LINEAR MATRIX INEQUALITIES APPROACH FOR A LINEAR TIME-DELAY SYSTEMS

K. Ratchagit

Department of Mathematics, Faculty of Science, Maejo University, Chiang Mai 50290, Thailand

Abstract: This paper is concerned with the problem of delay-dependent stability analysis for discrete-time systems with interval-like time-varying delays. The problem is solved by applying a novel Lyapunov functional, and an improved delay-dependent stability criterion is obtained in terms of a linear matrix inequality. The reduced conservatism of the proposed stability result is shown through a numerical example.

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1. INTRODUCTION

Recently, the problem of delay-dependent stability analysis for time-delay systems has received considerable attention, and lots of significant results have been reported; see, for example, Chen, Guan, and Lu (2003), He, Wang, Lin, and Wu (2007), Lin, Wang, and Lee (2006), Park (1999) and Xu and Lam (2005), and the references therein. Among these references, we note that the delay-dependent stability problem for discrete-time systems with interval-like time-varying delays (i.e., the delay $d(k)$ satisfies $0 < d_m \le d(k) \le d_M$) has been studied in Fridman and Shaked (2005), Gao and Chen (2007), Gao, Lam, Wang, and Wang (2004) and Jiang, Han, and Yu (2005), where some LMI-based stability criteria have been presented by constructing appropriate Lyapunov functionals and introducing free-weighting matrices. It should be pointed out that the Lyapunov functionals considered in these references are more restrictive due to the ignorance of the term $\sum\limits_{j=-h_M}^{j=-h_m-1} \sum\limits_{i=k+j}^{i=k-1} [x(i+1) - x(i)]^T R[x(i+1) - x(i)]$. Moreover, the term $\sum\limits_{i=k-h_m}^{i=k-1} x(i)^T Q_2 x(i)$ is also ignored in Gao and Chen (2007) and Gao *et al*. (2004). The ignorance of these terms may lead to considerable conservativeness.

On the other hand, in the study of stabilization for the discrete-time linear systems, traditional idea of the control schemes is to construct a control signal according to the current system state (Garcia, Bernussou, & Arzelier, 1994). However, as pointed out in Xiong and Lam (2006), in practice there is often a system that itself is not time-delayed but time-delayed may exist in a

channel from system to controller. A typical example for the existence of such delays is the measurement and the network transmission of signals. In this case, a time-delayed controller is naturally taken into account. It is worth noting that the closed-loop system resulting from a delayed controller is actually a time-delay system. Therefore, stability results of time-delay systems could be applied to design time-delayed controller.

The present study a new Lyapunov functional, an improved delay-dependent stability criterion for discrete-time systems with time-varying delays is presented in terms of LMIs. It is shown that the obtained result is less conservative than those in Fridman and Shaked (2005), Gao and Chen (2007), Gao *et al*., (2004), and Jiang *et al*., (2005). Example is providing, respectively, to demonstrate the reduced conservatism of the proposed stability criterion and the effectiveness of the proposed design method.

2. PRELIMINARIES

Lemma 2.1 [1]: The zero solution of difference system is asymptotic stability if there exists a positive definite function $V(x(k)) : \mathbb{R}^n \rightarrow \mathbb{R}^+$ such that

$$\exists \beta > 0 : \Delta V(x(k)) = V(x(k+1)) - V(x(k)) \leq -\beta \| x(k) \|^2,$$

along the solution of the system. In the case the above condition holds for all $x(k) \in V_\delta$, we say that the zero solution is locally asymptotically stable.

3. IMPROVED STABILITY CRITERION

In this section, we give a novel delay-dependent stability condition for discrete-time systems with interval-like time-varying delays. Now, consider the following system

$$x(k+1) = Ax(k) + Bx(k - h(k)), \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, A and B are known constant matrices, and $h(k) > 0$ is a time-varying delay satisfying $0 < h(k) \leq h$, where h is a positive integer.

Theorem 3.1: Give integer $h > 0$. Then, the discrete time-delay system (1) is asymptotically stable for any time delay $h(k)$ satisfying $0 < h(k) \leq h$, if there exist symmetric positive definite matrices P, G, W satisfying the following matrix inequalities:

$$\psi = \begin{pmatrix} A^T P A + hG + W - P & A^T P B \\ B^T P A & B^T P B - W \end{pmatrix} < 0, \tag{2}$$

Proof: Consider the Lyapunov function $V(y(k)) = V_1(y(k)) + V_2(y(k)) + V_3(y(k))$, where

$$V_1(y(k)) = x^T(k) P x(k),$$

$$V_2(y(k)) = \sum_{i=k-h(k)}^{k-1} (h(k) - k + i) \, x^T(i) G x(i),$$

$$V_3(y(k)) = \sum_{i=k-h(k)}^{k-1} x^T(i)Wx(i),$$

where *P*, *G*, *W* being symmetric positive definite solutions of (2).

Then difference of $V(x(k))$ along trajectory of solution of (1) is given by

$$\Delta V(x(k)) = \Delta V_1(x(k)) + \Delta V_2(x(k)) + \Delta V_3(x(k)),$$

where

$$\begin{aligned}
\Delta V_1(x(k)) &= V_1(x(k+1)) - V_1(x(k)), \\
&= [Ax(k) + Bx(k-h(k))]^T P [Ax(k) + Bx(k-h(k))] - x^T(k)Px(k) \\
&= x^T(k)[A^TPA - P]x(k) + x^T(k)A^TPBx(k-h(k)) + x^T(k-h(k))B^TPAx(k) \\
&\quad + x^T(k-h(k))B^TPBx(k-h(k)),
\end{aligned}$$

$$\begin{aligned}
\Delta V_2(x(k)) &= \Delta\left(\sum_{i=k-h(k)}^{k-1} (h(k) - k + i)\, x^T(i)Gx(i) \right), \\[2mm]
&= \sum_{i=k-h(k)+1}^{k} (h(k) - (k+1) + i)\, x^T(i)Gx(i) \\[2mm]
&= -\sum_{i=k-h(k)}^{k-1} (h(k) - k + i)\, x^T(i)Gx(i),
\end{aligned}$$

$$\begin{aligned}
\Delta V_3(x(k)) &= \Delta\left(\sum_{i=k-h(k)}^{k-1} x^T(i)Wx(i) \right), \\[2mm]
&= x^T(k)Wx(k) - x^T(k-h(k))Wx(k-h(k)).
\end{aligned} \tag{3}$$

Then we have

$$\begin{aligned}
\Delta V(x(k)) &= x^T(k)[A^TPA + G + W - P]x(k) \\
&\quad + x^T(k)A^TPBx(k-h(k)) + x^T(k-h(k))B^TPAx(k) \\
&\quad + x^T(k-h(k))(B^TPB - W)x(k-h(k)) + hx^T(k)Gx(k) \\
&\leq y^T(k)\psi\, y(k),
\end{aligned}$$

where

$$y(k) = [x(k), x(k-h)]$$

and

$$\psi = \begin{pmatrix} A^TPA + hG + W - P & A^TPB \\ B^TPA & B^TPB - W \end{pmatrix}.$$

On the above estimation we use: $h(k) \leq h$, and

$$h(k)x^T(k)Gx(k) \leq x^T(k)Gx(k).$$

By the condition (2), $\Delta V(y(k))$ is negative definite, namely there is a number $\beta > 0$ such that $\Delta V(y(k)) \leq \beta \| (y(k)) \|^2$, and hence, the asymptotic stability of the system immediately follows from Lemma 2.1. This completes the proof.

Example 3.1: Let us consider a discrete-time systems with interval-like time-varying delays (1), given by the system

$$x(k+1) = Ax(k) + Bx(k-h(k)),$$

where the matrices are:

$$A = \begin{pmatrix} -0.2 & 0.04 \\ 0.03 & -0.5 \end{pmatrix}, \qquad B = \begin{pmatrix} -0.06 & 0.001 \\ 0.003 & -0.05 \end{pmatrix} \quad \text{and} \quad h = 0.5.$$

Using the LMI Toolbox in MATLAB, we found that the LMIs in Theorem 3.1 are feasible and

$$P = \begin{pmatrix} 44.1489 & 0.0117 \\ 0.0117 & 44.5195 \end{pmatrix}, \quad G = \begin{pmatrix} 20.1983 & 0.2960 \\ 0.2960 & 17.6270 \end{pmatrix}, \quad W = \begin{pmatrix} 20.2779 & 0.2914 \\ 0.2914 & 17.6826 \end{pmatrix},$$

are set of solutions to the LMIs equation (2).

It follows from Lemma 2.1 that the zero solution of discrete-time systems with interval-like time-varying delays is asymptotically stable.

6. CONCLUSIONS

In this paper, an improved delay-dependent stability condition for discrete-time linear systems with interval-like time-varying delays has been presented in terms of an LMI.

ACKHOWLEDGMENT

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