parameter m and fin profile parameter ë on temperature variation and fin efficiency were investigated. It was found that increasing S_h by increasing either m or λ decreases dimensionless temperature profile as well as fin efficiency. Furthermore, it could be concluded that OHAM has a great reliability inasmuch as it is effective tool in solving nonlinear differential equation arising in convective porous fin with variable cross section.

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