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ANALYTICAL HEAT TRANSFER INVESTIGATION OF NON-NEWTONIAN FLUID FLOW IN AN AXISYMMETRIC CHANNEL WITH A POROUS WALL

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Abstract: *In this paper, heat transfer of non-Newtonian fluid flow in an axisymmetric channel with porous wall, is analyzed. Porous wall channels are used in turbines for cooling purposes. Homotopy Perturbation Method (HPM) is employed to get complete analytic solution for velocity and temperature profiles. Results show an acceptable agreement between this method and numerical solutions. Also the effects on different parameters are discussed through graphs.*

Keywords: *Homotopy perturbation method (HPM), Porous medium, Axisymmetric channel.*

1. INTRODUCTION

Behavior of Non-Newtonian fluid flow has become a noteworthy problem recently, because it has miscellaneous applications in different engineering fields. Especially the heat transfer problem in non-Newtonian fluid flow is interested significantly. Extrusion of plastics, hot rolling, flow in journal bearings, mud flow in oil drilling and shock absorbers are some scant instances of this fluid flow application. Flow of non-Newtonian fluid in a channel and the related heat transfer has been the focus of considerable researches and mathematical modeling with the purpose of finding the temperature distribution and the consequent behavior of fluid flow has taken into consideration [1-8].

A large number of engineering problems are nonlinear, especially ones with heat transfer equations, thereupon some of them are solved by using numerical solution and some by different analytic methods, such as perturbation method, variational iteration method and homotopy perturbation method that is introduced by He[9]. Many different methods have recently introduced some ways to eliminate the small parameter. One of the semi-exact methods which does not need small parameters is the Homotopy Perturbation Method. This method proposed by He for the first time in 1998 and was further developed and improved by him [10]. In the most cases, This method yields a very rapid convergence of the solution series and acceptable answers. The HPM proved its ability to solve a vast class of nonlinear problems efficiently, accurately and easily with rapid convergence to solution. Commonly, a few iterations lead to

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high accuracy solution. This method is employed for many researches in engineering sciences recently [11-15].

2. MATHEMATICAL FORMULATION

In this study, simultaneous development of flow and heat transfer of non-Newtonian viscoelastic fluid flow is investigated, that is applicable on the turbine disc for cooling purposes. The problem is shown schematically in Fig. 1. The y-axis is normal to surface of externally heated disk and the x-axis is parallel to it. The perforated disc of the channel is located at $y = + L$. Non-Newtonian fluid is injected from the other porous wall uniformly in order to cool the heated wall that coincides with the x-axis.

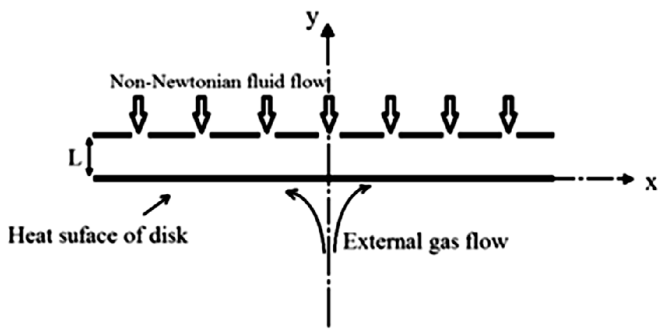

Figure 1: Schematic Diagram of the Physical System

It is supposed that the flow field is assumed to be stagnation point flow with injection. For steady, two-dimensional non-Newtonian fluid flow that is axisymmetric, equations which govern the flow and heat transfer are proposed by Kurtcebe and Erim, as follow [8]:

$$-2ff''' = \frac{f^{iv}}{\text{Re}} - K_1(4f''f''' + 2f'f^{iv}) - K_2(4f''f''' + 2f'f^{iv} + 2ff^{v}), \tag{2.1}$$

$$q_n'' - \text{Pr}.\text{Re}\,(nf'q_n - 2fq_n') = 0, \qquad (n = 0, 2, 3, 4, ...), \tag{2.2}$$

where $K_1 = \frac{\phi_1}{\rho L^2}$, $K_2 = \frac{\phi_2}{\rho L^2}$, Re is the injection Reynolds number and Pr is Prandtl number. The boundary conditions are:

$$f(0) = 0, \qquad f'(0) = 0, \qquad f(1) = 1, \qquad f'(1) = 0, \tag{2.3}$$

$$q_n(0) = 1, \qquad q_n(1) = 0. \tag{2.4}$$

Eqations. (2.1) and (2.2) are solved by Kurtcebe and Erim [8] for $K_2 = 0$ with the boundary conditions (2.3) and (2.4). In this study, these equations are considered as:

$$f^{iv} + 2\,\text{Re}\,ff''' - K_1\,\text{Re}\,(4f''f''' + 2f'f^{iv}) = 0, \tag{2.5}$$

$$q_n'' - \text{Pr}.\text{Re}\,(nf'q_n - 2fq_n') = 0, \qquad (n = 0, 2, 3, 4, ...), \tag{2.6}$$

and solved by Homotopy perturbation Method.

3. ANALYSIS OF THE HOMOTOPY PERTURBATION METHOD

In order to describe the basic ideas of this method, consider the following equation:

$$A(u) - f(r) = 0, \qquad r \in \Omega, \tag{3.1}$$

with the boundary condition of:

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \qquad r \in \Gamma, \tag{3.2}$$

where A is a general differential operator, B a boundary operator, $f(r)$ a known analytical function and Γ is the boundary of the domain Ω.

A can be divided into two parts which are L and N, where L is linear and N is nonlinear. Eq. (3.1) can therefore be rewritten as follows:

$$L(u) + N(u) - f(r) = 0, \qquad r \in \Omega. \tag{3.3}$$

Homotopy perturbation formula is introduced as below:

$$H(\nu, p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0 \tag{3.4}$$

where

$$\nu(r, p): \Omega \times [0, 1] \rightarrow R. \tag{3.5}$$

In Eq. (3.5), $p \in [0, 1]$ is an embedding parameter and u_0 is the first approximation that satisfies the boundary condition. We can assume that the solution of Eq. (25) can be written as a power series in p, as following:

$$\nu = \nu_0 + p\nu_1 + p^2\nu_2 + \ldots, \tag{3.6}$$

and the best approximation for solution is:

$$u = \lim_{p \rightarrow 1} \nu = \nu_0 + \nu_1 + \nu_2 + \ldots. \tag{3.7}$$

4. IMPLEMENTATION OF THE METHOD

According to the homotopy-perturbation method (HPM), homotopy suppose is constructed and the solution of Eq. (3.1) has the form:

$$H(f, p) = (1-p)(f^{iv} - f_0^{iv}) + p\,(f^{iv} + 2\mathrm{Re}\, ff''' - K_1 \mathrm{Re}\,(4f''f''' + 2f'f^{iv})) = 0 \tag{4.1}$$

$$H(\theta, p) = (1-p)(q_n'' - q_{n_0}'') + p\,(q_n'' - \mathrm{Pr}.\mathrm{Re}\,(f'q_n - 2fq_n')) = 0$$

f and q_n are considered as blow:

$$f(\eta) = f_0(\eta) + pf_1(\eta) + \ldots = \sum_{i=0}^{n} p^i f_i(\eta),$$

$$q_n = q_{n_0}(\eta) + pq_{n_1}(\eta) + \ldots = \sum_{i=0}^{n} p^i q_{n_i}(\eta). \tag{4.2}$$

With substituting f, q_n from Eq. (4.2) into Eq. (4.1) and doing some simplification and rearranging based on powers of p-terms, it can be obtained

$$\begin{aligned}
p^0 : & f^{iv} = 0, \\
& q_n'' = 0, \\
& f_0(0) = 0,\ f_0'(0) = 0,\ f_0(1) = 1,\ f_0'(1) = 0, \\
& q_{n_o}(0) = 1,\ q_{n_0}(1) = 0.
\end{aligned} \tag{4.3}$$

$$\begin{aligned}
p^1 : & f_1^{iv} - 2K_1 \mathrm{Re}\, f_0^{iv} f_0' + 2\,\mathrm{Re}\, f_0 f_0''' - 4k_1 \mathrm{Re}\, f_0'' f_0''' - 2K_1 \mathrm{Re}\, f_0^{iv} f_0' = 0, \\
& q_{n_1}'' - \mathrm{Re}\,\mathrm{Pr}\, n f_0' q_{n_0} - 2\,\mathrm{Re}\,\mathrm{Pr}\, q_{n_0}' = 0, \\
& f_1(0) = 0,\ f_1'(0) = 0,\ f_1(1) = 1,\ f_1'(1) = 0, \\
& q_{n_1}(0) = 0,\ q_{n_1}(1) = 0 \\
& \vdots
\end{aligned} \tag{4.4}$$

Eqs. (4.3) and (4.4) are solved with relevant boundary conditions:

$$\begin{aligned}
& f_0(\eta) = -2\eta^3 + 3\eta^2, \\
& q_{n0}(\eta) = -\eta + 1,
\end{aligned} \tag{4.5}$$

$$\begin{aligned}
f_1(\eta) = {} & \mathrm{Re}(-0.05714285714\eta^7 + 0.2\eta^6 + 4.8n\eta^5 - 12n\eta^4 \\
& - 0.5142857144\eta^3 + 9.6n\eta^3), \\
q_{n1}(\eta) = {} & \mathrm{Pr}.\mathrm{Re}\,(0.3n\eta^5 - n\eta^4 + n\eta^3 + \eta^2 - 0.3n\eta - \eta) \\
& \vdots
\end{aligned} \tag{4.6}$$

When $i \le 2$, the terms $f_i(\eta)$ and $q_{ni}(\eta)$ are too large that are illustrated graphically. Finally, when $p \to 1$, the following expressions for $f(\eta)$ and $q_n(\eta)$ are available:

$$f(\eta) = f_0(\eta) + f_1(\eta) + \ldots = \sum_{i=0}^{n} f_i(\eta)\,. \tag{4.7}$$

5. RESULTS AND CONCLUSION

In this paper, HPM method has successfully employed in order to determine the exact solutions of heat transfer problem of non-Newtonian fluid flow in a channel with one porous wall for turbine cooling application. This problem also has solved by a Numerical Method (fourth-order Runge-Kutta) and the consequent results of the two different methods, Namely, HPM and NM are compared in Table1 and Table 2 and Figs. 2, 3 and 4. These results show that this analytical method is a very powerful and efficient technique for solving different kinds of problems arising in various fields of science and engineering and gives a rapid convergence for the solutions.

Table 1
Comparison Between Numerical Results and HPM Solution at: Re = 0.5, K_1 = 0.01, Pr = 1, n = 0

	f			q_n		
η	*NM*	*HPM*	*%Error*	*NM*	*HPM*	*%Error*
0	0	0	1	1	0	
0.05	0.00769	0.007669	0.280991	0.942347	0.942174	0.019501
0.1	0.029627	0.029561	0.221484	0.88472	0.884444	0.035265
0.15	0.06411	0.064002	0.168202	0.827181	0.826856	0.047538
0.2	0.109442	0.109309	0.121141	0.769829	0.769494	0.056593
0.25	0.163932	0.1638	0.080264	0.712792	0.712474	0.062724
0.3	0.225897	0.225794	0.045492	0.656223	0.655937	0.066242
0.35	0.293668	0.293619	0.016698	0.600289	0.600046	0.067466
0.4	0.365597	0.36562	0.006305	0.545172	0.544974	0.066718
0.45	0.440058	0.440163	0.023762	0.491054	0.490898	0.064317
0.5	0.515454	0.51564	0.035987	0.438113	0.437996	0.060572
0.55	0.590225	0.590481	0.043362	0.386518	0.386435	0.055774
0.6	0.662849	0.663156	0.046347	0.336423	0.336366	0.050193
0.65	0.731849	0.732182	0.045477	0.28796	0.287923	0.044072
0.7	0.795795	0.796124	0.041379	0.241237	0.241215	0.037627
0.75	0.853309	0.853606	0.034776	0.196334	0.196322	0.031038
0.8	0.903067	0.903307	0.026508	0.153305	0.1533	0.024454
0.85	0.943801	0.943967	0.017565	0.112175	0.112172	0.017991
0.9	0.974297	0.974386	0.00913	0.072938	0.072937	0.011731
0.95	0.993397	0.993424	0.002658	0.035564	0.035564	0.005726
1	1	1	0	0	0	0

Table 2
Comparison Between Numerical Results and HPM Solution at: Re = 0.5, K_1 = 0.1, Pr = 1, n = 2

	f			q_n		
η	*NM*	*HPM*	*%Error*	*NM*	*HPM*	*%Error*
0	0	0	0	1	1	0
0.05	0.007694	0.007694	0.000225	0.929831338	0.929659	0.019964
0.1	0.029642	0.029642	0.00069	0.860381004	0.860117	0.035611
0.15	0.06414	0.06414	0.001145	0.792249157	0.791952	0.04731
0.2	0.10949	0.10949	0.001417	0.725916781	0.725624	0.055478
0.25	0.163997	0.163997	0.001409	0.661756105	0.661491	0.060555
0.3	0.225979	0.225978	0.001092	0.600040916	0.599814	0.062987
0.35	0.293765	0.293764	0.000493	0.540956929	0.540772	0.063209
0.4	0.365704	0.365704	0.000314	0.484612278	0.484468	0.061631
0.45	0.440171	0.440169	0.001227	0.431048078	0.430939	0.058632
0.5	0.515569	0.515563	0.00213	0.38024899	0.38017	0.054553
0.55	0.590337	0.590327	0.002907	0.332153638	0.332099	0.049689
0.6	0.662954	0.662938	0.00346	0.286664715	0.286628	0.044297
0.65	0.731942	0.731922	0.003717	0.243658621	0.243636	0.038588
0.7	0.795873	0.79585	0.003645	0.202994454	0.202981	0.032734
0.75	0.853371	0.853348	0.003253	0.164522228	0.164515	0.026869
0.8	0.903113	0.903091	0.002597	0.128090214	0.128087	0.021095
0.85	0.94383	0.943814	0.001779	0.093551336	0.09355	0.015484
0.9	0.974311	0.974302	0.000943	0.060768598	0.060768	0.010084
0.95	0.993401	0.993398	0.000277	0.029619573	0.02962	0.00492
1	*1*	*1*	*0*	*0*	*0*	*0*

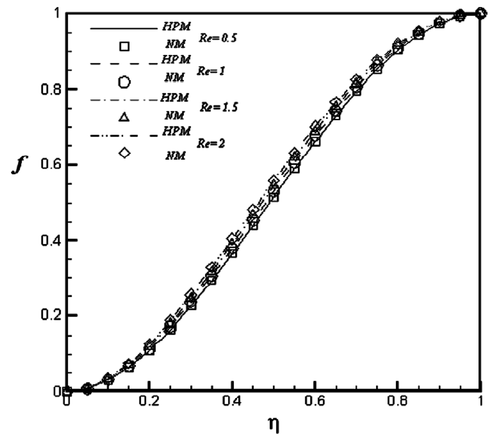

Figure 2: Velocity Component Profile (f) for Variable Re at $K_1 = 0.01$

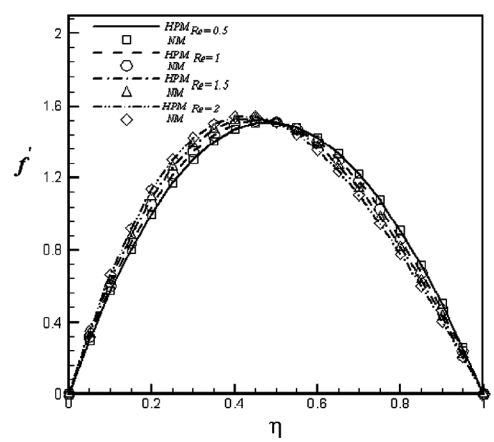

Figure 3: Velocity Component Profile (f') for Variable Re at $K_1 = 0.01$

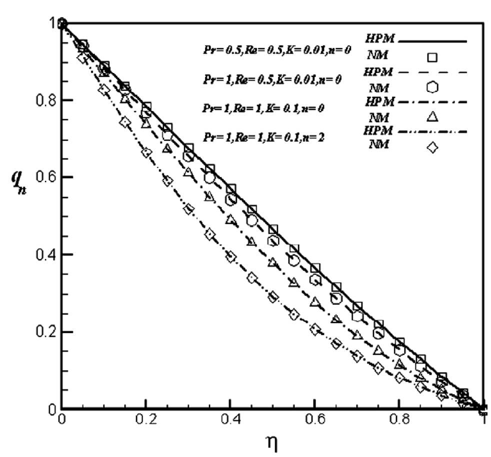

Figure 4: Temperature Profile (q_n) for Variable Active Parameter

Also it has illustrated that a coefficient namely Nusselt number $(\theta'(0))$ changes effectively with some parameters. Nu is enhanced by increasing the non-Newtonian fluid power coefficient, *n*, and Prandtl number in special value of *Re*, as shown in Figs. 5 and 6.

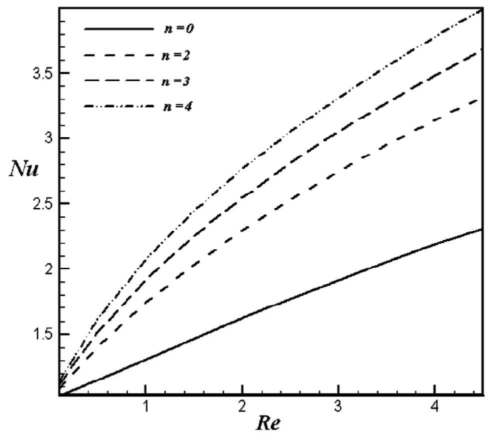

Figure 5: Nusselt Number for Variable n at $K_1 = 0.01$, Re = 1

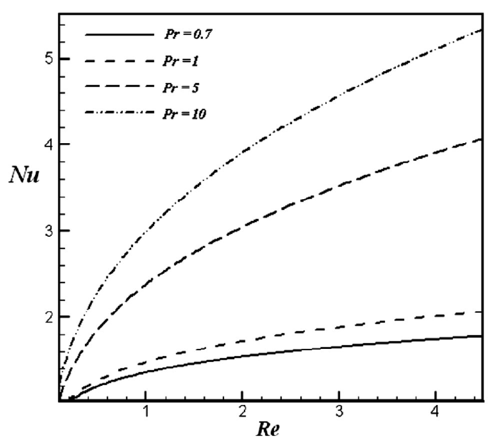

Figure 6: Nusselt Number for Variable Re at $K_1 = 0.01$, $n = 0$

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