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LINEAR AND NONLINEAR ANALYTICAL INVESTIGATION OF IMPACT ON TWO ADJACENT STRUCTURES

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Abstract: *accurate Dynamic response modeling with respect to a displacement component is not possible in most structural systems. Generally, dynamic response of a structure can not be approximated by a single degree of freedom system (SDOF) and it is more close to Multi Degree of Freedom (MDOF).That's, because usually, displacement function varies with time, in addition to response amplitude time dependency. In these systems, to identify structural mass displacement in each time interval, more independent displacement coordinates are required. At this paper, the assumptions and limits of SDOF and MDOF systems are be studied, and then impact phenomenon theories considering a linear and nonlinear model would be discussed. The impact structural computer software would be presented next and finally, the results of software analysis would be compared to the manual analysis. The results show that, there is a good compatibility between software and manual results for low rise buildings.*

Keywords: *Impact, Adjacent, Analysis, Nonlinear, Single Degree of Freedom, Multi Degree of Freedom*

1. INTRODUCTION

The first research on impact effect in structural engineering was done by Anagnostopoulos in 1988[8]. He assumed the structure as a Single Degree of Freedom System and since there are not enough data of the story response quantities, the model could not describe the performance of a multi story building. Skrikerud and Wolf studied the response of a nuclear tank structure against impacts in 1980[9]. Weatermo also studied impact prevention between buildings by the type of the connection between them in 1989[10]. Maison and Kasai presented formulation and MDOF systems equation of motions solution for a special type of impact in 1990[4]. They have also presented a software named SLAM. They have also presented the formulation andsolution of MDOF equations of motion for two multi story building in 1992[13].Filatrault et. al. put forward the step by step nonlinear dynamic modeling of adjacent buildings in 1994 by ANSR-1 software [15]. They also investigated the impact effect between two steel frames

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of 8 and 3 story buildings on shaking table in 1995.[17]. Jeng presented the gap distance between the adjacent buildings applying random vibration theory in 1997[21]. He also studied the impact effect in tall buildings in 2000[2]. in this research, the impact effect for two adjacent buildings has been studied by SDOF and MDOF modeling of the buildings and considering the linear and nonlinearity, the results have been compared.

1.1. Impact Principle

Impact is a physical phenomenon that happens when two bodies strike each other. In structural engineering, when two structures hit each other during strong ground motion, the force between these two structures which are not far enough from each other is known as impact. The two structures have two different dynamic properties, usually. Generally, different phase oscillations cause the adjacent structures to strike one another. According to the damages of structures due to impact forces between adjacent buildings, especially in commercial and crowded cities, are more common in tall buildings,impact has been a very important issue for the researchers and governments. As an example, in Mexico City earthquake in 1985, more than 15 percentages of the 330 structures which were partially or fully damaged had undergone the impact effect [3].

Impact force is a kind of motion quantity force which increases the available stress to the values more than design limit that will make the structure overturn. The impact seismic damages in adjacent structures have been seen in different parts of structures.

The research on the recent earthquake impact structural damages indicates that the impact can be categorized in four different kinds, including mid-Column impact, heavier adjacent building impact, taller adjacent building impact and eccentric impact. Mid-Column impact happens in adjacent buildings floors are not in the same level therefore they hit each other in the columns. Some adjacent buildings differ each other in the weight which might be because of structural type or structural usage; therefore the lighter structure would have more displacements due to the hit from the heavier one. This effect is known as heavier adjacent building impact. When two buildings beside each other have different heights the taller one would be affected by the shorter building impact which causes taller adjacent building impact. Finally the eccentric impact occurs when two adjacent structures have eccentricity from each other and one edge of one of them would hit the other building's edge [2].

In this paper the structural modeling of this phenomenon has been considered and different analysis methods have been presented. The assumptions have been described with impact phenomena theories and to better analyze it, the process has been applied to SDOF and MDOF systems. Finally the available software is presented and used to analyse the problem.

2. STUDY THE IMPACT EFFECT IN ADJACENT BUILDINGS AS A SDOF

The following assumptions have been considered at this method [18]:

1. Each building has its own mass, stiffness and damping.
2. Each building is supposed only as linear SDOF.

Figure1 shows the ideal modeling of two adjacent buildings.

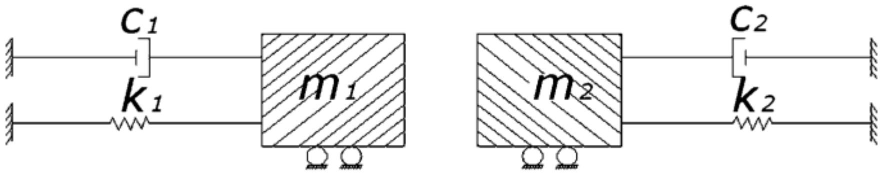

Figure.1: Ideal Modeling of Two Adjacent Buildings

To determine the adequate distance between two buildings the equations of motions can be written as Eq.(1).

$$m_i\ddot{x}_i(t) + c_i\dot{x}_i(t) + k_i x_i(t) = -m_i\ddot{x}_g \quad (1)$$

c_i and k_i can be shown as Eq.(2).

$$C_i = 4\pi\xi_i m_i/T_i, \qquad k_i = 4\pi^2 m_i/T_i^2 \quad (2)$$

Subsituting Eq.(2) in Eq.(1), Eq.(3) is gained.

$$\ddot{x}_i(t) + (4\pi\xi_i/T_i)\dot{x}_i(t) + (2\pi/T_i)^2 x_i(t) = -\ddot{x}_g(t) \quad (3)$$

Where ξ_i is damping percentage and T_i is vibration period of each building. This equation is mass independent. solving Eq.(1), step by step, the values of x_i, $\dot{x}_i$ and $\ddot{x}_i$ as displacement, velocity and acceleration can be determined in time intervals.

Two buildings with different periods have been studied and each building response has been calculated. The absolute value of two structure displacement, in each time interval could be determined and the maximum value can be described as two building separation distance.

To determine the tension pressure connection, when the buildings are connected to each other, the modeling can be assumed as Fig.2 and Fig.3.

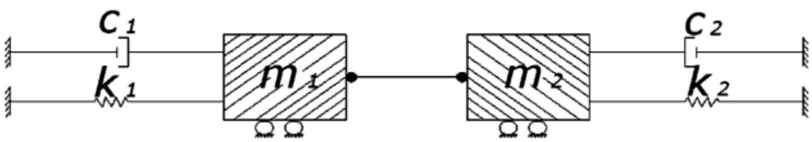

Figure.2: Two Structures with Tension-pressure Connection Modeling

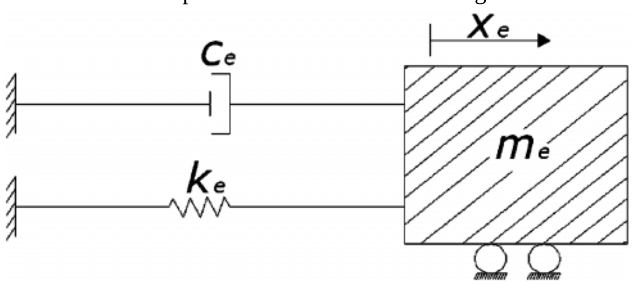

Figure.3: Equivalent Model for Fig.2

By tension-pressure connection of two buildings to each other, the buildings would work as one whole mass and the equation of motion can be written as Eq.(4) or Eq.(5).

$$(m_1 + m_2)\ddot{x}_e(t) + (c_1 + c_2)\dot{x}_e(t) + (k_1 + k_2)x_e(t) = -(m_1 + m_2)\ddot{x}_g(t) \quad (4)$$

$$m_e\ddot{x}(t) + c_e\dot{x}_e(t) + k_e x_e(t) = -m_e\ddot{x}_g(t) \quad (5)$$

Eq.(5) can be solved step by step and the values of x_e, $\dot{x}_e$ and $\ddot{x}_e$ which present displacement, velocity and acceleration of equivalent structures respectively can be calculated in determined time interval and the connection force can be calculated as Eq.(6) and Eq.(7).

$$p(t) - k_1 x_e(t) - c_1 \dot{x}_e(t) = m_1 a(t) \tag{6}$$

$$a(t) = \ddot{x}_e(t) + \ddot{x}_g(t) \tag{7}$$

where $p(t)$ is the connection force between two buildings which can be determined in each time interval. This model is an approximation of two buildings, general performance and has better results for the structures who dominant mode is the first mode for instance low rise and symmetric buildings it also has more simple calculations than other methods. Because the impact and it's effects have not been considered, the data of structure responses like stories lateral displacements and story shear can not be obtained.

3. STUDY THE IMPACT EFFECT IN ADJACENT BUILDINGS AS A MDOF

Practically, the buildings are multi degree of freedom; therefore MDOF modeling of structures would give more accurate results than SDOF and shows that structure hase a better performance against impact.

3.1. Assumptions and Limitations of MDOF Model

Although the important properties of the problem are available, to control and get better results, the following assumptions should be considered:

1. The two considered buildings which hit each have identified mass and stiffness other laterally.
2. As, tall buildings usually have concrete floor systems and their plain stiffness is big, it is better to consider the floors as a rigid system, so every floor has three degrees of freedom including two transformations and one rotation around vertical axis which is considered in the floor mass center.
3. Because, the floor diaphragm has been considered rigid, the impact forces would distribute between all structural members connected to the floor, therefore impact would affect the general response of the structure.

3.2. Linear Time History Analysis Considering Two Separate

The following assumptions have been considered in this method:

1. Pounding is available in a floor height in any building and the situation is already known. The building geometry is like it has a definite contact point definitely. It should be mentioned that, application of these methods for case that, there is more than one contact point leading to the non exact results.
2. To model the contact between the two buildings linear elastic spring has been applied to consider the small deformations of the impact location. It should be considered that the performance of the structure during impact is complex and the real response may include pression wave with high frequencies in floor diaphragm and energy absorption in inelastic phase.

3. Impact problem is idealized in two cases. In case1 the building oscillates itself and in case 2 the connected buildings oscillate.

3.2.1. Analysis of Idealized Model

Analysis can be studied by the following two cases.

Case 1 Separated Building: In this case the buildings do not have any contact with each other and just oscillate themselves. The equation of motion for a MDOF under earthquake can be written as Eq.(8).

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) \;=\; -mr\ddot{u}_g(t) \tag{8}$$

where $u(t)$, $\dot{u}(t)$ and $\ddot{u}(t)$ are floor displacement, velocity and acceleration as a column vector in t incident. m is the diagonal mass matrix, c is the damping matrix assumed as a linear combination of mass and stiffness, k is the stiffness matrix, r is the column vector of earthquake efficiency coefficient including dynamic degrees of freedom displacement up to unique displacement in building base and $\ddot{u}_g(t)$ is the earthquake acceleration at building base in t incident.

The equations are linear and coupled and could be uncoupled applying modal properties as Eq.(9).

$$\ddot{Z}_i(t) + 2\xi_i\omega_i\dot{Z}_i(t) + \omega_i^2 Z_i(t) \;=\; -P_i\ddot{u}_g(t) \tag{9}$$

Were Z parameters are generalized displacement, velocity and acceleration of i'th mode in t incidence. ω_i is the angular frequencies of i mode, ξ_i is the i mode damping ratio and ϕ_i is i mode column vector. The independent equations could be solved applying step by step method that ground acceleration time steps varies linearly along the step and finally the generalized responses of displacement and velocity in t_2 is written as Eq.(10).

$$\begin{aligned} Z_i(t_2) \;=\; & e^{-\xi_i\omega_i\Delta t}(\{Z_i(t_1) + P_i[\ddot{u}_g(t_1)/\omega_i^2 - 2\xi_i R/\omega_i^3]\}\cos\omega_{Di}\Delta t + 1/\omega_{Di} \\ & \{\dot{Z}_i(t_1) + \xi_i\omega_i Z_i(t_1) + P_i[\xi_i\ddot{u}_g(t_1)/\omega_i - R(2\xi_i^2 - 1)/\omega_i^2]\}\sin\omega_{Di}\Delta t) + \\ \dot{Z}_i(t_2) = & e^{-\xi_i\omega_i\Delta t}\{[\dot{Z}_i(t_1) + P_i R/\omega_i^2]\cos\omega_{Di}\Delta t + 1/\omega_{Di}\; P_i[-\ddot{u}_g(t_1)/\omega_i^2 + 2\xi_i R/\omega_i^3 - R\Delta t/\omega_i^2] \\ & [-\xi_i\omega_i\dot{Z}_i(t_1) - \omega_i^2 Z_i(t_1) - P_i u_g(t_1) + P_i\xi_i R/\omega_i]\sin\omega_{Di}\Delta t\} - P_i R/\omega_i^2 \end{aligned} \tag{10}$$

To calculate the other steps responses, each response is the initial values for the other step and so on. The general displacement can be written as Eq.(11).

$$u(t) \;=\; \sum_{i=1}^{n}\phi_i Z_i(t) \tag{11}$$

Case 2. Connected Building

The buildings contact each other and the equation of motion can be written as Eq.(12).

$$m\ddot{u}(t) + \bar{c}\dot{u}(t) + \bar{k}u(t) \;=\; -mr\ddot{u}_g(t) + b \tag{12}$$

Where $\bar{c}$ is the damping matrix including of mass and stiffness matrix and $\bar{k}$ is the stiffness matrix at initial state plus local elastic spring stiffness defined for the connection between the buildings and can be defined as Eq.(13).

$$
\begin{aligned}
k_{ii} &= k,\ k_{ik} = k_{ki} = -k \\
k_{ij} = k_{ji} &= -ky_A\ ,\ k_{il} = k_{li} = ky_B \\
k_{jj} &= ky_A^2\ ,\ k_{jk} = k_{kj} = ky_A \\
k_{kk} &= k\ , k_{jl} = k_{lj} = -ky_A y_B \\
k_{kl} &= k_{lk} = -ky_B, k_{ll} = ky_B^2
\end{aligned} \tag{13}
$$

Where b is the column vector of static preloading forces to calculate the distance effect at inertia state and can be calculated as Eq.(14).

$$
\begin{aligned}
b_i &= ks,\ b_k = -ks \\
b_j &= -ksy_A, b_l = ksy_B
\end{aligned} \tag{14}
$$

where i and k indexes are for transformation release degree in building A and B and j and l are the torsion release degree in building A and B nespectinely.The equations can be normalized as Eq .(15) which the second status modal properties have been applied.

$$\ddot{Z}_i(t) + 2\xi_i \dot{Z}_i(t) + \omega_i^2 Z_i(t) = -P_i \ddot{u}_g(t) + B_i \tag{15}$$

The uncoupled equations of motion can be solved the same as the case B. The only difference is in case B,where the generalized displacement should be separated in two main components of static and dynamic displacements as Eq.(16). and finally the response in case B can be written as Eq.(17).

$$Z_i^s = \frac{B_i}{\omega_i^2}\ ,\ Z_i(t) = Z_i^s + Z_i^d(t) \tag{16}$$

$$u(t) = \sum_{i=1}^{n} \phi_i Z_i(t) \tag{17}$$

3.2.2. Nonlinear Time History Analysis

The nonlinear modal time history analysis done by SAP2000 software is a progressive model of Vilson rapid method. It is a very strong method and is designed for structural systems that basically have linear performance with a few limited members of nonlinear performance. In FNA method analysis the nonlinearity of of structure is just in link members. The applied link member used to study the impact phenomena is the link GAP element that can present the element properties as linear or nonlinear as follows:

1. The element has the linear properties including linear damping and stiffness.
2. The element has the nonlinear properties including nonlinear stiffness and open parameter that reveals the element performance compact boundary as shown in Fig.4.

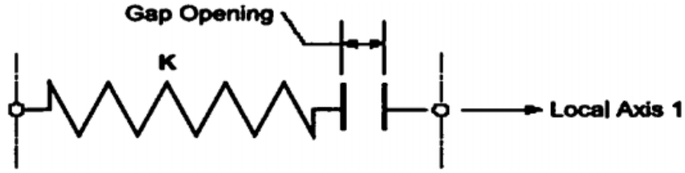

Figure.4: GAP Element and its Performance

To prevent nonlinear properties of link element in structural vibration modes, the values of linear damping and stiffness for the element is assumed to be zero. SAP2000 capability in defining these two properties separately, makes it possible to estimate the required time history modes more accurately.

Eq.(18) shows the dynamic equilibrium equation of a linear elastic structure with defined nonlinear link members.

$$M\ddot{u}(t) + c\dot{u}(t) + k_L u(t) + R_{NL}(t) \; = \; R(t) \tag{18}$$

K_L is the stiffness matrix of linear elastic members except link member, C is the respective damping matrix, M is the diagonal mass matrix, $R_{NL}(t)$ is the nonlinear degrees of freedom force vector of link elements.U, U' and U'' are the respective displacement, velocity and acceleration to the ground respectively and $R(t)$ is the external force vector. The modal analysis has been done based on the general stiffness matrix of K and general mass matrix of M. Applying standard methods, equation 19 is obtained that presents modal equilibrium equation.

$$I\ddot{Y}(t) + A\dot{Y}(t) + \Omega^2 Y(t) \;\; = \;\; q(t) - q_{NL}(t) \tag{19}$$

The elements of the equation 19 include structural frequencies square matrices, damping matrix A, mass matrix I, input modal force matrix $q(t)$, nonlinear elements modal forces matrix $q_{NL}(t)$, modal displacement coefficient vector $Y(t)$ and mode shape matrix Φ.

$$\begin{aligned}
\Omega^2 &= \Phi^t k\Phi, \\
\Lambda &= \Phi^T C\Phi \\
I &= \Phi^T M\Phi \\
q(t) &= \Phi^T R(t) \\
q_{NL}(t) &= \Phi^T [R_N(t) - k_e u(t)] \\
u(t) &= \Phi Y(t)
\end{aligned}$$

It should be mentioned that, the above equations are dependent on each other unlike the linear analysis. Generally, nonlinear forces make the modes independent on each other because they are dependant on modal displacements. Along the time history analyze of Structure, the vibration modes modify in a manner that the analysis results could be calculated based on real stiffness and other defined nonlinear parameters. It should be mentioned that solving above equations is related to accurate definition of nonlinear forces by modal forces. Therefore mass moment of inertia should be available in all nonlinear degrees of freedom and the possible modes of vibration should be calculated by modal analysis with eigen values or Ritz vector method.

The solution procedure is in such a way that, nonlinear modal equations vary linearly in all time intervals as trial error and accurate method like real integration are applied to solve the equations. The trial error goes till convergence of analysis is gained. If the analysis does not come to a convergence, the time intervals should be divided to smaller sub procedure and the equation solving procedure should be followed again. The control parameters of trial error procedure are force values convergence control, energy values convergence control, maximum and minimum values of sub procedures and maximum number of trial errors.

Each time interval with specified length of dt is divided to smaller sub procedures to gain the convergence. In each sub procedure the analysis equation are solved until right hand variations of the equations get to the smaller values than acceptable tolerances presented.If in maximum identified trial error, it gets to unacceptable results, the sub procedure should be divided into half and the trial error should be done again.

To control the convergence based on energy values, the nonlinear forces work is compared to the total work of all forces. If the difference of these two elements is bigger than the allowable tolerances, the sub procedure should be divided into half and the analysis should be continued.

If the results of analysis are acceptable along a sub procedure in both energy and force, he next sub procedure is considered by doubling the length of the previous sub procedure but the length should not exceed the allowable values and the same philosophy governe the minimum values too.

The maximum number of trial error is between two identified numbers and the real value of this parameter is considered automatically by the software in such a way that a sufficient balance is available between trial errors and dividing time intervals. The more the values of sub procedure are the more would be the values of trial error .

4. GENERAL PROCESS OF IMPACT ANALYSIS FOR SDOF SYSTEMS

The effect of impact theory process on SDOF systems would be discussed at this part.Fig.5 and Fig.6 show the properties of the systems in two statuses.

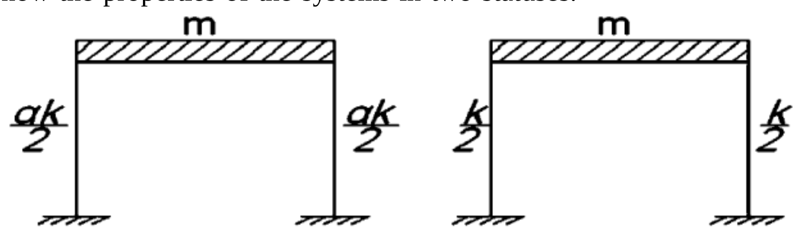

Figure.5: SDOF Disconnected Systems

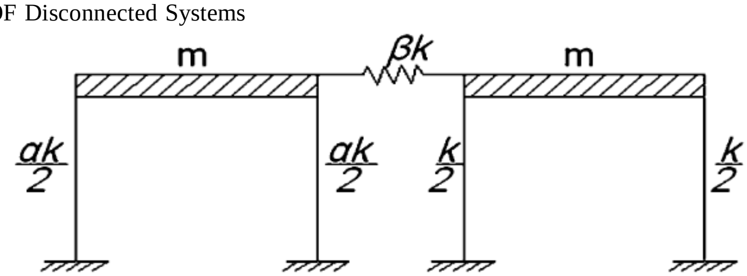

Figure.6: SDOF Systems in Impact Status

Eq.(20) shows un-damped free vibration equation of motion of a MDOF system as a matrix equation.

$$[M]\{\ddot{u}\}+[K]\{u\} = \{0\} \tag{20}$$

In free vibration, the system motion is simple therefore, displacement vector can be written as Eq.(21).

$$\{u\} = \{\bar{u}\}\sin\omega t \tag{21}$$

Eq.(22) is obtained by double derivation with respect to time.

$$\{\ddot{u}\} = -\omega^2\{u\} \tag{22}$$

Replacing Eq20 in Eq18, the eigen value equation is obtained as Eq.(23).

$$([K]-\omega^2[M])\{u\} = \{0\} \tag{23}$$

Classically, homogenous equilibrium equations set have an inevitable response unless the coefficient matrix determinant is equal to zero as Eq.(24).

$$\det([K]-\omega^2[M]) = \{0\} \tag{24}$$

Eq.22 is called system frequencies equation. Determinent distribution for a N- degree of freedom system would result in a N degree of freedom equation with respect to frequency parameter ω^2. N roots of this equation present N possible vibration modes of system. The mode with the least frequency is the first mode and the next bigger mode is the second and so on.

SDOF systems equations of motion due to acceleration can be written as Eq.(25) and Eq.(26).

$$m\ddot{u}+ku = -m\ddot{u}_g(t) \tag{25}$$

$$\alpha m\ddot{u}+\alpha ku = -\alpha m\ddot{u}_g(t) \tag{26}$$

α is the numerical coefficient shown in Fig.5.

To transform equations25 and 26 to normal coordinate, eigen vectors should be found at first. SDOF systems have just one angular frequency; the values can be obtained easily as Eq.(27) and Eq.(28).

$$|k-\omega^2 m| = 0 \Rightarrow \omega^2 = \frac{k}{m} \tag{27}$$

$$|\alpha k-\omega^2\alpha m| = 0 \Rightarrow \omega^2 = \frac{k}{m} \tag{28}$$

As the equal values of angular frequency for each building as Figure 6, normal coordinate system equation of motion can be written as Eq.(29).

$$\ddot{Z}_1(t)+\omega_1^2 Z_1(t) = -\ddot{u}_g(t) \tag{29}$$

Considering ground motion accelerometer shape and solving equation Eq.(29) applying duhammel integral or other numerical methods response history displacement of system can be gained. As both systems frequencies are equal the equation of motion is the same for both systems and they move like each other hence they don’t strike each other.

In second status, the systems contact each other and the mass and stiffness matrices of equation 20 can be written as Eq.(30) and replacing in Eq.(24) , Eq.(31) can be obtained.

$$[k] = \begin{bmatrix}\alpha+\beta & -\beta \\ -\beta & \beta+1\end{bmatrix}k \quad [m] = \begin{bmatrix}\alpha & 0 \\ 0 & 1\end{bmatrix}m \tag{30}$$

$$\begin{vmatrix} (\alpha+\beta)k-\omega^2\alpha m & -\beta k \\ -\beta k & (\beta+1)k-\omega^2 m \end{vmatrix} = 0 \tag{31}$$

Distributing Eq.(31), a second order equation is obtained solving which results in Eq.(32) as angular frequencies.

$$\omega_2^2 = \left[1+\beta(\frac{1-\alpha}{\alpha})\right](\frac{k}{m}) , \ \omega_1^{\ 2} = 1(\frac{k}{m}) \tag{32}$$

Replacing angular frequency values in Eq.(25) the eigen vector or mode shape matrix can be obtained as Eq.(33).

$$\phi = \begin{bmatrix} 1 & 1 \\ 1 & -\alpha \end{bmatrix} \tag{33}$$

MDOF equation of vibration without damping in normal coordinate can be written as Eq.(34), where $p(t)$ is the product of mass to the ground motion acceleration.

$$\phi_n^T m\phi_n \ddot{Z}_n + \phi_n^T k\phi_n Z_n = \phi_n^T p(t) , \tag{34}$$

$$p(t) = mr\ddot{u}_g(t)$$

Therefore equation of motion for the two modes of vibration according to the Eq.(34) can be written as Eq.(35).

$$\ddot{z}_1(t) + \frac{k}{m} Z_1(t) = \ddot{u}_g(t)$$

$$\ddot{Z}_2(t) + \left[1+\beta(\frac{1+\alpha}{\alpha})\right]\frac{k}{m} Z_2(t) = 0 \tag{35}$$

Eq.(33) shows that the displacements are due to the first mode because the movement force of the second mode is equal to zero. In other words, the corporation coefficient in the second mode is zero, because the first mode has a shape vector of $\Phi = [1,1]$ which indicates that both m and αm masses move together and don't contact each other. Displacement can be written as Eq.(36) in this case.

$$u(t) = \sum_{n=1}^{2} \{\phi_n\} Z_n(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} Z_1(t) = \begin{bmatrix} Z_1(t) \\ Z_1(t) \end{bmatrix} \tag{36}$$

According to the previous equations, it can be said that, if the period of two SDOF adjacent buildings are the same and zero distance at inertia state, there is no impact between them.

5. STRUCTURAL IMPACT ANALYSIS SOFTWARE

SLAM is a well-known computer software to analyze structural impact. The software has been produced by Maison and Kasai at Berkeley California and the software is based on part 3.2 theory. Structural Dynamic properties can be produced in a software called SUPER ETABS. This software has another limitation beside the limits mentioned in 3.2 that is, to be more precise it considers the impact between two adjacent buildings just one link member at the top of the shorter building to model the connection which can not be a good assumption to predict the impact effects.

SAP200 is a software that can analyze the impact problem in addition to analyze and design of structures. This software has an element called GAP which is a pure pressure element that analyzes the model by nonlinear time history analysis.SAP2000 can analyze the impact effect accurately and fast. This software has the ability of choosing multiple contact points in two adjacent buildings which reduces the impact effect prediction accuracy in adjacent buildings.SAP2000 has been used in this paper.

5.1. Software Modeling of an Impact between Two SDOF and Results Control

To study and control the impact phenomena modeling results applying SAP2000, two examples have been considered which are discussed in the next parts and their results are validated by the manual calculations.

5.2. Comparison of Results for Manual and Software Analysis

Example 1: Two SDOF with specified mass and stiffness adjacent to each other has been considered as a sample. The properties of the two systems are mentioned as m_1, m_2 and k_1 and k_2. The initial distance between them is S = 0.4(in) and also the local spring stiffness at the place of impact is K_s = 50,000(kip/in).Fig.7 and Fig.8 show the modeling of the system.

$$m_2 = 60(\frac{kip-\sec^2}{in}),\ m_1 = 39(\frac{kip-\sec^2}{in})$$

$$k_1 = k_2 = 1200\,(\frac{kip}{in})$$

Figure.7: Two SDOF System Beside Each other in Without Impact Status

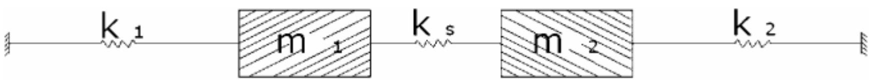

Figure.8: Two SDOF Systems Beside Each other in Impact Status

The two systems have been connected to each other by GAP connection element with nonlinear stiffness of K_s = 50,000(Kip/in), to study the impact between two free vibration systems with initial conditions of $U_1(0)$ and $U'_1(0)$. As natural frequency of system 1 is ω = 5.55(rad/sec) and T = 1.13sec the equation f system1 is written as:

$$\dot{u}_1(0) = 0(\frac{in}{\sec})\ ,$$

$$u_1(0) = -0.81(in)$$

$$m_1\,\ddot{u}_1(t) + k_1 u_1(t) = 0$$

According to the following equations, the impact time of two buildings is:

$$u_1(t) = -0.81\ \cos 5.55t$$

$$u_1(t') = 0.4\,(in) \quad \Rightarrow \quad t' = 0.376\sec$$

mass1 has hadedisplacement and velocity of $u_1(0)$ and $\dot{u}_1(0)$. According to the mentioned initial conditions, the two systems equation of motion at impact status are shown in Fig.9

$$u_1(0.376) \;=\; 0.4(in)$$

$$\dot{u}_1(0.376) \;=\; 4(\frac{in}{\sec})$$

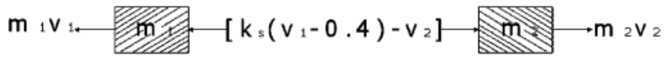

Figure. 9: The Two SDOF Systems at Impact Status

$$m_1\ddot{u}_1(t) + (k_1 + k_s)u_1(t) - k_s u_2(t) = 0.4k_s$$

$$m_2\ddot{u}_2(t) - k_s u_1(t) + (k_s + k_2)u_2(t) = -0.4k_s$$

To transform the following equations to the normal coordinate, the eigen value and vectors should be calculated. According to the following equations, the mass matrix and stiffness matrices are equal to *m* and *k* and the frequencies and natural mode of the system are as follows:

$$k \;=\; \begin{bmatrix} 51200 & -50000 \\ -50000 & 51200 \end{bmatrix} \quad m = \begin{bmatrix} 39 & 0 \\ 0 & 60 \end{bmatrix}$$

$$w_1 = 4.92\left(\frac{rad}{\sec}\right), \qquad w_2 = 46.3\left(\frac{rad}{\sec}\right), \qquad \phi = \begin{bmatrix} 1 & 1 \\ 1 & -0.65 \end{bmatrix}$$

According to the following equations, two vibration modes of uncoupled systems can be written as:

$$\ddot{Z}_1(t) + 24.21Z_1(t) \;=\; 0$$

$$\ddot{Z}_2(t) + 2143.69Z_2(t) \;=\; 512.82$$

And finally the responses of systems are equal to:

$$u_1(t) \;=\; 0.32\sin 4.92t + 0.16\cos 4.92t + 0.052\sin 46.3t + 0.24$$

$$u_2(t) \;=\; 0.32\sin 4.92t + 0.16\cos 4.92t - 0.034\sin 46.3t - 0.156$$

Table1 shows the results of different methods as by software and manually. It can be seen that the error is just a bit and the results are in good agrement.

Table1
Comparison between Manual and Software Analysis

	T(sec)	*Manual analysis*	*SAP2000 Analysis*	*Error*
mass 1 displacement U1(in)	0.2	–0.36	–0.354	–1.67%
mass 2 displacement U2(in)	0.2	0	0	0
Spring Force 1 V1(kip)	0.2	432	425.1	–1.59%
Impact Force F(kip)	0.2	0	0	0
mass 1 displacement U1(in)	0.41	0.503	0.498	0.99%
mass 2 displacement U2(in)	0.41	0.021	0.02	–4.76%
Spring Force 1 V1(kip)	0.41	603.6	597.88	–0.95%
Spring Force 2 V1(kip)	0.41	25.2	24.24	–3.81%
Impact Force F(kip)	0.41	4100	3901.9	–4.83%

Example 2: To study more aspects in example1, the two masses are replaced with each other and vibrating the second mass as m2 with initial conditions as bellow and the impact effect is studied. The general procedure is the same as example1.

$$m_2 = 60(\frac{kip - \sec^2}{in})$$

$$\dot{u}_1(0) = 0\left(\frac{in}{\sec}\right),\ u_1(0) = -0.81(in)$$

Considering the natural frequencies of system 1 that is ω = 4.47 (rad/sec) the equation of motion for system 1 can be written as:

$$m_1\,\ddot{u}_1(t) + k_1 u_1(t) = 0$$

According to the following equation, the impact time can be obtained as:

$$u_1(t) = -0.81\ \cos\ 4.47\ t$$

$$u_1(t') = 0.4(in) \qquad \Rightarrow \qquad t' = 0.467 \quad \sec$$

At impact time mass1 has displacement and velocity of $u_1(0.376) = 0.4$(in) and $\dot{u}_1(0.376)$ = 3.15(in/sec)respectively. According to the mentioned initial conditions impact status equations of motion due to figure10can be written as:

$$m_1\ddot{u}_1(t) + (k_1 + k_s)u_1(t) - k_s u_2(t) = 0.4\,k_s$$

$$m_2\ddot{u}_2(t) - k_s u_1 + (k_s + k_2)u_2 = -0.\ 4k_s$$

To transform the above equations to the normal coordinate, the eigen values and eigen vectors should be calculated at first. According to the equations the *m* matrix and *k* matrix can be calculated as:

$$k = \begin{bmatrix} 51200 & -50000 \\ -50000 & 51200 \end{bmatrix} \quad m = \begin{bmatrix} 60 & 0 \\ 0 & 39 \end{bmatrix}$$

The frequencies and natural modes of the system can be obtained as:

$$w_1 = 4.92\left(\frac{rad}{\sec}\right)$$

$$w_2 = 46.3\left(\frac{rad}{\sec}\right)$$

$$\phi = \begin{bmatrix} 1 & 1 \\ 1 & -0.65 \end{bmatrix}$$

Due to the above equations the uncoupled equations of motion of two vibration modes can be discussed as:

$$\ddot{Z}_1(t) + 24.21 Z_1(t) = 0$$

$$\ddot{Z}_2(t) + 2143.69\, Z_2(t) = 512.82$$

And finally, the response of the systems in impact status are:

$$u_1(t) = 0.39 \sin 4.92t + .24 \cos 4.92t + 0.053 \sin 46.3t + 0.07 \cos 46.3t + 0.24$$

$$u2(t) = 0.39 \sin 4.92t + 0.24 \cos 4.92t - 0.034 \sin 46.3t - .045 \cos 46.3t - 0.156$$

Table.2 displays the comparison between the results with software and manually. It shows that the manual results and software calculations are approximately the same.

Table 2
Comparison between Manual and SAP2000 Results

	t(sec)	*Manual analysis*	*SAP2000 Analysis*	*Error*
mass 1 displacement U1(in)	0.2	0.507	–.517	1.97%
mass 2 displacement U2(in)	0.2	-	-	-
Spring Force 1 V1(kip)	0.2	608.76	620.99	2%
Impact Force F(kip)	0.2	-	-	-
mass 1 displacement U1(in)	0.5	0.539	0.581	2.02%
mass 2 displacement U2(in)	0.5	0.108	0.105	2.78%
Spring Force 1 V1(kip)	0.5	711.6	698.2	1.88%
Spring Force 2 V1(kip)	0.5	129.6	126.5	2.39%
Impact Force F(kip)	0.5	4250	3850.2	9.41%

6. CONCLUSION

According to this research, it can be concluded that, impact makes more critical loading conditions than vibration without impact, therefore neglecting probabilistic effects of impact, leads to unstable design of structures and since impact is an instant force and destroys the structure instantaneously, controling and preventing this effect is so important in structural design. The presented formulation and method in this paper have good compatibility with the results of the nonlinear dynamic software and it shows that there is a little difference between the software results and manual calculation results, therefore the formulation can be used for the real structures too but only for it's suggested the method not be applied for high rise structures as tall buildings but for low rise buildings that's. Because for high rise structures, the equations get more complex by increasing the degrees of freedom and also strong ground motion effects, hence the equations should be reconsidered for tall buildings.

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