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STEADY LAMINAR NATURAL CONVECTION IN ISOSCELES TRIANGULAR ENCLOSURES FOR DIFFERENT BOUNDARY CONDITIONS

S. M. Moghimi[1], Mirgolbabae*[2] & M. Hosseini[1]
[1]*Department of Mechanical Engineering, Islamic Azad University, Ghaemshahr branch, Ghaemshahr,P.O. Box163, Iran*
[2]*Department of Mechanical Engineering, Islamic Azad University, Jouybar branch, Jouybar, Iran*

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Abstract: *This study is concerned with steady state laminar natural convection in air-filled, 2-D isosceles triangular enclosures subject to either cooling or heating at inclined walls corresponding to the heating or cooling at base wall, respectively. The simulation is performed by developing a FORTRAN code based on finite volume method. As regards the symmetrical shape of the cavity, the study addresses just half of the full domain, using adiabatic condition for the geometrical symmetry plane. Constant temperatures for hypotenuse and base walls have been assumed. SIMPLEC scheme is adopted for pressure-velocity coupling. Various cavity aspect ratios, and the Grashof numbers based on the cavity height, in the range 1.00e3-5.00e5, and 0.25-1, respectively, are considered to discuss on their influence on the flow patterns, the temperature distributions and the heat transfer rates. Finally, the results are compared to the measured data of the former studies and excellent agreement is observed.*

Keywords: Natural Convection, Triangular Enclosure, Heat Transfer, Numerical Simulation

1. INTRODUCTION

Natural convection in enclosures has been extensively studied both experimentally and numerically, finding applications in many domestic and industrial systems as well as in geophysical flows. One application which ahs attracted a great deal of attention is natural convection heat transfer through attics of buildings. The geometry of the attics of domestic and industrial buildings usually has an isosceles triangular cross-section with two inclined surfaces (the roof) and a horizontal surface (the ceiling or base). Considering the thermal isolation of doors and window's of buildings, Batchelor [1] perceived that three prevailing factors on convection heat transfer are aspect ratio, Prandtle number and Rayleigh number. Flack and Witt [2] investigated the effect of varying the angle between inclined sides of the triangular enclosures experimentally, and obtained the velocity gradient. In their other study,

* Corresponding Author: *hesam_peeter@yahoo.com*

Flack et al. [3] concluded that under summer conditions (hot top, cold base) the air flow remains laminar, convection near the walls is minimum and the aspect ratio at a given Grashof number has very little influence over Nusselt number. As for the winter conditions (cold top, hot base), Flack shows the flow changes from laminar to turbulent as Gr increments and most of the heat transfer through the base is realized via convection. Polikakus and Bejan [4] studied the triangular enclosures with small height experimentally, considering the cold condition for inclined wall and investigated the temperature and velocity profiles. Ghasemi and Roux [5] studied natural convection numerically in triangular enclosures considering different angles for hypotenuse wall ranging between 30° and 60° and Grashof number between 10^3 and 10^6. They considered both summer and winter conditions and investigated the velocity and temperature field as well as Nusselt number.

The objective of the present study is to investigate the effect of Grashof number and aspect ratio on the heat transfer rate and flow pattern into the air-filled, 2-D isosceles triangular enclosures subject to different thermal boundary conditions; summer day and winter day boundary conditions. Lei et al. [6] considered transient natural convection in a water-filled isosceles triangular enclosure subject to cooling at the inclined surfaces and simultaneous heating at the base. They visualized the unsteady flows over a range of Grashof numbers using a shadowgraph technique, and corresponding numerical simulations were carried out using a Finite Volume Method. They found that for a fixed aspect ratio of 0.5, a transition of the unsteady flow from symmetric to asymmetric structures occurs for Grashof numbers above 2.95×10^4. Moreover, their heat transfer calculations indicate that the average Nusselt number over the inclined and horizontal surfaces approximately scales with $Gr^{0.2}$.

Mathematical Modeling

If we are to analyze the symmetrical shape, it's sensible to analyze just half of the domain, reducing the amount of calculations should be performed at a time. Although a full isosceles triangular domain was adopted in some early studies [7, 8], a number of investigations [9-13] have adopted a right-angled triangle with an adiabatic vertical wall, which effectively constituted only half of the full domain. A presumption of the half-domain model was that the flow in the attic was symmetric about the geometric mid-plane (Fig. 1). Fundamentally, the isolation and thermal boundary conditions are defined the way we could examine the heat transfer through ceiling in both winter and summer conditions. Since there is no agent into enclosure to circulate the air stream, heat transfer is due to natural convection.

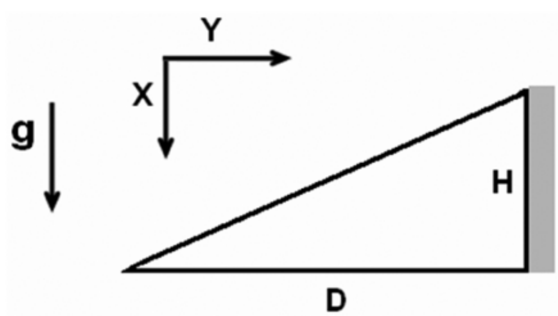


Figure. 1: Sketch of the Geometry and Coordinate System of the Cavity

Two different boundary conditions are mainly considered: a) summer condition: the temperature of the hypotenuse wall (T_H) is greater than the base temperature (T_L), b) winter condition: the temperature of the base wall (T_H) is greater than the hypotenuse temperature (T_L). Therefore, according to the winter day and summer day condition, heat is transferred from the base wall through the air to the inclined wall, and inclined wall to the base wall, respectively, by combined conduction(adjacent to the walls) and convection processes. The study is aimed at obtaining the variation of temperature and velocity contour corresponding to change of aspect ratio, $Ar = \frac{H}{2D}$ and Grashof number. Aspect ratios considered into the study are in the range 0.25-1.00 and the Grashof number based on height of the enclosure is in the range 1.00e3-5.00e5. Prandtl number value of air at atmospheric pressure and temperature 35°C is assumed 0.71 for all models [14].

Fundamental equations for fluid flow, continuity, and energy are solved using FORTRAN programming software. The buoyancy-driven flow is considered to be two dimensional, steady and laminar. The fluid is assumed to be incompressible, with constant physical properties and negligible viscous dissipation. We have the following set of governing equations, for which the Boussinesq assumption [15] has been made:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{1}{\rho}\frac{\partial P}{\partial X} + \upsilon\left[\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right] + g\beta\left(T - T_c\right) \tag{2}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{1}{\rho}\frac{\partial P}{\partial Y} + \upsilon\left[\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right] \tag{3}$$

$$U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \alpha\left[\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right] \tag{4}$$

With the following boundary conditions

$$X = H;\ 0 < Y < D : U = V = 0,\ T = T_C \ \ or\ \ T_H \tag{5}$$

$$X = YH/D;\ Y = DX/H : U = V = 0,\ T = T_H \ \ or\ T_C \tag{6}$$

$$Y = D;\ \ 0 < X < H : U = V = 0,\ \frac{\partial T}{\partial Y} = 0 \tag{7}$$

The nondimensional parameters are introduced as follows:

$$x = \frac{X}{H},\ y = \frac{Y}{H},\ u = \frac{U}{\upsilon/H},\ v = \frac{V}{\upsilon/H},\ \theta = \frac{T - T_c}{T_h - T_c},\ p = \frac{P}{\rho g\left(T_h - T_c\right)\beta H} \tag{8}$$

And using these non dimensional variables, the following set of governing equations is obtained:

$$\frac{1}{Gr_H}\frac{\partial}{\partial x}\left[uu - \frac{\partial u}{\partial x}\right] + \frac{1}{Gr_H}\frac{\partial}{\partial y}\left[uv - \frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} - \theta \tag{9}$$

$$\frac{1}{Gr_H}\frac{\partial}{\partial x}\left[uv-\frac{\partial v}{\partial x}\right]+\frac{1}{Gr_H}\frac{\partial}{\partial y}\left[vv-\frac{\partial v}{\partial y}\right] = -\frac{\partial p}{\partial y} \tag{10}$$

$$\frac{\partial}{\partial x}\left[u\theta-\frac{1}{pr}\frac{\partial\theta}{\partial x}\right]+\frac{\partial}{\partial y}\left[v\theta-\frac{1}{pr}\frac{\partial\theta}{\partial y}\right] = 0 \tag{11}$$

Through the introduction of the non dimensional parameter into the physical boundary conditions mentioned above the following boundary conditions are obtained:

$$x = Ar;\ 0 < y < 1 : u = v = 0, \theta = 1 \text{ or } 0 \tag{12}$$

$$x = Dx/H\ ;\ y = Hy/D : u = v = 0, \theta = 1 \text{ or } 0 \tag{13}$$

$$y = 1;\ 0 < x < Ar : u = v = 0, \frac{\partial\theta}{\partial y} = 0 \tag{14}$$

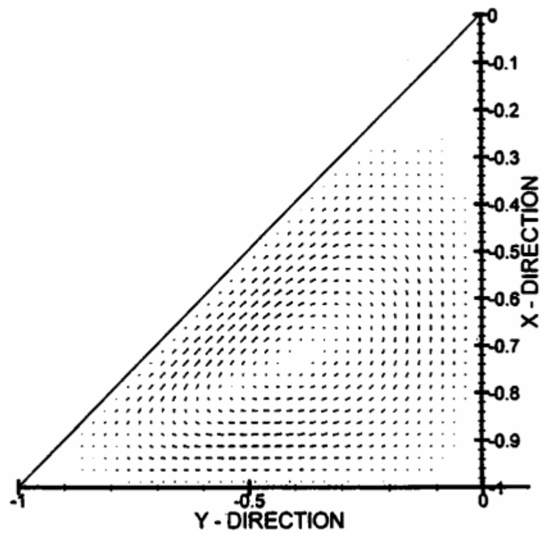
where $Gr_H = g\beta(T_H - T_C)H^3/\upsilon^2$ is the Grashof number based on the cavity height. To obtain the velocity and temperature fields the above mentioned governing equations, Equations (9)–(11), with the specified boundary conditions are solved using a finite volume method. A power law scheme is adopted for the convection–diffusion formulation [16]. Pressure-velocity coupling is handled adopting the SIMPLER algorithm (semi implicit method for pressure linked equations revised) described in full details by Patankar [16]. For the discretization of the flow domain, quadrilateral meshes are constructed within the flow domain. The discretized equations obtained are solved iteratively, using a line-by-line application of the TDMA algorithm [16]. Under relaxation factor is used to ensure the convergence of the solution procedure, that its value for both momentum and energy equation was 0.4 and for pressure correction equation is 0.5. The solution is considered to be fully converged when the absolute value of the average residual for each grid is smaller than a prescribed value, i.e., 10^{-7}. Grid-dependence tests have been conducted and it has been concluded that the grid size of 20×20 is acceptable one so that all calculations for models are performed using this number of grids.

2. RESULTS AND DISCUSSION

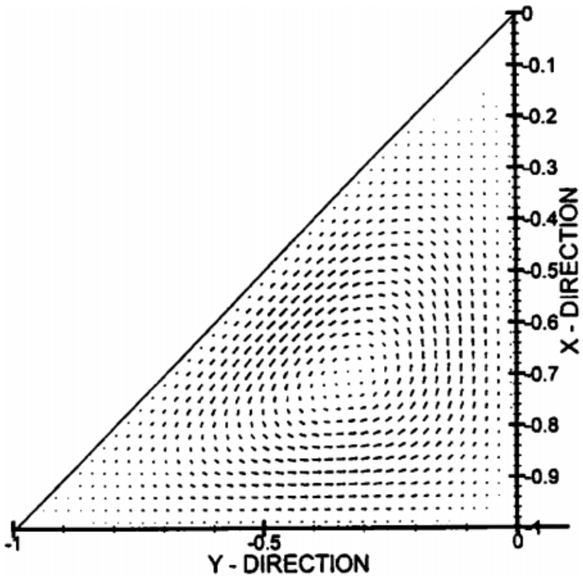
Numerical simulations are performed for Pr = 0.71 (air is the working fluid) and different values of both the Grashof number in the range $10^3 \le Gr_H \le 10^6$, and the aspect ratio of the cavity ranging $0.25 \le Ar \le 1$. In order to point out the influence of Gr_H and Ar upon the flow structure type and the temperature distributions throughout the cavity, results for three different qualities for Gr_H, 1.00e5, 4.00e4, 5.00e3, and three different Ar are reported in terms of temperature gradient and streamlines.

Streamline Investigation

Considering the summer thermal conditions for enclosure, the air adjacent to the inclined wall warms up and because of the buoyancy driven force, it moves up and then go down along the vertical wall (symmetric geometry plane)(Fig. 2). In left hand bottom tip, because of the adjacency of two walls and consequently the influence of two boundary layers on each other, the stream velocity decreases. The velocity vectors for the model with Ar = 1 and two different boundary conditions are shown in figure 2. As it's seen the flow pattern in different conditions are the same and just the direction of the flows is inverse.



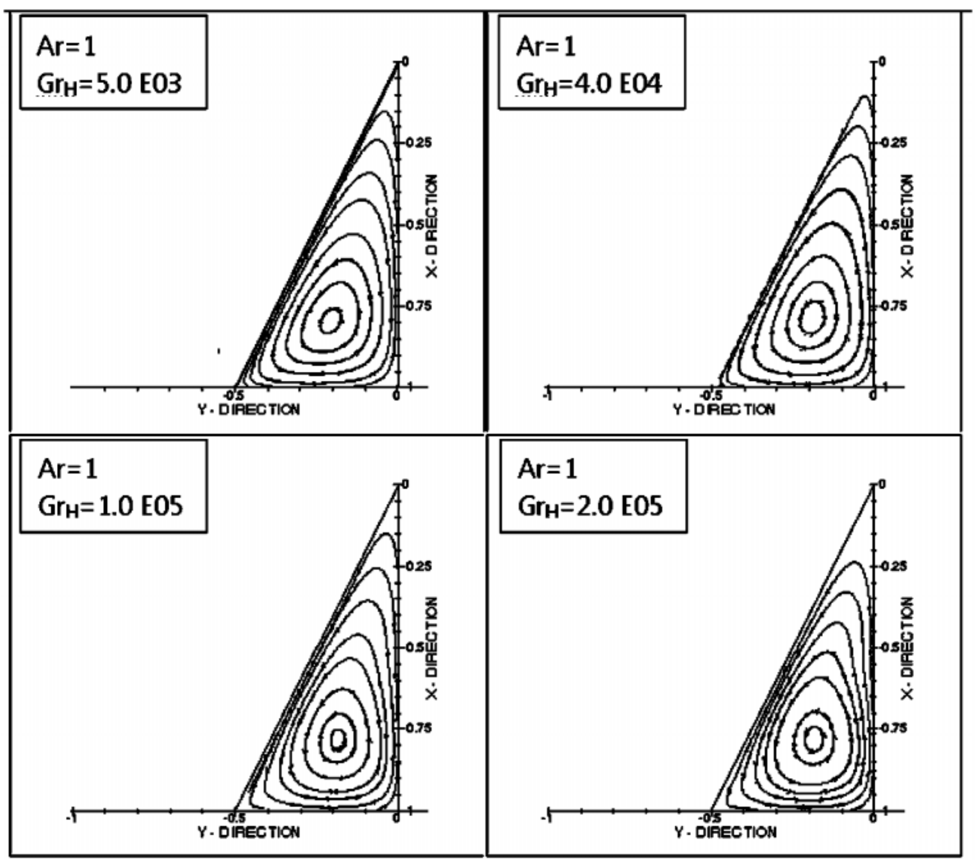
(a)

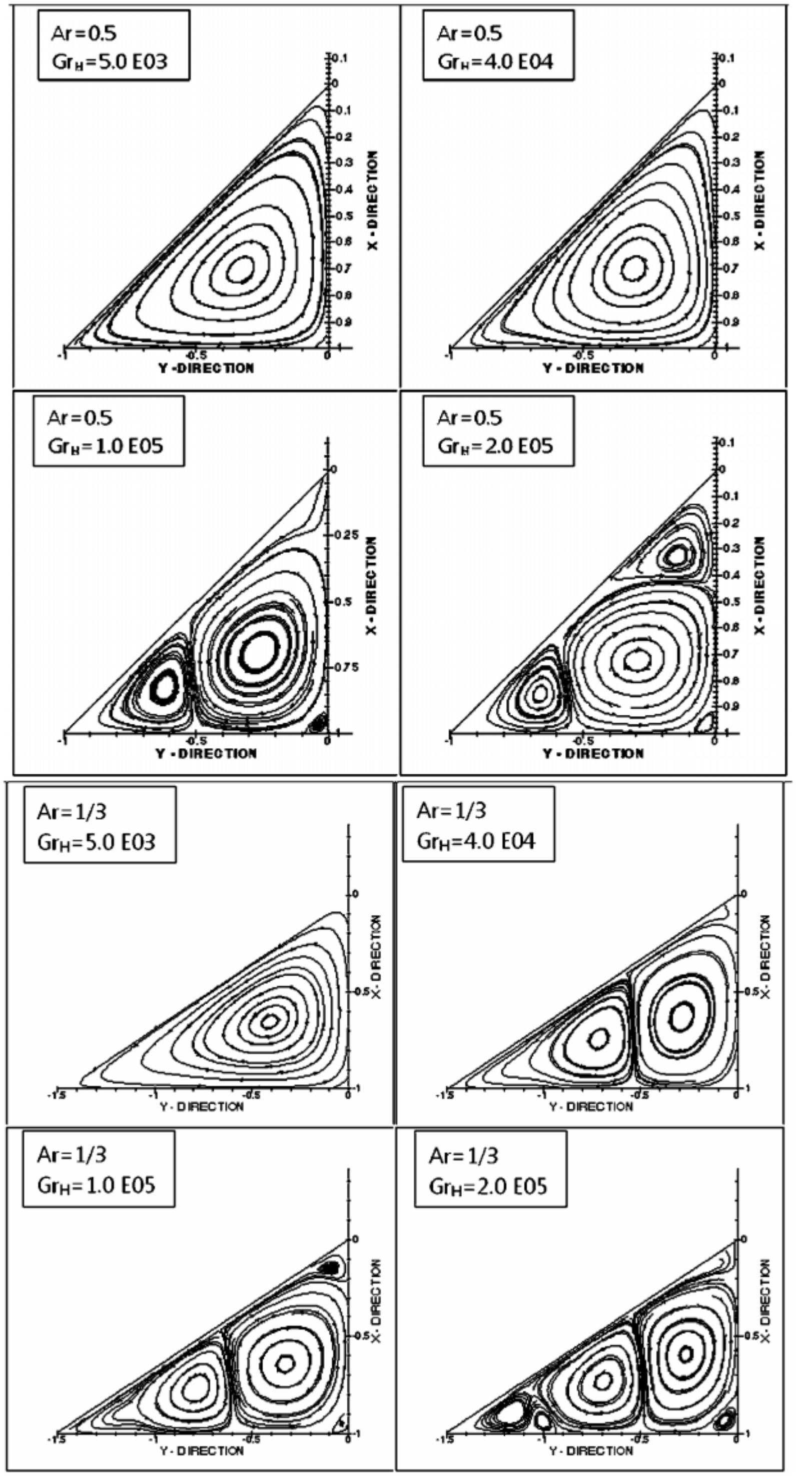


(b)

Figure 2: Velocity Vector for the Cavity with Ar = 0.5 and Gr_H = 1 E 03, in a) Summer Condition, b) Winter Condition

In winter day condition, the air near the hypotenuse wall is cooled down moving down because of the gravity effect (Fig. 3). On the other hand, the air close to the vertical wall moves up and replaces the latter stream. The upward and downward motion of air brings about the appearance of circulation cell. For the model with *Ar* = 1.00 and *Gr* = 5.00*e*3 there is one cell that is spinning about center of the enclosure. As the Grashof number increases, the streamlines extend to the tip and apex. For the case Ar = 0.5, firstly as the Grashof number increases to 4.00e4, elongation of the streamlines occurs toward the tip and apex. Further increase of Grashof number, gives result in the formation of distinct cell close to the bottom tip and also some deviation appears near the upper apex which further increase of Grashof number results in formation of a new cell as well. This occurrence may be explained as a result of the increase in fluid velocity, because of Grahof number increase, and decrease in the boundary layer effects near the tip and apex. On the whole, the enclosure center is more stable than other parts of that. These results are in good agreement with the result of former study [5]. In all diagram outlined in Fig. 3, the vertical and horizontal axes present the non dimensional coordinate systems in x and y direction, respectively. Considering Fig.3, as the aspect ratio decreases the number of the circulation cells increased; with decreasing the aspect ratio, the likelihood of hot and cold columns of air occurrence increases and giving result into increasing the appearance of more cells. Also the increment in cells numbers may be interpreted as a result of the breakdown of the regular boundary layers near apex of the enclosure, in small aspect ratios. It's noteworthy that to present the results of the streamline in more obvious manner the pictures are not n scaled features. For the summer condition the same e results for the streamline could be observed with the inverse direction of the flow.




Ar=0.5
Gr_H=5.0 E03
Ar=0.5
Gr_H=4.0 E04
Ar=0.5
Gr_H=1.0 E05
Ar=0.5
Gr_H=2.0 E05
Ar=1/3
Gr_H=5.0 E03
Ar=1/3
Gr_H=4.0 E04
Ar=1/3
Gr_H=1.0 E05
Ar=1/3
Gr_H=2.0 E05
Y - DIRECTION
X - DIRECTION

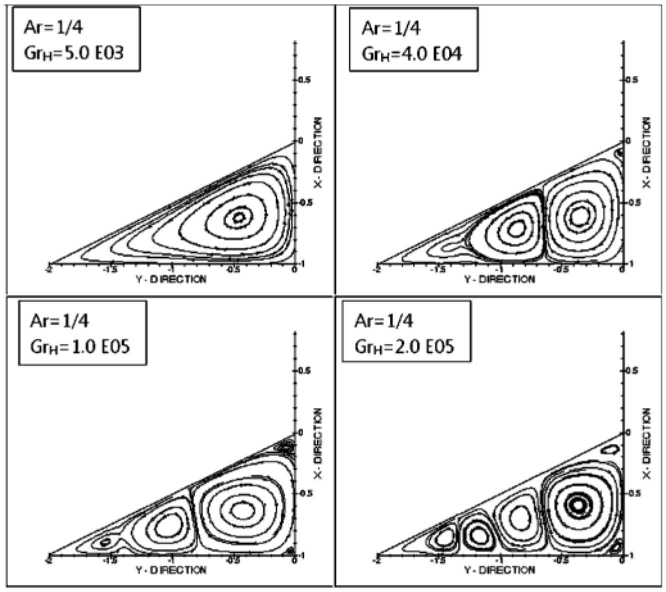
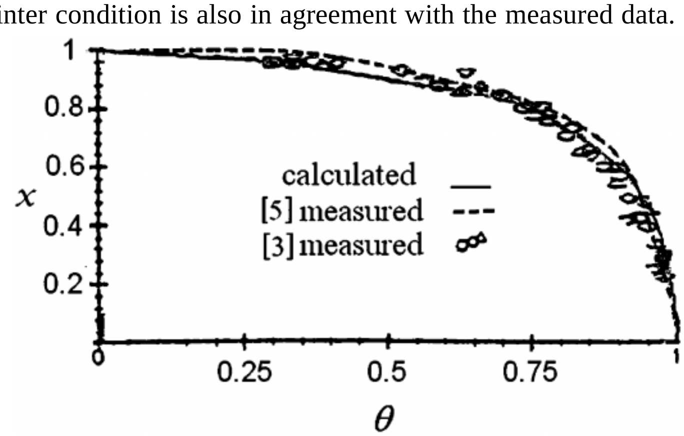


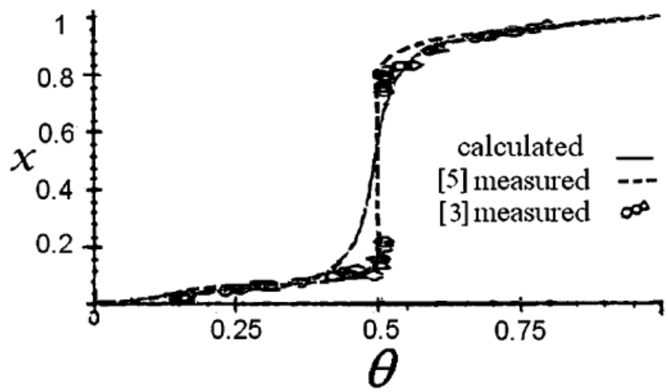
Figure 3: Streamline of Air Flow for Enclosures with different Aspect Ratios and Grashof Number

Temperature Profile

The temperature profile along the symmetric plane the enclosure for *Ar=1.00*, and *Gr=5.00e3*, for winter and summer conditions is shown in Figure 4. In Fig. 4.a where the inclined wall is hot, temperature profile shows temperature decrease near the base wall and monotonic increase near the inclined wall. For the winter day condition, Fig. 4.b, the temperature profile indicates that the temperature increases along the centerline of the enclosure toward the horizontal wall. Especially a remarkable increase is obvious in the middle region of the cavity. The same conclusion is applicable to other models with different aspect ratios and Grashof numbers values except. The results f the temperature gradients are compared with the measured data of [3] and [5]. As can be seen the results of the summer conditions are in excellent agreement with the experimental data. Except for a small region near the center point of the enclosure, the results of the winter condition is also in agreement with the measured data.



(a)



(b)

Figure 3: None-dimensional Centerline Temperature Profile along the Vertical Bisector of the Horizontal Edge, a) Summer Condition, b) Winter Condition

Nusselt Number Variation

The Nusselt number for the horizontal wall is calculated using the following equation:

$$Nu = \frac{QH}{kD\left(T_H - T_C\right)} = -Ar\int \frac{\partial\theta}{\partial y}\Big|_{wall}\, dx \tag{15}$$

The following results are obtained considering the Nusselt number for different Grashof numbers and aspect ratios. As the Grashof number based on the vertical distance of the enclosure increases the Nusselt number increases. However, this increase is more intensified for the small values of Nusselt number (Figure 4). As the aspect ratio increases the Nusselt number decreases.

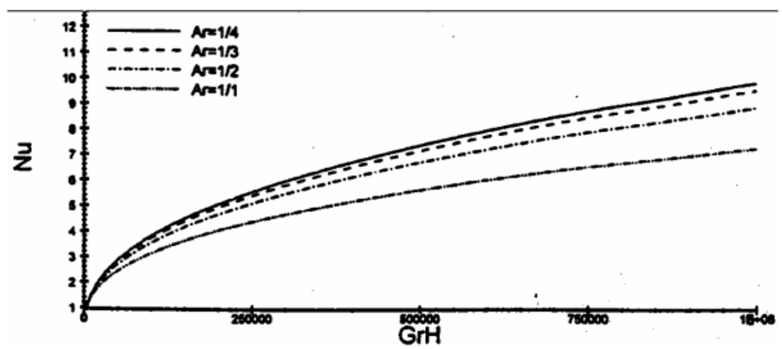


Figure 4: Nusselt Number Variation against Grashof Number in different Aspect Ratios

3. CONCLUSION

In this article the natural convection heat transfer in a triangular cavity filled with air has been modeled using two dimensional CFD code written in FORTRAN. The simulation has been performed assuming two thermal boundary conditions; summer day and winter day conditions. The streamline into the enclosure has been found thoroughly similar for the two mentioned thermal conditions. The results have also been compared with ones from former studies and it is found that the numerical simulation is reliable to predict the thermal and fluid flow characteristics of the physical model. Using the developed model, the effect of aspect ratio and Grashoaf number on the streamline of the air in the enclosure has been discussed and it has been concluded that as the aspect ratio decreases the number of the circulating cells increase. Also as the Grashof number increases, the streamlines extend to the enclosure tip and apex and further increment of the Grashof number will result in the appearance of the new cells. The temperature profile for both thermal conditions has been investigated. It has been found that, for the hot inclined wall, the temperature gradient in the middle of the centerline is smaller compared to the regions near the tip, while for the model with cold inclined wall the temperature gradient in the middle of the centerline is greater than the other regions along the centerline. Also, Nusselt number variation against the Grashof number for different aspect ratios has been presented.

REFERENCES

[1] Batchelor, G. H., "Heat Transfer by Free Convection Across a Closed Cavity between Vertical Boundaries at different Temperature", *Quarterly Journal of Heat Transfer*, **12**, pp. 209-233, 1954.

[2] Flack, R. D., Witt, C. L. "Velocity Measurement in Two Natural Convection Air Flows using a Laser Velocimeter" *ASME Journal of Heat Transfer*, **101**, No. 2, pp. 256-260, 1979.

[3] Flack R. D., "The Experimental Measurement of Natural Convection Heat Transfer in Triangular Enclosures Heated or Cooled from Below", *ASME Journal of Heat Transfer*, **102**, pp.770-772, 1980.

[4] Poulikekos, D., Bejan, A., "Natural Convection Experimental a Triangular Enclosure", *Journal of Heat Transfer, Transactions of the ASME*, **105 (3)**, pp. 652-655, 1983.

[5] Ghassemi, M., Roux, J. A. "Numerical Investigation of Natural Convection within a Triangular Shaped Enclosure", *National Heat Transfer Conference HDT*, **107**, pp. 169-174, 1989.

[6] Lei , C., S. W. Armfield, J. C. Patterson, "Unsteady Natural Convection in a Water-filled Isosceles Triangular Enclosure Heated from Below", *International Journal of Heat and Mass Transfer*, **51,** 2008, 2637–2650.

[7] Campo, E. M. d., M. Sen, E. Ramos, "Analysis of Laminar Natural, Convection in a Triangular Enclosure", *Numerical Heat Transfer,* **13**, 1988.

[8] Y.E. Karyakin, Y.A. Sokovishin and O.G. Martynenko, "Transient Natural Convection in Triangular Enclosures", *Int. J. Heat Mass Transfer* **31** (9) (1988), pp. 1759–1766.

[9] D. Poulikakos and A. Bejan, "Natural Convection Experiments in a Triangular Enclosure", *J. Heat Transfer, Trans. ASME*, **105,** 1983, pp. 652–655.

[10] D. Poulikakos and A. Bejan, "The Fluid Dynamics of an Attic Space", *J. Fluid Mech,* **131,** 1983, pp. 251–269.

[11] V.A. Akinsete and T.A. Coleman, "Heat Transfer by Steady Laminar Free Convection in Triangular Enclosures", *Int. J. Heat Mass Transfer,* **25** (7), (1982), pp. 991–998.

[12] H. Salmun, "Convection Patterns in a Triangular Domain", *Int. J. Heat Mass Transfer,* **38 (2)**, 1995, pp. 351–362.

[13] P.M. Haese and M.D. Teubner, "Heat Exchange in an Attic Space", *Int. J. Heat Mass Transfer,* **45,** 2002, pp. 4925–4936.

[14] Incropera, F. P., De witt, D. P., "Introduction to Heat Transfer", 3rd Edition, John Wiley & Sons Inc., USA, 1996.

[15] Bejan, A., "Convective Heat Transfer", 2nd Edition, Wiley Interscience, Canada, 1994.

[16] Patankar, S. V., "Numerical Heat Transfer and Fluid Flow", Hemisphere Publishing Corp., USA, 1980.