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TEMPERATURE DISTRIBUTION DEFINITION IN ONE–DIMENSIONAL TRANSIENT COOLING IN A THREE–LAYER SLAB USING ORTHOGONAL EXPANSION TECHNIQUE

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Received: 17th January 2019, Accepted: 27th April 2019

Abstract: *In this paper, temperature distribution definition of a three–layer slab with one–dimensional transient cooling was presented via the analytical solution of Orthogonal Expansion Technique (OET), which results in the exact solution for this form of heat conduction problem. A three–layer slab consisting of three different materials with different thermal diffusivities and conductivities is considered and the results of OET were compared with those of the Crank-Nicolson method which is a Numerical method. Results show that the Orthogonal Expansion Technique(OET) is strongly capable of solving problems of temperature distribution definition of one-dimensional transient cooling of multi-layer media.*

Keywords: *Three layer slab, Orthogonal Expansion Technique, Transient Heat Conduction, Exact solution, Crank-Nicolson, Numerical method.*

1. INTRODUCTION

It is of interest for engineering calculations to propose simple mathematical formulations to deal with heat conduction problems. The transient–temperature distribution in a composite medium consisting of several layers in contact has numerous applications in industry [1-3]. Typical applications may be found in the thermal study of building envelopes, metallurgical processes involving heat treatment of steel strips and in many other industries. These media are categorized into three groups; “slab”, “cylinder” and “sphere”.

In this paper, the mathematical formulation of one–dimensional transient heat conduction in a composite medium consisting, three parallel layers of slabs is presented. The analysis of one-dimensional transient heat condition in a composite slab consisting of several different layers in contact may be performed following different analytical approaches:

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- *The Orthogonal Expansion Technique [4-7]:* It is particularly suitable for solving transient problems of composite medium of finite thickness without internal heat generation (exact closed-form solution).This technique was developed first by Vodicka[4].
- *The Quasi-Orthogonal Expansion Technique [8,9]:* It solves the same composite domain problems listed before (exact closed-form solution). This technique was independently developed first by Tittle[8].
- *The Laplace Transform method [10]:* This method is convenient for the solution of unsteady problems of composite medium involving regions of infinite and semi infinite thickness (exact closed-form solution).
- *The Green's function approach [10-13]:* Transient problems on conduction of heat in composite solids with energy generation are usually best solved by this approach (exact closed-form solution).
- *The Galerkin procedure [14-16]:* This procedure may be used for accurately solving several transient conduction problems in composite geometries. Although it gives an approximateclosed-form solution (quite accurate), it offers the benefits and limitations one expects from an exact solution.
- *The Finite Integral Transform Technique [17]:* It allows the unsteady problem for a multi-region medium with time-dependent heat transfer coefficient to be solved (approximate closed-form solution).

The book by Ozisik [18] contains a detailed review (in Chapter 8) of one-dimensional composite media, focusing on orthogonal expansions, Green's functions and Laplace transform techniques. Recently, F.de Monte [19] has developed a solution for the two-layer problem using a 'natural' eigenfunction expansion method. He also provides a detailed and wellwritten introduction in which various solution methods are described and compared. The global (or 'natural') eigenfunction expansion method has the advantage of making ''the solution consistent with the physical reality of the problem'' because the ''transient response the solid to changes in the outer boundary conditions is strictly linked to the thermal diffusivity'' [20]. The method is both ''efficient and simple'' in the sense that the problem formulation is intuitive and its solution by eigenfunction expansions is straightforward.

We generalized this solution method to the three-layer problem. Our three-layer model describes the simplified case in which all heat transfer occurs only by conduction (no convection, no radiation, no heat generation, no combustion [21]).

The layers are initially in temperatures with the distributions respectively defined by $F_1(x)$, $F_2(x)$ and $F_3(x)$. We want to find temperature distribution for each layer when the boundaries of the slab are at specific temperatures. The mathematical formulation of this heat conduction problem is given by[18]:

$$\begin{cases} \alpha_i \dfrac{1}{x^p}\dfrac{\partial}{\partial x}\left(x^p \dfrac{\partial T_i}{\partial x}\right) = \dfrac{\partial T_i(x,t)}{\partial t} \\ x_i < x < x_{i+1} \\ t > 0, i = 1,2,3 \end{cases} \tag{1}$$

where,

$$p = \begin{cases} 0 & \text{slab} \\ 1 & \text{cylinder} \\ 2 & \text{sphere} \end{cases}$$

And

$\alpha =$ $\alpha = \frac{k_i}{c_i \rho_i}$ where, i, is the number of each layer, α, is thermal diffusivity, $K\left(\frac{W}{m.C^\circ}\right)$ is the heat conductivity, $c\left(\frac{J}{Kg.C^\circ}\right)$ is the specific heat and $\rho\left(\frac{Kg}{m^3}\right)$ is the density.

It will be shown that the orthogonal expansion technique (OET) will lead us to transient temperature distribution in the slab for some different eigenvalues.

2. BASIC CONCEPTS

Consider an S–layer slab in which the S layers are in perfect thermal contact. Initially, the layers are at temperatures $F_1(x)$, $F_2(x)$, $F_3(x)$,… $FS(x)$. For times $t > 0$, the two outer boundaries at $x = 0$ and $x = x_s$ are kept at T^*. Now, we try to obtain temperature distribution in the medium[22].

From Eq. (1), we have

$$\alpha_i \frac{\partial^2 T}{\partial X^2} = \frac{\partial T_i(x,t)}{\partial t}, \tag{2}$$

$x_i < x < x_{i+1}$,

$t > 0$, $i = 1, 2, \ldots, S$.

Subject to the initial and boundary conditions, we have:

$$T(x, 0) = F(x) \tag{3}$$

$$T_1(0, t) = T^*, \; t > 0 \tag{4}$$

$$T_S(x_{s+1}, t) = T^*, \; t > 0, \tag{5}$$

$$T_i(x_{i+1}, t) = T_{i+1}(x_{i+1}, t), \; t > 0, \tag{6}$$

$$K_i \frac{\partial T_i(x,t)}{\partial x} = K_{i+1} \frac{\partial T_{i+1}(x,t)}{\partial x}, \; t > 0, \text{ at the interface } x = x_{i+1}, \tag{7}$$

We know that Eq. (6) implies the continuity of temperature or perfect thermal contact at the interfaces.

To solve the above heat conduction problem, the variables are separated as follows:

$$T_i(x, t) = \psi_i(x)\Gamma(t) \tag{8}$$

When Eq. (8) is substituted in Eq. (2), we obtain:

$$\alpha_i \frac{1}{\psi_i(x)} \frac{d}{dx}\left(\frac{d\psi_i}{dx}\right) = \frac{1}{\Gamma(t)} \frac{d\Gamma(t)}{dt} \equiv -\beta^2 \tag{9}$$

where, β is the separation constant.

The separation given by Eq. (9) results in the following ordinary differential equations to determine the functions $\Gamma(t)$ and $\psi_i(\beta, x)$:

$$\frac{d\Gamma(t)}{dt} + \beta_n^2 \Gamma(t) = 0, \qquad t > 0. \tag{10}$$

$$\frac{d^2\psi_{in}}{dx^2} + \frac{\beta_n^2}{\alpha_i}\psi_{in} = 0, \quad x_i < x < x_{i+1}, \qquad i = 1, 2, \ldots, S. \tag{11}$$

where, $\Psi_{in} = \Psi_i(\beta_n, x)$. The subscript n indicates that there is an infinite number of discrete values of the eigenvalues $\beta_1 < \beta_2 .... < \beta_n < ....$ and the corresponding eigenfunctions ψ_{in}.

The boundary conditions for Eq (11) are obtained by substituting Eq. (8) in the boundary conditions (11) constitute an eigenvalue problem to determine the eigenvalues β_n and the corresponding eigenfunctions ψ_{in} The solution for the time–variable function $\Gamma(t)$ is simply obtained through Eq. (10), in the form of:

$$\Gamma(t) = e^{-\beta_n^{2t}} \tag{12}$$

And the general solution for the temperature distribution *Ti* (*x*, *t*) in any region *i*, is constructed as:

$$T_i(x, t) = \sum_{n=1}^{\infty} o_n e^{-\beta_n^{2t}} \psi_{in}(x), \qquad i = 1, 2, \ldots S. \tag{13}$$

We now constrain the solution to satisfy the initial condition of Eq. (3), and obtain the following equation:

$$F_i(x) = \sum_{n=1}^{\infty} o_n \psi_{in}(x), \qquad x_i < x < x_{i+1}, \qquad i = 1, 2, \ldots S. \tag{14}$$

Operating $\frac{k_i}{\alpha_i}\int_{x_i}^{x_{i+1}} \Psi_{ir}(x)dx$ on both sides of Eq. (14) and summing up the three resulting expressions and also knowing that [7]:

$$\sum_{i=1}^{s} \frac{k_i}{\alpha_i}\int_{x_i}^{x_{i+1}} \psi_{in}(x)\psi_{ir}(x)dx = \begin{cases} 0 & n \neq r \\ N_n & n = r \end{cases} \tag{15}$$

the coefficients *cn* can be determined as:

$$c_n = \frac{1}{N_n}\sum_{i=1}^{s} \frac{k_i}{\alpha_i}\int_{x_i}^{x_{i+1}} \psi_{in}(x)F_i(x)dx$$

Now, the solution takes the form of:

$$T_{i(x,\,t)} = \sum_{n=1}^{\infty} e^{-\beta_n^{2t}} \frac{1}{N_n}\psi_{in}(x)\sum_{j=1}^{s}\frac{k_j}{\alpha_j}\int_{x_j}^{x_{j+1}} \psi_{jn}(x)F_j(x)dx, \qquad \begin{matrix} x_i < x < x_{i+1} \\ i = 1, 2, \ldots, S \end{matrix} \tag{16}$$

where the norm *Nn* is defined as in Eq. (15).

We need to define the eigenvalues, β_n, and the eigenfunctions, ψ_{in}, to determine *Ti*(*x*, *t*) absolutely. The general solution, ψ_{in}, of the eigenvalue problem given by Eqs. (4-7) and (11) can be written in the form of:

$$A_{in}\phi_{in}(x) + B_{in}\theta_{in}(x), \qquad \begin{matrix} x_i < x < x_{i+1} \\ i = 1, 2, \ldots, S \end{matrix} \tag{17}$$

where, $\phi_{in}(x)$ and $\theta_{in}(x)$ are the two linearly independent solutions of Eq. (11), and finally, Ain and Bin are the coefficients. Table 1 lists the functions $\phi_{in}(x)$ and $\theta_{in}(x)$ for slabs, cylinders and spheres. The heat conduction problem of an S–layer composite medium, generally, involves S solutions in the form given by Eq. (17). Thus, there are 2S arbitrary coefficients of A_{in} and B_{in}, i = 1,2,…, S to be determined. The boundary conditions (4-7) provide us with them. For simplicity, Ain is assumed to be unity without the loss of generality. Finally an additional relation is needed to define the eigenvalues, β_n. The determinant of these 2S coefficients is to be vanished to result in nontrivial solutions so that the above additional relation is obtained. All the existing parameters and variables of Eq. (16) are defined to consequently determine $Ti(x, t)$.

Table 1
Linearly Independent Solutions, $\phi_{in}(x)$ and $\theta_{in}(x)$, of Eq. (11) for Slabs, Cylinders and Spheres.

Geometry	$\phi_{in}(x)$	$\theta_{in}(x)$
Slab	$\sin\left(\frac{\beta_n}{\sqrt{\alpha_i}}x\right)$	$\cos\left(\frac{\beta_n}{\sqrt{\alpha_i}}x\right)$
Cylinder	$J_0\left(\frac{\beta_n}{\sqrt{\alpha_i}}x\right)$	$Y_0\left(\frac{\beta_n}{\sqrt{\alpha_i}}x\right)$
Sphere	$\frac{1}{x}\sin\left(\frac{\beta_n}{\sqrt{\alpha_i}}x\right)$	$\frac{1}{x}\cos\left(\frac{\beta_n}{\sqrt{\alpha_i}}x\right)$

3. APPLICATION

Now we consider a specific form of three–layer slab consisting of the first layer $0 \le x(meter) \le 0.12$, the second layer $0.12 \le x(meter) \le 0.22$ and the third layer $0.22 \le x(meter) \le 0.3$ which are in perfect thermal contact (figure 1). Initially, let $F_1(x) = 1666.67x + 350$, $F_2(x) = -1000x + 670$, and $F_3(x) = -1250x + 725$. When $t > 0$, the two outer boundaries at $x = 0$ and $x = 0.3(m)$ are kept at $350(K)$. Now, we try to obtain temperature distribution in the medium.

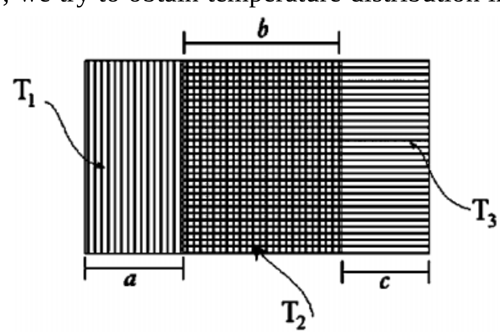

Figure 1: A Schematic View of a Three-layer Slab

According to table 2 which lists the physical properties of each layer, the equations now take the following form:

Table 2
Physical Properties of each Layer

	Material	$\rho(kg/m^3)$	$k(J/m.s.K)$	$c(J/Kg.K)$	$l(m)$
Layer 1	Chrome steel 5%	7833	40	460	0.12
Layer 2	Pure Iron	7897	73	452	0.1
Layer 3	Pure Aluminum	2707	204	896	0.8

From Eq. (1), we have:

$$\begin{cases} \alpha_i \dfrac{\partial^2 T_i}{\partial x^2} = \dfrac{\partial T_i(x,t)}{\partial t} \\ x_i < x < x_{i+1} \\ t > 0, i = 1,2,3 \end{cases} \tag{18}$$

Subject to the initial and boundary conditions, we have:

$$T_i(x,\ 0) = F_i(x), \tag{19}$$

$$T_1(0,\ t) = 350K, \qquad t > 0, \tag{20}$$

$$T_3(0.3,\ t) = 350K, \qquad t > 0, \tag{21}$$

$$T_1(0.12,\ t) = T_2(0.12,\ t), \qquad t > 0, \tag{22}$$

$$T_2(0.22,\ t) = T_3 = (0.22,\ t), \qquad t > 0, \tag{23}$$

$$K_1 \frac{\partial T_1(x,t)}{\partial x} = K_2 \frac{\partial T_2(x,t)}{\partial x},\ t > 0, \text{ at the first interface } x = 0.12, \tag{24}$$

$$K_2 \frac{\partial T_2(x,t)}{\partial x} = K_3 \frac{\partial T_3(x,t)}{\partial x},\ t > 0, \text{ at the second interface } x = 0.22, \tag{25}$$

Since I = 1,2,3 and considering Eq. (20) and Eq. (17) takes the form of:

$$\psi_{1n}(x) = \sin\left(\frac{\beta_n}{\sqrt{\alpha_1}} x\right) \tag{26}$$

$$\psi_{2n}(x) = A_{2n} \sin\left(\frac{\beta_n}{\sqrt{\alpha_2}} x\right) + B_{2n} \cos\left(\frac{\beta_n}{\sqrt{\alpha_2}} x\right) \tag{27}$$

$$\psi_{3n}(x) = \alpha_i = \frac{k_i}{c_i \rho_i}. \tag{28}$$

Where $\alpha_i = \dfrac{k_i}{c_i \rho_i}$.

According to Eqs. (26-28) and (12) the boundary conditions (20-25) change into the matrix form of:

$$\Delta . x = \Omega \tag{29}$$

where,

$$\Delta = \begin{bmatrix} 0 & 0 & \sin\left(\frac{0.3\beta_n}{\sqrt{\alpha_3}}\right) & \cos\left(\frac{0.3\beta_n}{\sqrt{\beta_3}}\right) \\ \sin\left(\frac{0.22\beta_n}{\sqrt{\alpha_2}}\right) & \cos\left(\frac{0.22\beta_n}{\sqrt{\alpha_2}}\right) & -\sin\left(\frac{0.22\beta_n}{\sqrt{\alpha_3}}\right) & -\cos\left(\frac{0.22\beta_n}{\sqrt{\alpha_3}}\right) \\ -\frac{K_2}{\sqrt{\alpha_2}}\cos\left(\frac{0.12\beta_n}{\sqrt{\alpha_2}}\right) & \frac{K_2}{\sqrt{\alpha_2}}\sin\left(\frac{0.12\beta_n}{\sqrt{\alpha_2}}\right) & 0 & 0 \\ \frac{K_2}{\sqrt{\alpha_2}}\cos\left(\frac{0.22\beta_n}{\sqrt{\alpha_2}}\right) & -\frac{K_2}{\sqrt{\alpha_2}}\sin\left(\frac{0.22\beta_n}{\sqrt{\alpha_2}}\right) & -\frac{K_3}{\sqrt{\alpha_3}}\cos\left(\frac{0.22\beta_n}{\sqrt{\alpha_3}}\right) & \frac{K_3}{\sqrt{\alpha_3}}\sin\left(\frac{0.22\beta_n}{\sqrt{\alpha_3}}\right) \end{bmatrix} \tag{30}$$

$$X = \begin{bmatrix} A_{2n} \\ B_{2n} \\ A_{3n} \\ B_{3n} \end{bmatrix} \tag{31}$$

$$\Omega \begin{bmatrix} 0 \\ 0 \\ -\frac{K_1}{\sqrt{\alpha_1}}\cos\left(\frac{0.12\beta_n}{\sqrt{\alpha_1}}\right) \\ 0 \end{bmatrix} \tag{32}$$

From Eq. (29), A2n, B2n, A3n and B3n are found through Kramer’s method[18]:
where

$$B_{2n} = \frac{10^8}{|\Delta|}\cos(36.02\beta)[2.67\cos(48.65\beta)\cos(87.23\beta) - 1.94\sin(48.65\beta)\sin(87.23\beta)] \tag{33}$$

$$B_{2n} = -\frac{10^8}{|\Delta|}\cos(36.02\beta)[2.67\sin(48.65\beta)\cos(87.23\beta) + 1.94\cos(48.65\beta)\sin(87.23\beta)] \tag{34}$$

$$A_{3n} = \frac{1.94*10^8}{|\Delta|}\cos(36.02\beta)\cos(111.22\beta) \tag{35}$$

$$B_{3n} = -\frac{1.94*10^8}{|\Delta|}\cos(36.02\beta)\sin(111.22\beta) \tag{36}$$

$$|\Delta| = 3.91*10^8 \cos(87.23\beta)\cos(22.11\beta) - 2.61*10^8 \sin(22.11\beta)\sin(87.23\beta) \quad (37)$$

To determine eigenvalues, β_n, we set $|\Delta| = 0$. Hence, all the parameters and variables are found and can be inserted into the following equation to determine $Ti\ (x, t)$.

$$N_n = \sum_{i=1}^{3} \frac{k_i}{\alpha_i} \int_{x_1}^{x_{i+1}} \left(\psi_{in}(x)\right)^2 dx. \quad (38)$$

$$T_{i(x,t)} = \sum_{n=1}^{\infty} e^{-\beta_n^2 t} \frac{1}{N_n} \psi_{in}(x) \sum_{j=1}^{3} \frac{k_j}{\alpha_j} \int_{x_j}^{x_{j+1}} \psi_{jn}(x') F_j(x') dx', \begin{matrix} x_i < x < x_{i+1} \\ i = 1,2,3 \end{matrix} \quad (39)$$

4. THE COMPARISON OF THE RESULTS OF NUMERICAL METHOD AND ORTHOGONAL EXPANSION TECHNIQUE(OET)

Eqs. (18-25) have been solved using Crank-Nicolson method [23] as a numerical method for four time intervals of 1.5s ($\Delta t = 1.5s$), taking 16 nodes ($x = 0.02m$) and resulting in 64 pieces of data. One can now list the results as in the form of table 3 as the theoretical temperatures. To have numerical values from OET, the first 50 roots of Eq. (37) were calculated as eigenvalues and substituted into Eq. (39), and for four different times of $t = 0$, $t = 1.5$s, $t = 3$s, and $t = 4.5$s, the results of Eq. (39) were compared with those of the numerical method and shown in Figure 2. Finally, it was shown that OET is appropriately applicable to problems of temperature distribution definition in one dimensional heat transient cooling of three-layer slabs to obtain the exact solution.

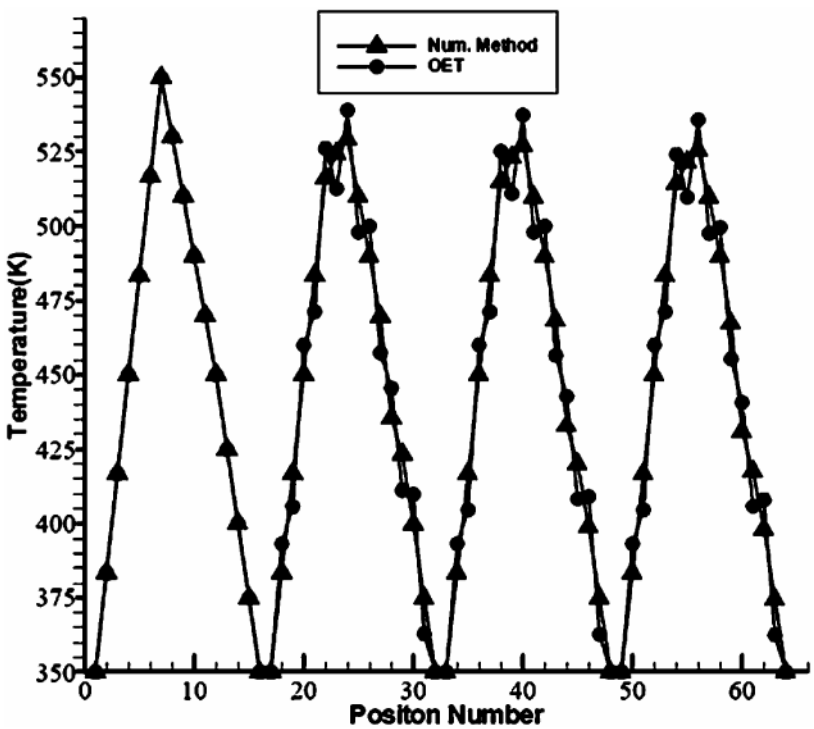

Figure 2: The Comparison of the Results of Orthogonal Expansion Technique and the Numerical Method in Solving Eqs. (18.25).

Table 3
Theoretical Temperatures(k) and Label of Nodes used for Eqs. (20-25)

X(mm)	*T at t = 0s*	*T at t = 1.5s*	*T at t = 3s*	*T at t = 4.5s*
0	u_1 = 350	u_{17} = 350	u_{33} = 350	u_{49} = 350
20	u_2 = 383.33	u_{18} = 383.3304	u_{34} = 383.3307	u_{50} = 383.331
40	u_3 = 416.67	u_{19} = 416.6696	u_{35} = 416.6692	u_{51} = 416.6688
60	u_4 = 450	u_{20} = 449.9998	u_{36} = 449.9986	u_{52} = 449.9952
80	u_5 = 483.33	u_{21} = 483.3202	u_{37} = 483.2808	u_{53} = 483.2046
100	u_6=516.67	u_{22} = 516.1601	u_{38} = 515.151	u_{54} = 514.1628
120	u_7 = 550	u_{23} = 524.5138	u_{39} = 523.0289	u_{55} = 521.6375
140	u_8 = 530	u_{24} = 529.0911	u_{40} = 527.3455	u_{56} = 525.7332
160	u_9 = 510	u_{25} = 509.9669	u_{41} = 509.8405	u_{57} = 509.6041
180	u_{10} = 490	u_{26} = 489.9803	u_{42} = 489.9014	u_{58} = 489.7447
200	u_{11} = 470	u_{27} = 469.4799	u_{43} = 468.4199	u_{59} = 467.3392
220	u_{12} = 450	u_{28} = 435.4154	u_{44} = 432.8605	u_{60} = 430.8286
240	u_{13} = 425	u_{29} = 423.2256	u_{45} = 420.1359	u_{61} = 417.7636
260	u_{14} = 400	u_{30} = 399.7842	u_{46} = 399.0706	u_{62} = 397.9959
280	u_{15} = 375	u_{31} = 374.9741	u_{47} = 374.8492	u_{63} = 374.5698
300	u_{16} = 350	u_{32} = 350	u_{48} = 350	u_{64} = 350

5. CONCLUSIONS

In this paper, by considering the results of the comparison of applying the Crank-Nicolson method as a numerical method and Orthogonal Expansion Technique (OET) to temperature distribution definition in one–dimensional transient cooling in a three–layer slab, it was shown that the orthogonal expansion technique (OET) is strongly capable of solving problems of temperature distribution definition of one–dimensional transient cooling of multi–layer media, and that increasing the number of the eigenvalues will converge to the exact solution for this specific form of problem.

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