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AN ANALYTICAL STUDY ON EFFECT OF PARTICLE SPHERICITY ON FALLING PROCESS OF NON-SPHERICAL PARTICLES

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Abstract: *The explanation of the unsteady falling motion of immersed bodies in fluids is present in several manufacturing processes. In the current study, the acceleration motion of a non-spherical particle in an incompressible Newtonian environment has been studied using a drag of the form presented by Chien for a wide range of Reynolds numbers. An analytical expression is gained for velocity, acceleration and position of the particle, using homotopy perturbation method. Equation of motion was solved generally and for some practical conditions. Results were compared with a numerical method and very good agreements were obtained. Present investigation shows the effectiveness of HPM and exhibit a new application of this method for nonlinear problems.*

Keywords: *Non-spherical Particle, Analytical Solution, Drag Coefficient, Acceleration Motion.*

NOMENCLATURE

a,b,c,d	constants	m	particle mass (kg)
A	general differential operator	N	nonlinear part of equation
A_s	Surface of the spherical particle (m^2)	P	embedding parameter
A_{ns}	Surface of the non-spherical particle(m^2)	Re	Reynolds number
Acc	acceleration (m/s^2)	t	time (s)
B	boundary operator	u	velocity (m/s)
C_D	drag coefficient	ϕ	Sphericity
D	particle diameter (m)	μ	dynamic viscosity (kg/ms)
G	acceleration due to gravity(m/s^2)	ρ	fluid density (kg/m^3)
HPM	Homotopy Perturbation Method	ρ_s	particle density (kg/m^3)
L	linear part of equation	Ω	domain
		Γ	boundary of domain

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1. INTRODUCTION

There are many situations in engineering applications when a reliable prediction of the acceleration motion and terminal falling velocity of particles in stationary fluids is required. In contrast to motion of particles at terminal velocity, much less has been reported about the unsteady motion of spherical objects or particles in fluids. Common examples include classification, centrifugal and gravity collection or separation of liquid–solid mixtures, fixed and fluidized bed reactors, petroleum engineering applications, etc. In these applications, it is often necessary to determine the trajectory of particle accelerating in a fluid for proposes of design or improved operation process [1]. Also for other particular situations, like viscosity measurement using the falling-ball method or rain-drop terminal velocity measurement it is necessary to know the time and distance required to reach terminal velocity for a given sphere-fluid combination.

Drag coefficient and terminal velocities of particles are most important design parameters in mentioned applications. During past decays, a vast body of knowledge has been accumulated on the steady state motion of spheres in incompressible Newtonian fluids and extensive sets of data were collected which resulted in several theoretical and empirical correlations for the drag coefficient, C_D, in the terms of the Reynolds number, *Re*. These relationships for spherical bodies were reviewed by many authors in review papers by Clift et al. [2], Khan and Richardson [3] and Chhabra [4]. A comparison between most of these correlations for spheres by Hartman and Yutes [5] demonstrated relatively low deviations.

However, in practice, particles are non-spherical and the particle motion is affected by particle shape and particle orientation. In the case of non-spherical particles, less information is available in the literature. Some evaluations of the available data against the predictive correlations have been reported by Chhabra et al. [6], Tang et al. [7] and Yow et al. [8] for regular shaped particles falling in Newtonian media.

Heywood [9] developed an approximate technique to calculate the terminal velocity of a non-spherical particle. The method was a revision of his method for spheres. He used an experimental coefficient, to account for deviations from the spherical shape. Haider and Levenspiel [10] presented a C_D – *vs* – Re correlation for non-spherical particles as follow:

$$C_D = \frac{24}{\mathrm{Re}}\left(1+e^{\left(2.3288-6.4581\phi+2.4486\phi^2\right)}\,\mathrm{Re}^{(0.0964+0.5565\phi)}\right)+\frac{73.69\,\mathrm{Re}\,e^{(-5.0748\phi)}}{\mathrm{Re}+5.378e^{(6.2122\phi)}} \quad (1)$$

In above Equation, The Reynolds number and drag coefficient are defined as follow:

$$\mathrm{Re} = \frac{uD}{\mu} \quad (2)$$

$$C_D = \frac{F_D}{\frac{\pi}{8}\rho u D^2} \quad (3)$$

Where F_D denotes the drag force. They also reported a relationship to calculate explicitly terminal velocities for particles of different shapes. The authors used the concept of sphericity, ϕ, described by Wadell [11], to account for the particle shape. Wadell [11] described the degree of sphericity as:

$$\phi = \frac{A_s}{A_{ns}} \tag{4}$$

Where A_s is the surface of a sphere having the same volume as the particle, and A_{ns} is the actual surface area of the particle. According to this definition, the sphericity of a true sphere is equal to 1. The more the aspect ratio departs from unity, the lower is the sphericity. Predictions by Haider and Levenspiel [10] showed relatively poor accuracy for particles with $\phi < 0.67$, therefore, some authors [12-13] try to improve the accuracy of the Haider and Levenspiel [10] correlations. Chien [12] and Hartman et al. [13] used the sphericity as shape factor. Chien [12] used the data available in the petroleum engineering and processing literature and proposed the following expression for drag:

$$C_D = \frac{30}{\text{Re}} + 67.289e^{(-5.03\phi)} \tag{5}$$

where C_D and are again based on the equal volume sphere diameter. Eq. 5 was stated to be valid in the ranges $0.2 < \phi < 1$ of and $\text{Re} < 5000$.

The comparison of Chien [12] and Haider and Levenspiel [10] correlations is presented in figure 1.

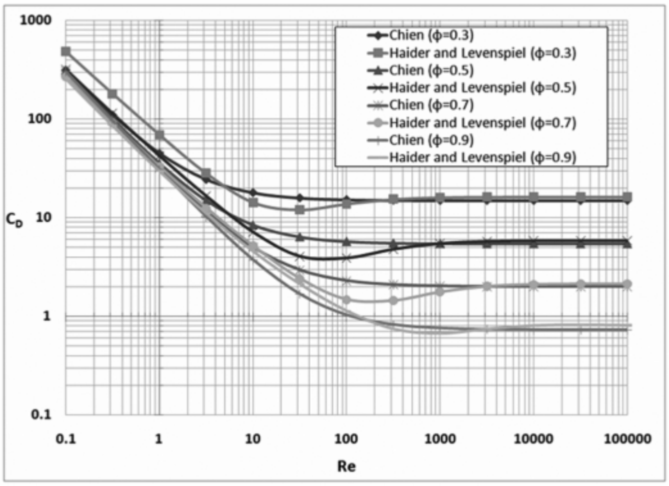

Figure. 1: Drag Coefficients Versus Reynolds Numbers

Most of previous studies were performed experimentally or numerically. However, an analytical expression is more convenient for engineering calculations, and is also the obvious starting point for a better understanding of the relationship between the physical properties of the solid-liquid combination and the accelerated motion of the particle. The aim of current study is to analytically investigation of acceleration motion of a falling non-spherical particle. We employed the correlation suggested by Chien [12] for different particle sphericity. Homotopy Perturbation Method (HPM) was used as an analytical method to gain an exact solution for bubbles in acceleration and steady movements. Investigation and solution of falling objects' equation of motion is a new application for HPM which was used for some engineering problems such fluid flows and heat transfer [14-32].

2. PROBLEM DEFINITION

Consider a non-spherical particle with the sphericity of ϕ falling from the rest in a Newtonian viscose medium, under the influence of gravity. Particle will accelerate until the gravitational force is exactly balanced by the resistance force that includes buoyancy and drag. The constant velocity reached at that stage is called the "terminal velocity" or "settling velocity". The resistive drag force depends upon drag coefficient which is defined in Eq. (3). Figure 2 illustrates the schematic view of current problem while a cylindrical particle was considered as an example of non-spherical particles.

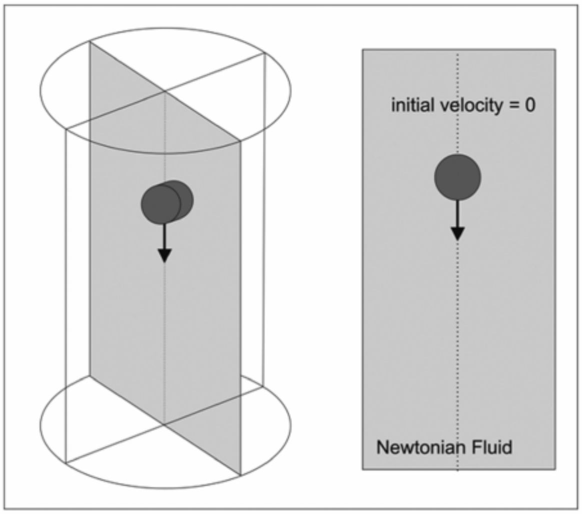

Figure. 2: Schematic View of Current Problem

It was assumed that:

(1) There is no mass transfer between two phases therefore the gravity force is constant;

(2) Effect of the confining walls on the particle motion are neglected;

(3) Particle falls rectilinearly;

(4) The drag (resistance) force on the particle is of a form similar to that under constant velocity conditions. This approximation is consistent with the previous studies in this field [1,33-37].

Under these assumptions, the motion of the particle in a fluid can be described by a force balance equation. For a dense particle falling in light liquids and by assuming $\rho << \rho_s$, Basset History force is negligible. Thus, the equation of motion is gained as follow:

$$m\frac{du}{dt} = mg\left(1-\frac{\rho}{\rho_s}\right)-\frac{1}{8}\pi D^2\rho C_D u^2-\frac{1}{12}\pi D^3\rho\frac{du}{dt} \tag{6}$$

Where C_D represents the drag coefficient. In the right hand side of the Eq. (5), the first term represents the buoyancy affect, the second one corresponds to resistance, drag, effect and the last one denotes the added-mass effect which is due to acceleration of fluid around the particle.

Substituting Eqs.(2) and (5) into Eq. (6) Following Expression is gained:

$$\left(m+\frac{1}{12}\pi D^3\rho\right)\frac{du}{dt} = mg\left(1-\frac{\rho}{\rho_s}\right)-\frac{1}{8}\pi D^2\rho\left(\frac{30\mu}{\rho u D}+67.289e^{(-5.03\phi)}\right)u^2 \quad (7)$$

The main difficulty in solution of above equation lies in the non-linear terms due to non-linearity nature of the drag coefficient, C_D. By rearranging parameters, Eq. (5) could be rewritten as follow:

$$a\frac{du}{dt}+bu+cu^2-d = 0, \quad u(0) = 0 \quad (8)$$

where a, b, c and d are constant and gained as follow:

$$a = \left(m+\frac{1}{12}\pi D^3\rho\right) \quad (9)$$

$$b = 3.75\pi D\mu \quad (10)$$

$$c = \frac{67.289e^{(-5.03\phi)}}{8}\pi D^2\rho \quad (11)$$

$$d = mg\left(1-\frac{\rho}{\rho_s}\right) \quad (12)$$

Equation (6) is a nonlinear ordinary differential equation which could be solved by numerical techniques such Runge–Kutta method. We employed HPM to gain an analytical expression for acceleration movement of the particle. Subsequently, we compared our results with numerical solution using 4th order Runge–Kutta method with absolute error of $1e-7$ for convergence.

3. ANALYTICAL METHOD

Present problem was modeled by a nonlinear ordinary differential equation. In most cases, classic analytical solutions cannot be used. So, the equation should be solved using special methods. In recent decay, much attention has been devoted to the newly developed methods to obtain an exact solution of nonlinear equations; such Variational Iteration Method (VIM) and Homotopy Perturbation Method (HPM) [14-32, 38-47]

The Homotopy perturbation method (HPM) is one of the well-known methods to solve various nonlinear equations that are established by He [38-47] and has been used by many authors in [14-32] and the references therein to handle a wide variety of scientific and engineering applications: linear and nonlinear, and homogeneous and inhomogeneous as well. It was shown that this method provides improvements over existing techniques. One of the advantages of HPM is that it provides an analytical expression which easily could be used for similar conditions.

Therefore in the present work we examine the nonlinear equation of motion for a single falling non-spherical particle and try to obtain its solution using the *HPM*.

The Homotopy perturbation method is a combination of the classical perturbation and Homotopy techniques. To explain the basic idea of the HPM for solving nonlinear differential equations, consider the equation:

$$A(u) - f(r) \;=\; 0,\; r \in \Omega \tag{13}$$

With the boundary conditions of:

$$B(u, \partial u / \partial n) \;=\; 0,\; r \in \Gamma \tag{14}$$

where A is a general differential operator, B denotes a boundary operator, $f(r)$ is a given analytical function, Γ is the boundary of domain Ω and $\partial u/\partial n$ denotes differentiation along the normal drawn outwards from Ω. The operator A can be divided into two parts: a linear part L and a nonlinear part N. Therefore, Eq. (13) can be rewritten as follows:

$$L(u) + N(u) - f(r) \;=\; 0 \tag{15}$$

In case that the nonlinear Eq. (8) has no "small parameter", we can construct the following Homotopy:

$$H(v, p) \;=\; (1 - p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0 \tag{16}$$

where,

$$v(r, p) : \Omega \times [0,1] \to R \tag{17}$$

In Eq. (16), $p \in [0, 1]$ is an embedding parameter and u_0 is the first approximation that satisfies the boundary condition. We can assume that the solution of considered equation can be obtained as a power series in p, as following:

$$v = \sum_{i}^{n} p^i v_i = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + p^4 v_4 + \cdots \tag{18}$$

and the best approximation for solution is:

$$u \;=\; \lim_{p \to 1} v = v_0 + v_1 + v_2 + v_3 + v_4 + \cdots \tag{19}$$

When, Eq. (16) correspond to Eq. (13) and Eq. (19) becomes the approximate solution of the considered nonlinear equation. By increasing the number of series terms in Eq. (18) the accuracy of the result will be augmented and result could be used as an exact solution. Convergence and stability of this method is shown in [22].

4. APPLICATION OF HPM

4.1. General Equation

Considering Eq. (8), according to the HPM, the homotopy, Eq.(16) is gained as follow:

$$H(v,\, p) \;=\; a\frac{dv}{dt} + bv - \left(a\frac{du_0}{dt} + bu_0\right) + p\left(a\frac{du_0}{dt} + bu_0\right) + p(cv^2 - d) = 0 \tag{20}$$

where in Eq. (16), linear parts of v and u_0 are as follows:

$$L(v) = a\frac{dv}{dt} + bv, \qquad L(u_0) = a\frac{du_0}{dt} + bu_0\,, \tag{21}$$

In our modeling it was assumed that the particle is accelerated from rest. Therefore, $\frac{du_0}{dt} = u_0 = 0$, and substituting v from Eq. (18) into Eq. (20) and some simplification and rearranging based on powers of p-terms, we have:

$$p^0 \quad : \quad a\frac{dv_0}{dt} + bv_0 = 0, \qquad t = 0, \qquad v_0 = 0 \tag{22}$$

$$p^1 \quad : \quad a\frac{dv_1}{dt} + bv_1 + cv_0^2 - d = 0, \qquad t = 0, \qquad v_1 = 0 \tag{23}$$

$$p^2 \quad : \quad a\frac{dv_2}{dt} + bv_2 + 2cv_0v_1 = 0, \qquad t = 0, \qquad v_2 = 0 \tag{24}$$

$$p^3 \quad : \quad a\frac{dv_3}{dt} + bv_3 + cv_1^2 + 2cv_0v_1 = 0, \qquad t = 0, \qquad v_3 = 0 \tag{25}$$

$$p^4 \quad : \quad a\frac{dv_4}{dt} + bv_4 + 2cv_1v_2 + 2cv_0v_3 = 0, \qquad t = 0, \qquad v_4 = 0 \tag{26}$$

$$p^5 \quad : \quad a\frac{dv_5}{dt} + bv_5 + cv_2^2 + 2cv_1v_3 + 2cv_0v_4 = 0, \qquad t = 0, \qquad v_5 = 0 \tag{27}$$

$$p^6 \quad : \quad a\frac{dv_6}{dt} + 2cv_0v_5 + 2cv_2v_3 + 2cv_1v_4 = 0, \qquad t = 0, \qquad v_6 = 0 \tag{28}$$

$$\vdots$$

By continuing above terms, higher accuracy will be gained. Solving Eqs. (22) to (28) considering appropriate initial conditions, we have:

$$v_0 = 0 \tag{29}$$

$$v_1 = \frac{d}{b} - \frac{de^{-\frac{bt}{a}}}{b} \tag{30}$$

$$v_2 = 0 \tag{31}$$

$$v_3 = -\frac{cd^2\left(\left(ae^{\frac{bt}{a}}/b\right) - 2t - \left(ae^{-\frac{bt}{a}}/b\right)\right)}{b^2a} \tag{32}$$

$$v_4 = 0 \tag{33}$$

$$v_5 = \frac{c^2d^3e^{-\frac{bt}{a}}}{b^5} - \frac{1}{a^2b^5}\left(c^2d^3\left(-2a^2e^{\frac{bt}{a}} + 2b^2t^2 + 2a^2e^{-\frac{bt}{a}} + 2abt + 4ae^{-\frac{bt}{a}}bt + a^2e^{-\frac{2bt}{a}}\right)e^{-\frac{bt}{a}}\right) \tag{34}$$

$$v_6 = 0 \tag{35}$$
$$\vdots$$

Thus, u will be gained as follow:

$$u = \lim_{p\to 1}(v_0 + pv_1 + p^2v_2 + p^3v_3 + p^4v_4 + p^5v_5 + p^6v_6 \ldots)$$
$$= 0 + \frac{d}{b} - \frac{de^{-\frac{bt}{a}}}{b} + 0 - \frac{cd^2\left(\left(ae^{\frac{bt}{a}}/b\right) - 2t - \left(ae^{-\frac{bt}{a}/b}\right)\right)}{b^2a} + 0 + \frac{c^2d^3e^{-\frac{bt}{a}}}{b^5}$$
$$-\frac{1}{a^2b^5}\left(c^2d^3\left(-2a^2e^{\frac{bt}{a}} + 2b^2t^2 + 2a^2e^{-\frac{bt}{a}} + 2abt + 4ae^{-\frac{bt}{a}}bt + a^2e^{-\frac{2bt}{a}}\right)e^{-\frac{bt}{a}}\right) + 0 + \ldots \tag{36}$$

As it is obvious, solution of terms vary periodically and in each step more duration of the particle motion is covered. We continued Eq. (36) up to 15th power of p to gain an exact solution for unsteady particle movement.

4.2. Real Conditions

Mentioned method was applied for real combination of solid-liquid. A single Aluminum particle of 3 (mm) equivalent diameter was assumed to fall in an infinity medium of water, glycerin or ethylene-glycol. Required physical properties of selected materials are given in Table 1.

Table 1
Physical Properties of Materials

Material	*Density [kg/m³]*	*Viscosity [Kg/m.s]*
Water	996.51	0.001
Ethylene-glycol	1111.40	0.0157
Glycerin	1259.90	0.799
Aluminum	2702.00	–

Inserting above properties into Eqs. (9) amd (12), three different combination are gained which are classified in Table 2.

Table 2
Selected Coefficient of Eq. (8)

Solid	*Fluid*	*a*	*b*	*c/e^(–5.03*φ)*	*d*
Aluminum	Water	0.00001086370637	0.00003534187500	0.2369819790	0.00002366559889
	Ethylene-glycol	0.00001167579198	0.0005548674375	0.2643041931	0.00002207137046
	Glycerin	0.00001272544566	0.02823815812	0.2996192666	0.00002001076534

By substituting above coefficients into Eq. (8), different nonlinear equations are achieved for different sphericity coefficient. Homotopy Perturbation Method was applied to gained equations and results were compared with numerical method. Figures 3 to 6 depict the falling velocity profiles of the particles versus time for different sphericity and fluids.

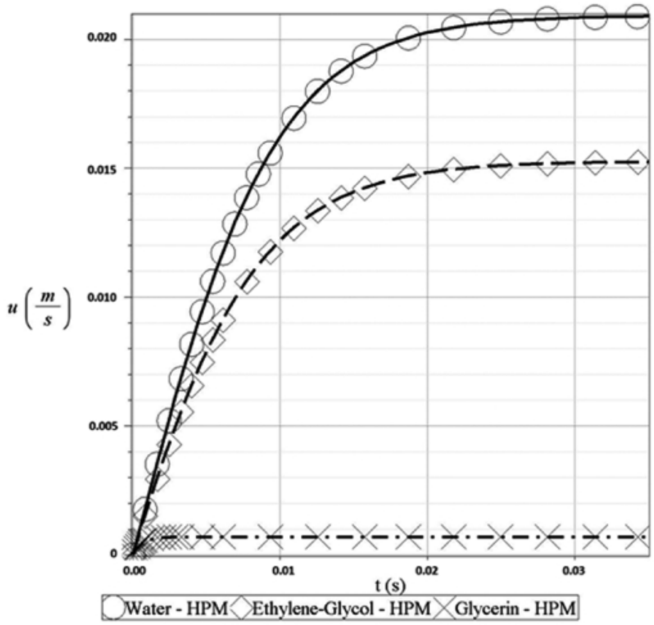

Figure. 3: Velocity Profiles of Particle with ϕ = 0.3 for Different Fluids, Lines Denote the Numerical Results: (Solid-line: Water, Dash-line: Ethylene-Glycol, Dash-Dot-line: Glycerin)

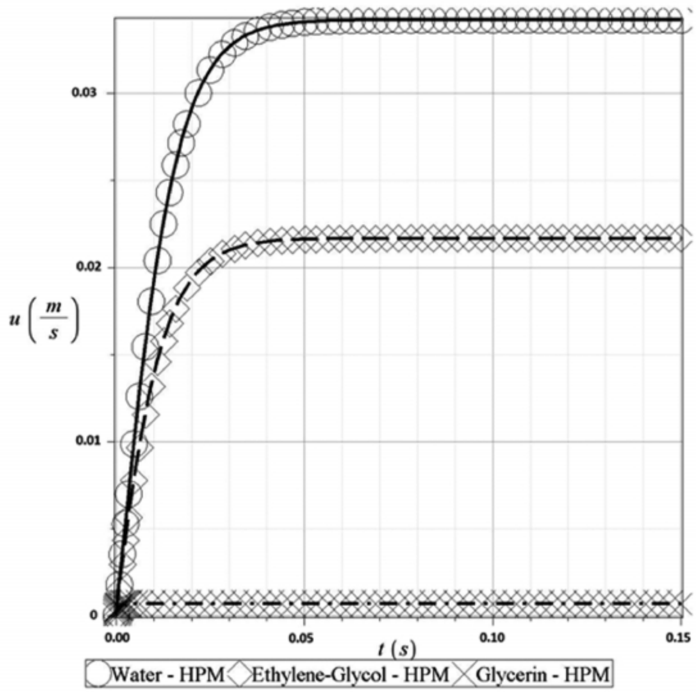

Figure. 4: Velocity Profiles of Particle with ϕ = 0.5 for Different Fluids Lines Denote the Numerical Results: (Solid-line: Water, Dash-line: Ethylene-Glycol, Dash-Dot-line: Glycerin)

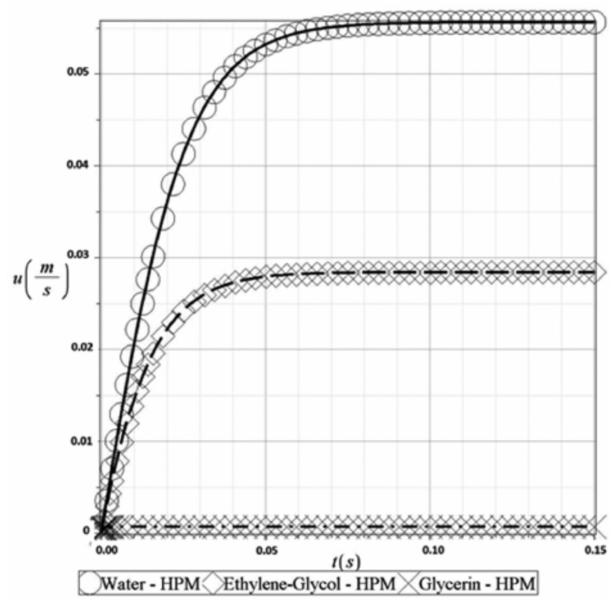

Figure. 5: Velocity Profiles of Particle with $\phi = 0.7$ for Different Fluids, Lines Denote the Numerical Results: (Solid-line: Water, Dash-line: Ethylene-Glycol, Dash-Dot-line: Glycerin)

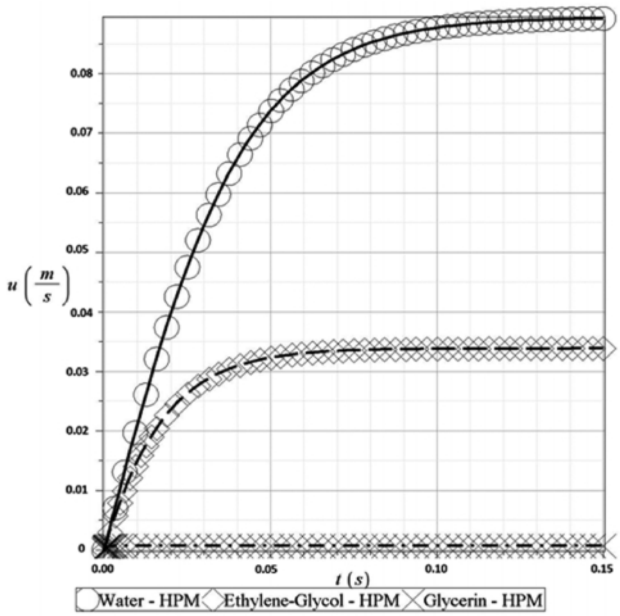

Figure. 6: Velocity Profiles of Particle with $\phi = 0.9$ for Different Fluids

Lines denote the numerical results: (Solid-line: Water, Dash-line: Ethylene-Glycol, Dash-Dot-line: Glycerin)

Presented results demonstrate an excellent agreement between HPM and numerical method. Initially, the gravity force makes the particle fall in the liquid. Velocity of particle is increased monotically while reaching the constant value of the terminal velocity. The velocity profiles show that the particle terminal velocities reduced gradually with augmenting the liquid viscosity. Also acceleration duration is increased by reduction of liquid viscosity. The effect of particle sphericity on velocity profile is shown in figures 6 to 8 for each analyzed liquid.

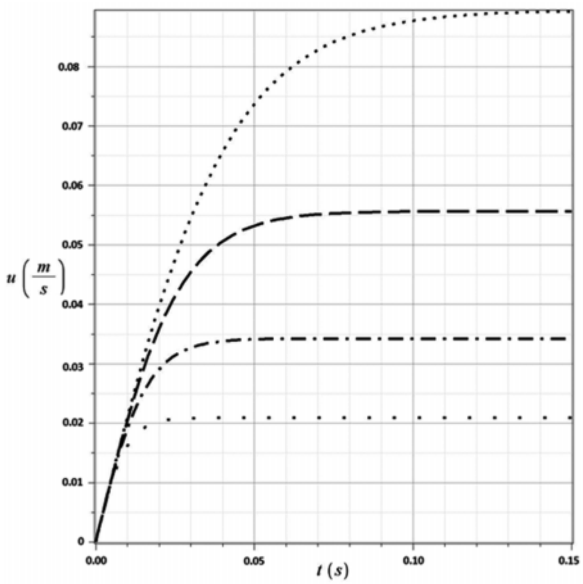

Figure. 7: Velocity Profiles of Particles with Different Sphericities Falling in Water Lines Denote the Numerical Results: (Dotted-line: $\phi = 0.9$, Dash-line: $\phi = 0.7$, Dash-Dot-line: $\phi = 0.5$, Space-Doted-line: $\phi = 0.3$)

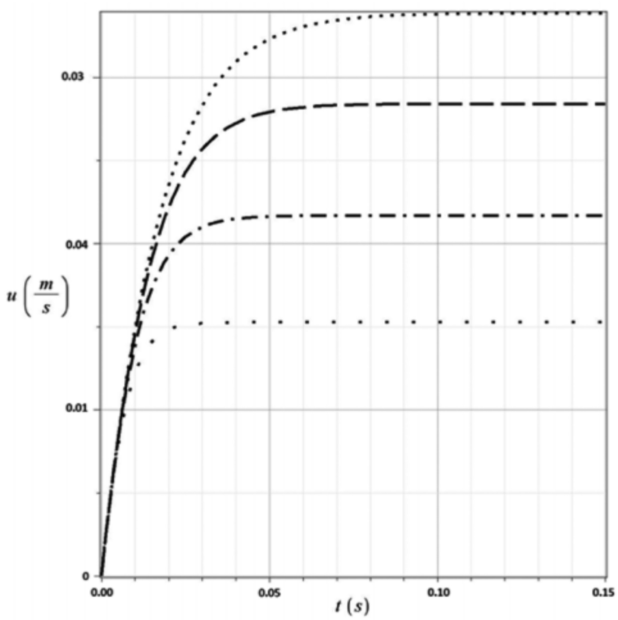

Figure. 8: Velocity Profiles of Particles with Different Falling in Ethylene-Glycol Lines Denote the Numerical Results: (Dotted-line: $\phi = 0.9$, Dash-line: $\phi = 0.7$, Dash-Dot-line: $\phi = 0.5$, Space-Doted-line: $\phi = 0.3$)

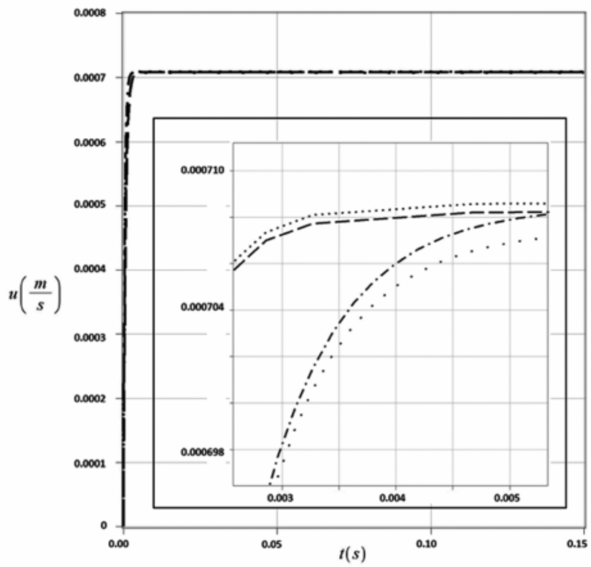

Figure. 9: Velocity Profiles of Particles with Different Falling in Glycerin (With Zoomed Picture for more detail.) Lines Denote the Numerical Results: (Dotted-line: $\phi = 0.9$, Dash-line: $\phi = 0.7$, Dash-Dot-line: $\phi = 0.5$, Space-Doted-line: $\phi = 0.3$)

With the increase of the particle sphericity, the terminal velocity is increased significantly. Also acceleration duration and displacement are larger for particle with bigger sphericity. Outcomes show that for high viscose mediums such glycerin, the difference between terminal velocities of particles is reduced significantly and could be neglected practical applications. Accelerations of particles during falling process for different conditions are illustrated in figures 10 to 12.

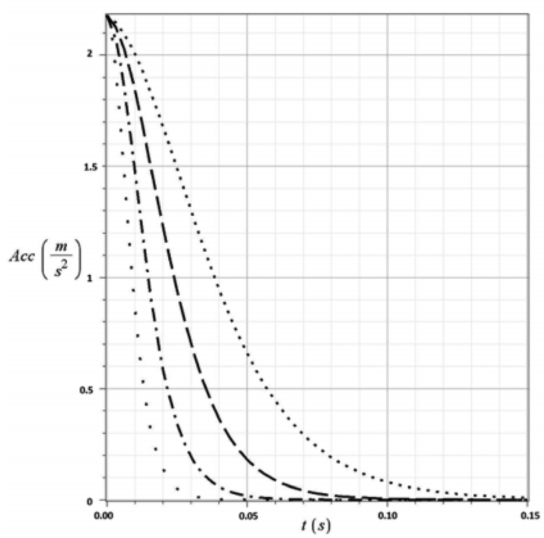

Figure. 10: Acceleration Variation for different Particle Sphericities Falling in Water .(Dotted-line: $\phi = 0.9$, Dash-line: $\phi = 0.7$, Dash-Dot-line: $\phi = 0.5$, Space-Doted-line: $\phi = 0.3$)

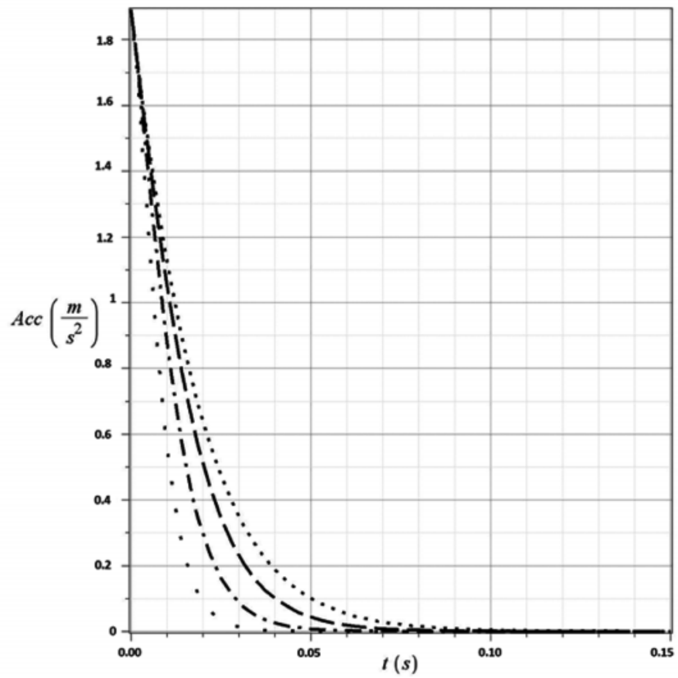

Figure. 11: Acceleration Variation for Different Particle Sphericities Falling in Ethylene-Glycol. (Dotted-line: $\phi = 0.9$, Dash-line: $\phi = 0.7$, Dash-Dot-line: $\phi = 0.5$, Space-Doted-line: $\phi = 0.3$)

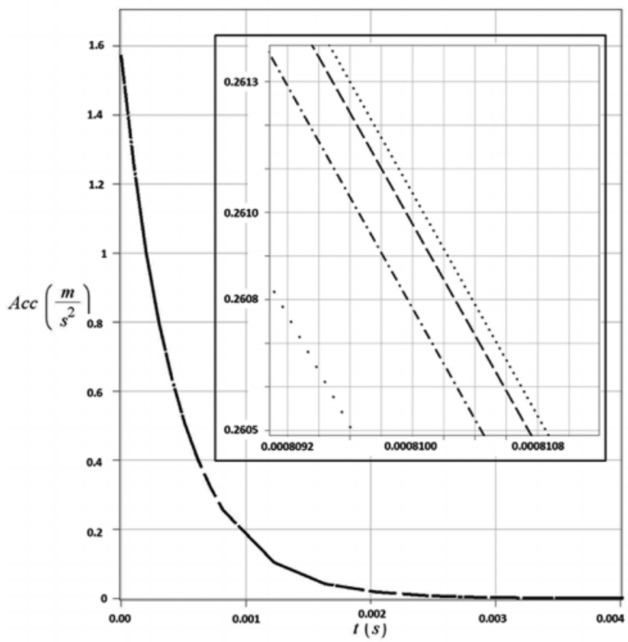

Figure. 12: Acceleration Variation for Different Particle Sphericities Falling in Glycerin. (with Zoomed Picture for more Details) (Dotted-line: $\phi = 0.9$, Dash-line: $\phi = 0.7$, Dash-Dot-line: $\phi = 0.5$, Space-Doted-line: $\phi = 0.3$)

Outcomes demonstrate that particle with higher sphericity brings higher acceleration duration and displacement. Also for a given time, before reaching the terminal velocity (zero acceleration), smaller liquid viscosity causes higher acceleration.

Using definite and indefinite integration for HPM solution, Eq. (36), the instantaneous value and general equation of displacement could be found for the bubble during rising procedure. Figure 13 illustrates the effect of liquid phase viscosity on unsteady particle movement while the sphericity of the particle is considered to be.

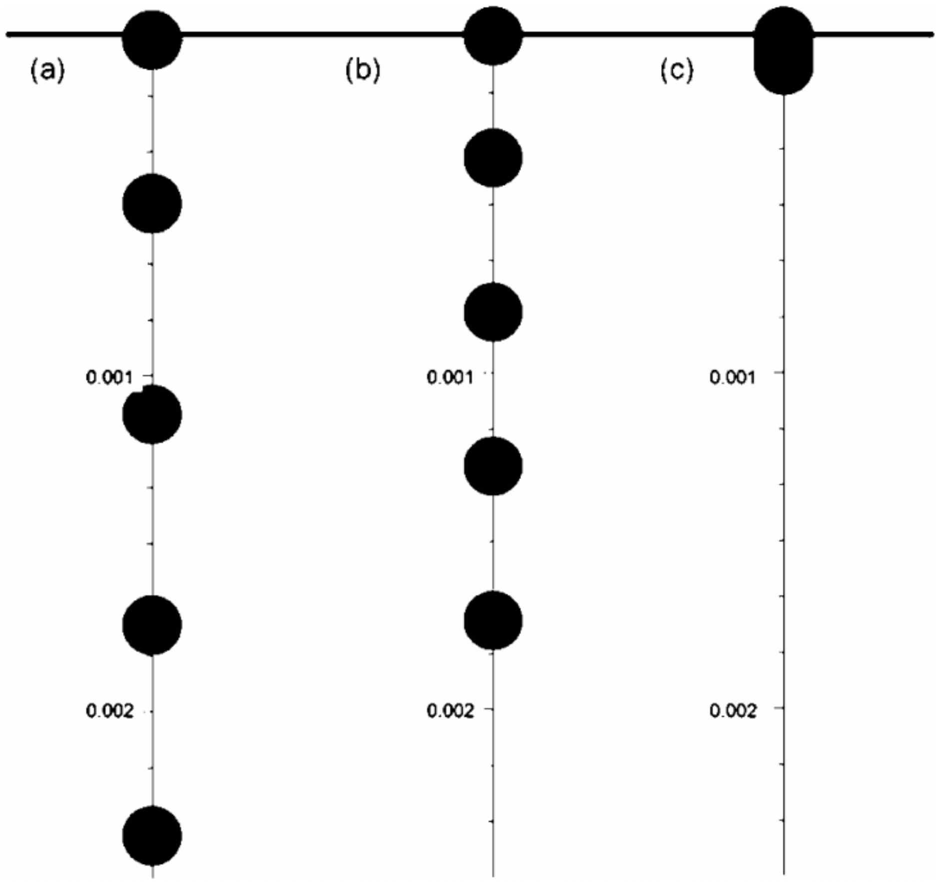

Figure. 13: Positions of Falling Particle for different Fluids, $\phi = 0.3$, Numbers are in Meter(m) Time Step = 0.03 [s], (a) Water, (b) Ethylene-glycol, (c) Glycerin

As it is obvious in figure 13, by increase of liquid viscosity, the falling velocity of the particle is decreased significantly and particle transverses smaller height of the liquid column. These value at $t = 0.003$(s) and for a non-spherical particle with $\phi = 0.3$, is approximately reduced from 0.00048(m) for water to 0.000020(m) for Glycerin. Figure 14 demonstrates the effect of particle sphericity on unsteady motion of the particle in water.

Results show that the particle with larger sphericity moves faster rather than smaller ones Figures 13 and 14 depict the quality and quantity of falling process for non-spherical particles appropriately.

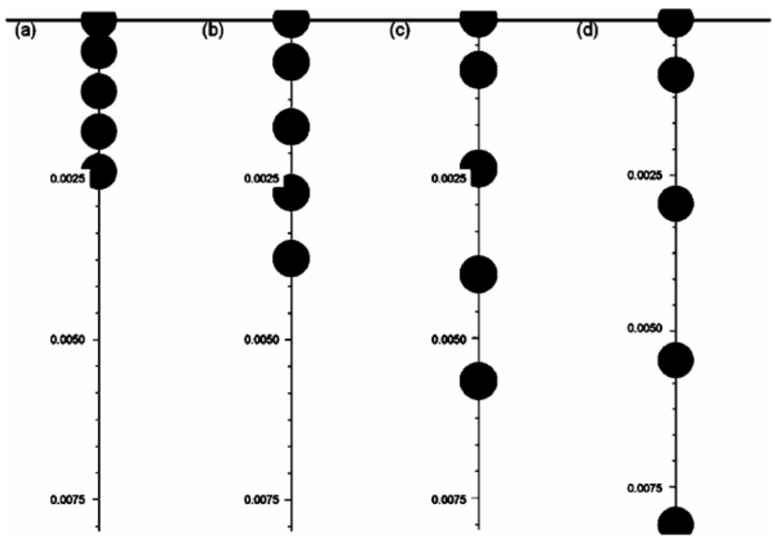

Figure. 14: Displacements for Particles with different Sphericity in Water, Time Step: 0.003 (s), (a) $\phi = 0.3$, (b) $\phi = 0.5$, (c) $\phi = 0.7$, (d) $\phi = 0.9$

5. CONCLUSIONS

In this paper, the analytical Homotopy Perturbation Method (HPM) is applied to obtain the solution of falling bubble nonlinear equation of motion with drag coefficient in form of Eq. (5). Equation was solved generally and solution was extended for some real combinations of solid-liquid. Instantaneous velocity, acceleration and position were obtained as results and outcomes were compared with numerical Runge–Kutta method solution. Very good agreement has been seen between numerical and the current analytical method. This work approved the simplicity and capability of homotopy perturbation method and explains an exact expression which is valuable for engineering problem in the field of powder technology. This method could be used in wide range of two-phase systems problems.

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