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Using Operations Research Models in Decision Making to Find the Node Influences in the Noordin Dark Network

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ABSTRACT

In a social and dark network analysis the SNA software output provided includes many measures and metrics. For each of these measures and metric, the output provides the ability to obtain a rank ordering of the nodes in terms of these measures. We might use this information in decision making concerning disrupting or deceiving a given network. All is fine when all the measures indicate the same node as the key or influential node. What happens when the measures indicate different key nodes? Our goal in this paper is to explore methodologies to identify the key players or nodes in a given network. We apply a priority average ranking scheme, AHP, and TOPSIS to analyze these outputs to find the most influential nodes as a function of the decision makers' inputs as a process to consider both subjective and objectives inputs through pairwise comparison matrices. We compare these methods by illustration using the Noordin Dark Network with seventy nine nodes. We discuss sensitivity analysis that should be applied to these methods because of their use of subjective inputs.

Key words: social network analysis, multi-attribute decision making, average priority ranks, Analytical hierarchy process (AHP), decision criterion, weighted criterion, TOPSIS, node influence, sensitivity analysis

INTRODUCTION TO SOCIAL NETWORK ANALYSIS AND DARK NETWORKS

Social network analysis (SNA) is the methodical analysis of social networks in general and dark networks in particular (Everton, 2012; Roberts, *et al.* 2011). Social network analysis is a collection of theories and methods that assumes that the behavior of actors (individuals, groups, organizations, etc.) is profoundly affected by their ties to others and the networks in which they are embedded. Rather than viewing actors as automatons unaffected by those around them, SNA assumes that interaction patterns affect what actors say, do, and believe. Networks contain *nodes* (representing individual actors or entities within the network) and edges and arcs (representing relationships between the individuals, such as friendship, kinship, organizational position, sexual relationships, communications, tweets, Facebook friendships, etc.). These networks are often depicted in two formats: graphically or matrix. We might call the graph a social network diagram or dark network diagram, where nodes are represented as *points* or *circles* and arcs are represented as lines that interconnect the nodes.

We will provide a brief background of social network analysis. More precisely, we introduce some of the more common metrics and measures as well as their definitions that are used for exploratory analysis of networks. In this paper, we assume decision makers are only looking for the *powerful* and *influential* players in a network. In the SNA literature there has been some discussion as to four main measures that might be sued to analysis the most influential person in a network (Newman, 2010) and these include only the following centrality measures: *degree, betweenness, closeness,* and *eigenvector.*

There are a multitude of measures (metrics) that are found in most SNA software. The software package that we used in this analysis is ORA. According to the documentation,

"From the ORA document manual (2010) it explains ORA is a network analysis tool that detects risk sorvulner abilities of an organization's design structure. The design structure of an organization is the relationship among its personnel, knowledge, resources, and task sentities. The seen tities and relationships are represented by the Meta-Matrix as well as graphical depiction. Measures that take as input a Meta-Matrix are used to analyze the structural properties of an organization for potential risk.

ORA contains over 100 measures which are categorized by which type of risk they detect. Measures are also organized by input requirements and by output. ORA generates for matted reports view able on screen or in log files, and reads and writes networks in multiple data formats to be interoperable with existing network analysis packages….The current version ORA1.2 software is available on the CASOS website".

We begin by defining a few metric terms or measures in social network analysis from ORA that we use including the main four centrality metrics.

Betweenness

Betweenness is a measure of the extent to which a node lies on the shortest path between other nodes in the network. This measure takes into account the connectivity of the node's neighbors, giving a higher value for nodes which *bridge clusters*. The measure reflects the number of people who a person is connecting indirectly through their direct links.

Centrality

Centrality is the measure which gives a rough indication of the social power of a node based on how well they "connect" the network. "Betweenness," "Closeness," "Degree," and "Eigenvector" are all measures of centrality.

Centralization

Centralization is the difference between the number of links for each node divided by maximum possible sum of differences. A centralized network will have many of its links dispersed around one or a few nodes, while a decentralized network is one in which there is little variation between the numbers of links each node possesses.

Closeness

Closeness is the degree an individual is near all other individuals in a network (directly or indirectly). It reflects the ability to access information through the "grapevine" of network members. Thus, closeness is the inverse of the sum of the shortest distances between each individual and every other person in the network. The shortest path may also be known as the "geodesic distance."

Degree

Degree is the count of the number of ties to other players in the network.

Density

Density is a measure of network cohesion that is equal to the actual number of ties in a network divided by the total possible number of ties, which means that density scores range from 0.0 to 1.0.

Eigenvector Centrality

Eigenvector centrality is a variation on degree centrality in that assumes that ties to central actors are more important than ties to peripheral actors and thus weights an actor's summed connections to others by their centrality scores. Google's Pagerank score is a variation on eigenvector centrality.

Degree Centrality

Degree centrality is defined as the number of links incident upon a node (i.e., the number of ties that a node has). The degree can be interpreted in terms of the immediate risk of a node for catching whatever is flowing through the network (such as a virus, or some information). In the case of a directed network (where ties have direction), we usually define two separate measures of degree centrality, namely *indegree* and *outdegree*.

For our analysis, we use the subset of the Noordin Top Terrorist Network drawn primarily from "Terrorism in Indonesia: Noordin's Networks," a 2006 publication of the International Crisis Group. It includes relational data on the 79 individuals listed that publication. The data were initially coded by Naval Postgraduate School students and our Common Research Environmental (CORE) Lab.

Previous work on using AHP and TOPSIS (Fox, *et al.* 2013; 2014) have shown the *proof of principle* approach using two basic social networks from the literature: The Kite and Knoke Networks.

Application of Technique of Order Preference by Similarity to the Ideal Solution (TOPSIS) in a Dark Network

Nomenclature for TOPSIS:

$(x_{ij})_{m \times n}$ Matrix of values for alternatives by criterion

$(r_{ij})_{m \times n}$ Matrix of normalized values for alternatives by criterion

$(t_{ij})_{m \times n}$ Matrix of weighted normalized values for alternatives by criterion

A_w Worst solution in the column

A_b Best solution in the column

d_{ib} L2 distance between the target and best solution

d_{iw} L2 distance between the target and worst solution

s_{iw} Ratio similarity to the ideal worst solution

s_{ib} Ratio similarity to the ideal best solution

C Final ranking

The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is a multi-criteria decision analysis method (Hwang and Yoon , 1981). It has been further developed and refined (Yoon, 1987; Hwang et al. 1993). TOPSIS is based on the concept that the chosen alternative should have the shortest geometric distance from the positive ideal solution and the longest geometric distance from the negative ideal solution. It is a method of compensatory aggregation that compares a set of alternatives by identifying weights for each criterion, normalizing the scores for each criterion and calculating the geometric distance between each alternative and the ideal alternative, which is the best score in each criterion. An assumption of TOPSIS is that the criteria are monotonically increasing or decreasing. Normalization is usually required as the parameters or criteria are often of incompatible dimensions in multi-criteria problems. Compensatory methods such as TOPSIS allow trade-offs between criteria, where a poor result in one criterion can be negated by a good result in another criterion. This provides a more realistic form of modeling than non-compensatory methods, which include or exclude alternative solutions based on hard cut-offs.

TOPSIS Background

We only desire to briefly discuss the elements in the framework of TOPSIS. TOPSIS can be described as a method to decompose a problem into sub-problems. In most decisions, the decision maker has a choice among several to many alternatives. Each alternative has a set of attributes or characteristics that can be measured, either subjectively or objectively. The attribute elements of the hierarchal process can relate to any aspect of the decision problem—tangible or intangible, carefully measured or roughly estimated, well- or poorly-understood—anything at all that applies to the decision at hand.

The TOPSIS process is carried out as follows:

Step 1 Create an evaluation matrix consisting of m alternatives and n criteria, with the intersection of each alternative and criteria given as x_{ij}, giving us a matrix $(\boldsymbol{X}_{ij})_{mxn}$.

$$D = \begin{array}{c c} & \begin{array}{cccccc} x_1 & x_2 & x_3 & \cdot & \cdot & \cdot \; x_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ \cdot \\ \cdot \\ \cdot \\ A_m \end{array} & \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdot & \cdot & \cdot & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdot & \cdot & \cdot & x_{2n} \\ x_{31} & x_{32} & x_{33} & \cdot & \cdot & \cdot & x_{3n} \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & & & & \cdot \\ x_{m1} & x_{m2} & x_{m3} & \cdot & \cdot & \cdot & x_{mn} \end{bmatrix} \end{array}$$

Step 2: The matrix shown as $\boldsymbol{D}$ above then normalized to form the matrix $\boldsymbol{R} = (\boldsymbol{R}_{ij})_{mxn}$,
using the normalization method

$$r_{ij} = \frac{x_{ij}}{\sqrt{\Sigma x_{ij}^2}}$$

for $i = 1,2\ldots,m;\ j = 1,2,\ldots n$

Normalization

Two methods of normalization that have been used to deal with incongruous criteria dimensions are linear normalization and vector normalization.

Linear normalization can be calculated as in *Step 2* of the TOPSIS process above. Vector normalization was incorporated with the original development of the TOPSIS method (Yoon, 1987), and is calculated using the following formula:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\Sigma x_{ij}^2}}$$

for $i = 1,2\ldots,m;\ j = 1,2,\ldots n.$

In using vector normalization, the non-linear distances between single dimension scores and ratios should produce smoother trade-offs (Huang, *et al.* 2011).

Step 3 Calculate the weighted normalized decision matrix. First we need the weights. Weights can come from either the decision maker or by computation.

Step 3 a. Use either the decision maker's weights for the attributes $x_1, x_2, .. x_n$ or compute the weights through AHP's decision maker weights method (Saaty, 1980) to obtain the weights as the eigenvector to the attributes versus attribute pairwise comparison matrix.

$$\sum_{j=1}^{n} w_j = 1$$

The sum of the weights over all attributes must equal 1 regardless of the method used.

Step 3b. Multiply the weights to each of the column entries in the matrix from *Step 2* to obtain the matrix, ***T***.

$$T = (t_{ij})_{m\times n} = (w_j r_{ij})_{m\times n}, i = 1, 2, \ldots, m$$

Step 4 Determine the worst alternative (A_w) and the best alternative (A_b) : Examine each attribute's column and select the largest and smallest values appropriately. If the values imply larger is better (profit) then the best alternatives are the largest values and if the values imply smaller is better (such as cost) then the best alternative is the smallest value.

$$A_w = \{\langle \max(t_{ij} \mid i = 1, 2, \ldots, m \mid j \in J_-\rangle, \langle \min(t_{ij} \mid i = 1, 2, \ldots, m) \mid j \in J_+\rangle\} \equiv \{t_{wj} \mid j = 1, 2, \ldots, n\},$$

$$A_{wb} = \{\langle \min(t_{ij} \mid i = 1, 2, \ldots, m \mid j \in J_-\rangle, \langle \max(t_{ij} \mid i = 1, 2, \ldots, m) \mid j \in J_+\rangle\} \equiv \{t_{bj} \mid j = 1, 2, \ldots, n\},$$

where,

$J_+ = \{j = 1, 2, \ldots n \mid j)$ associated with the criteria having a positive impact, and

$J_- = \{j = 1, 2, \ldots n \mid j)$ associated with the criteria having a negative impact.

We suggest that if possible make all entry values in terms of positive impacts.

Step 5 Calculate the L2-distance between the target alternative i and the worst condition A_w

$$d_{iw} = \sqrt{\Sigma_{j=1}^{n} (t_{ij} - t_{wj})^2}, i = 1, 2, \ldots, m,$$

and the distance between the alternative i and the best condition A_b

$$d_{ib} = \sqrt{\Sigma_{j=1}^{n} (t_{ij} - t_{bj})^2}, i = 1, 2, \ldots, m,$$

where d_{iw} and d_{ib} are L2-norm distances from the target alternative i to the worst and best conditions, respectively.

Step 6 Calculate the similarity to the worst condition:

$$s_{iw} = \frac{d_{iw}}{(d_{iw} + d_{ib})}, 0 \le s_{iw} \le 1, i = 1, 2, \ldots, m$$

$S_{iw} = 1$ if and only if the alternative solution has the worst condition; and

$S_{iw} = 0$ if and only if the alternative solution has the best condition.

Step 7 Rank the alternatives according to their value from S_{iw} $(i = 1, 2, \ldots, m)$.

For our models we explore two options for the weights from Step 3. First, the decision maker might actually have a weighting scheme that they want the analyst to use. In not, we suggest using Saaty's 9-Point pair-wise method developed for the Analytical Hierarchy Process (AHP) [see 10]. We briefly describe this pairwise method to obtain weights.

We build a numerical representation using a 1-9 point scale in a pairwise comparison for the attributes criterion and the alternatives. The goal is to obtain a set of eigenvectors of the system that measures the importance with respect to the criterion. The resulting eigenvectors are the weights. We can put these values into a matrix or table based on the following description:

Intensity of Importance in Pair-wise Comparisons	Definition
1	Equal Importance
3	Moderate Importance
5	Strong Importance
7	Very Strong Importance
9	Extreme Importance
2,4,6,8	For comparing between the above
Reciprocals of above	In comparison of elements *i* and *j* if *I* is 3 compared to *j*, then j is 1/3 compared to *i*.
Rational	Force consistency; measure values available

Several methods exist to obtain these eigenvectors. One methods uses discrete dynamical systems (Fox, 2012; Giordano, et al. 2008) and the other the power method for eigenvalues & eigenvectors (Burden and Faires, 2013).

Objective Statement ← This is the decision desired such as a rank order of key players in a dark network

Alternatives: *1, 2, 3, …, n*(the nodes or players)

For each of the alternatives there are attributes to compare.

Attributes: $a_1, a_2, \ldots, a_m$ (ORA output measures and metrics of interest)

Once the hierarchy is built, the decision maker(s) systematically evaluate its various elements pairwise (by comparing them to one another two at a time), with respect to their impact on an element above them in the hierarchy. In making the comparisons, the decision makers can use concrete data about the elements, but they typically use their judgments about the elements' relative meaning and importance. It is the essence of the TOPSIS that human judgments, and not just the underlying information, can be used in performing the evaluations.

TOPSIS converts these evaluations to numerical values that can be processed and compared over the entire range of the problem. A numerical weight or priority is derived for each element of the hierarchy, allowing diverse and often incommensurable elements to be compared to one another in a rational and consistent way. This capability distinguishes the TOPSIS from other decision making techniques.

In the final step of the process, numerical priorities or ranking are calculated for each of the decision alternatives. These numbers represent the alternatives' relative ability to achieve the decision goal, so they allow a straightforward consideration of the various courses of action.

Uses and Applications

While it can be used by individuals working on straightforward decisions, TOPSIS is most useful where teams of people are working on complex problems, especially those with high stakes, involving human perceptions and judgments, whose resolutions have long-term repercussions. It has unique advantages when important elements of the decision are difficult to quantify or compare, or where communication among team members is impeded by their different specializations, terminologies, or perspectives.

Decision situations to which the TOPSIS might be applied include:

- Choice - The selection of one alternative from a given set of alternatives, usually where there are multiple decision criteria involved.
- Ranking - Putting a set of alternatives in order from most to least desirable
- Prioritization - Determining the relative merit of members of a set of alternatives, as opposed to selecting a single one or merely ranking them
- Resource allocation - Apportioning resources among a set of alternatives
- Benchmarking - Comparing the processes in one's own organization with those of other best-of-breed organizations
- Quality management - Dealing with the multidimensional aspects of quality and quality improvement
- Conflict resolution - Settling disputes between parties with apparently incompatible goals or positions

Applications of TOPSIS to find influences in a dark network

We illustrate using the Noodin Network with its graphical network depicted in Figure 1.

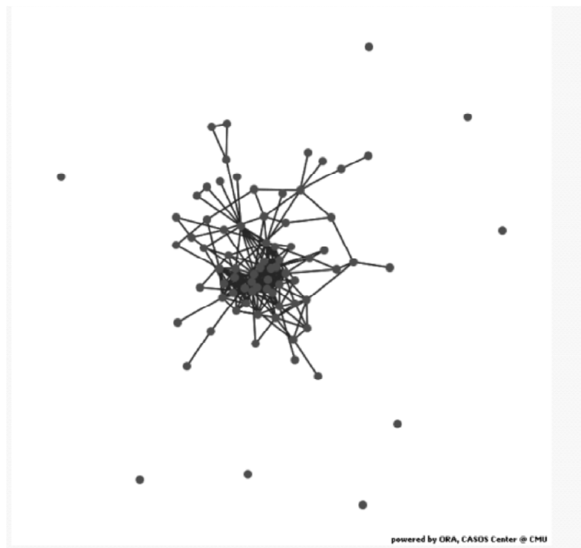

Figure 1: ORA's Trust Network from Noordin's seventy-nine Node Dark Network.

We obtained all the outputs from ORA and a summary of Key Node analysis as shown in Table 1. Table 1 shows different key nodes across the metrics.

We extracted the actual metric values from the output of ORA for the 20 nodes across these key metrics. Table 2 provides these metrics values from ORA for the top 20 nodes (size: 79 nodes, density: 0.0879585) for eight outputs initially identified by ORA. Next we performed the analysis with only the main eight centrality measures.

We use the decision weights from our AHP program (unless a real decision maker gives us their own weights) and find the eigenvectors for our eight metrics as shown in figure 2 displaying the pairwise comparison, the consistency ratio (less than 0.1) and the resulting eigenvectors.

We take all these output metrics from ORA and perform steps 2-7 of TOPSIS to obtain the following raw and then ordered outputs shown in Table 3.

Next, we repeated the analysis using only the four main centrality measures . Table 4 shows the decision matrix and weights with a CR = 0.02846 (less than 0.1).

Table 1: ORA's Key Nodes from the Noordin Dark Network.

Rank	*Betweenness centrality*	*Closeness centrality*	*Eigenvector centrality*	*Eigenvector centrality per component*	*In-degree centrality*	*In-Closeness centrality*	*Out-degree centrality*	*Total degree centrality*
1	N2	N2	A5	A5	A5	N2	A5	A5
2	I7	A5	M4	M4	N2	A5	N2	N2
3	A13	U	T	T	M4	U	M4	M4
4	A4	A13	N2	N2	A6	A13	A6	A6
5	A5	F	A6	A6	T	F	T	T
6	U6	M4	J	J	F	M4	F	F
7	A12	A6	F	F	J	A6	J	J
8	Z	A23	U	U	U	A23	U	U
9	D2	T	S8	S8	S8	T	S8	S8
10	M5	I7	A22	A22	A23	I7	A23	A23
11	S6	J	M3	M3	B	J	B	B
12	U	S8	B	B	A13	S8	A13	A13
13	J	B	S5	S5	A22	B	A22	A22
14	A6	D2	A23	A23	D2	D2	D2	D2
15	A23	A17	A2	A2	I7	A17	I7	I7
16	A16	A7	I6	I6	M3	A7	M3	M3
17	B	I2	D2	D2	S5	I2	S5	S5
18	P	I6	A17	A17	A17	I6	A17	A17
19	F	A22	I2	I2	S6	A22	S6	S6
20	A17	S5	A7	A7	A7	S5	A7	A7

	BETW	CC	EC	ECPC	IDC	ICC	ODC	TCC				Priority	Weights
Betw	1	2	2	2	2	3	4	5		CR	0.045377	betw	0.237698
cc	0.5	1	2	2	2	4	5	6				cc	0.214033
ec	0.5	0.5	1	2	2	3	4	5				ec	0.166106
ecpc	0.5	0.5	0.5	1	2	2	3	4				ecpc	0.125044
idc	0.5	0.5	0.5	0.5	1	2	3	4				idc	0.105366
icc	0.333333	0.25	0.333333	0.5	0.5	1	3	4				icc	0.076461
odc	0.25	0.2	0.25	0.333333	0.333333	0.333333	1	3				odc	0.046489
tcc	0.2	0.166667	0.2	0.25	0.25	0.25	0.333333	1				tcc	0.028802

Figure 2: Decision pairwise matrix and decision weights.

Table 2: Summary of ORA's output for Noordin Dark Network for the 8 criterion

Data	c1	c2	c3	c4	c5	c6	c7	c8
Agent	Betweeness Cent.	Closeness	Eigenvector Cent.	ECPC	In-degree	In-Closeness Cent	Out-Closeness Cent	Total Degree Cent.
a5	0.09	0.102	0.434	0.276	0.359	0.102	0.359	0.359
n2	0.182	0.103	0.35	0.222	0.333	0.103	0.333	0.333
m4	0	0.1	0.392	0.249	0.269	0.1	0.269	0.269
a6	0.033	0.1	0.325	0.206	0.256	0.1	0.256	0.256
t	0	0.099	0.376	0.239	0.256	0.099	0.256	0.256
f	0.025	0.1	0.313	0.199	0.231	0.1	0.231	0.231
j	0.034	0.099	0.32	0.203	0.231	0.099	0.231	0.231
u	0.038	0.101	0.305	0.194	0.231	0.101	0.231	0.231
s8	0	0.099	0.299	0.19	0.205	0.099	0.205	0.205
a23	0.032	0.1	0.257	0.163	0.192	0.1	0.192	0.192
b	0.028	0.099	0.279	0.177	0.192	0.099	0.192	0.192
a13	0.14	0.101	0	0	0.179	0.101	0.179	0.179
a22	0	0.098	0.289	0.184	0.179	0.098	0.179	0.179
d2	0.04	0.099	0.226	0.144	0.179	0.099	0.179	0.179
I7	0.163	0.099	0	0	0.179	0.099	0.179	0.179
m3	0	0	0.281	0.179	0.179	0	0.179	0.179
s5	0	0.098	0.264	0.168	0.179	0.098	0.179	0.179
a17	0.025	0.098	0.224	0.143	0.167	0.098	0.167	0.167
s6	0.039	0	0	0	0.167	0	0.167	0.167
a7	0	0	0.209	0.153	0.154	0	0.154	0.154

Table 3: Raw Output and Ordered Output from TOPSIS

a5	0.419676898		n2	0.965425053
n2	0.965425053		l7	0.798190393
m4	0.127914773		a13	0.693837087
a6	0.062025923		a5	0.419676898
t	0.126211599		s6	0.27177498
f	0.028309765		a7	0.237506156
j	0.064185335		m3	0.237393009
u	0.086940389		m4	0.127914773
s8	0.121464812		t	0.126211599
a23	0.047693103		s8	0.121464812
b	0.024764452		a22	0.121076687
a13	0.693837087		s5	0.120921061
a22	0.121076687		d2	0.097651096
d2	0.097651096		u	0.086940389
l7	0.798190393		j	0.064185335
m3	0.237393009		a6	0.062025923
s5	0.120921061		a23	0.047693103
a17	0.023880284		f	0.028309765
s6	0.27177498		b	0.024764452
a7	0.237506156		a17	0.023880284

Table 4. Decision matrix and weights with four key metrics

	A	B	C	D	E	F	G	H
1		TDC	Betw	Closeness	EC			
2	TDC	1	2	3	4		CR	0.028406
3	Betw	0.5	1	2	3			
4	Closeness	0.333333	0.5	1	2		Weights	Metrics
5	EC	0.25	0.333333	0.5	1		0.465819	TDC
6							0.27714	BETW
7							0.16107	Closness
8							0.09597	Eigen.
9								
10								
11								

Table 5 shows the raw and ordered TOPSIS output.

In comparison to using eight criteria, we note that the top 5 nodes do not change and the first change occurs in position number 6.

Sensitivity Analysis

In our analysis, we have utilized weights as applicable to the metrics for the nodes. Weights are subjective even if used in AHP and TOPSIS methodologies. Since

Table 5: Raw and ordered output from TOPSIS

	Raw			Ordered
a5	0.487222891		n2	0.97181449
n2	0.973982022		l7	0.791416446
m4	0.10722359		a13	0.688099762
a6	0.163571388		a5	0.443594317
t	0.097216293		s6	0.186941734
f	0.115675763		m3	0.175970038
j	0.157554624		m4	0.146982142
u	0.177490287		t	0.141756837
s8	0.056569571		s8	0.125520682
a23	0.133339299		s5	0.120812321
b	0.112782967		a22	0.120795198
a13	0.713101164		a7	0.119177118
a22	0.039541984		u	0.111160844
d2	0.174194014		a6	0.11064716
l7	0.807187014		d2	0.097135811
m3	0.139994824		j	0.094399316
s5	0.037311139		f	0.074373911
a17	0.089768404		a23	0.058285584
s6	0.212225078		b	0.042589095
a7	0.023265869		a17	0.015588124

these are subjective relationships, we should consider sensitivity analysis for the weights. We recommend equation (1) for doing the sensitivity analysis for adjusting weights (Alinezhad, *et al.*, 2011):

$$w_j' = \frac{1 - w_p'}{1 - w_p} w_j \tag{1}$$

where w_j' is the new weight and w_p is the original weight of the criterion to be adjusted and w_p' is the value after the criterion was adjusted.

The literature provides no direct sensitivity analysis procedures. We recommend, as a minimum, at least a numerical trial and error approach to sensitivity analysis. Not only do we recommend altering the criterion pair-wise comparison to measure the model's robustness but delving into break points is also useful.

In our four metric model, we find that the model is quite robust and that with major changes in priority and pairwise comparison the top 5 nodes are not affected.

SUMMARY AND COMPARISONS

We have also used the two other MADM methods to rank order our nodes in previous work: DEA (Charnes, *et al.*, 1978; Cooper, *et al.*, 2000; Zhenhua, 2009; Thanassoulis, 2011; Winston, 1995) and AHP (Krackhardt, 1990; Knoke, *et al.*, 1981,1982; ,Fox, 2012). Here we used TOPSIS, AHP, and our average priority method. We present our results in table 6.

Additionally, we applied equation 1 to our TOPSIS analysis in order to measure the effects of the altering of the criteria weights.

In the eight metric model, we again used equation (1) for adjusting decision maker weights. We plotted the top 10 alternatives using three major adjustments in criteria weighting each time insuring a different criterion was the most heavily weighted. It is seen from the graph, Figures 3 and 4, that the top 5 never changed position.

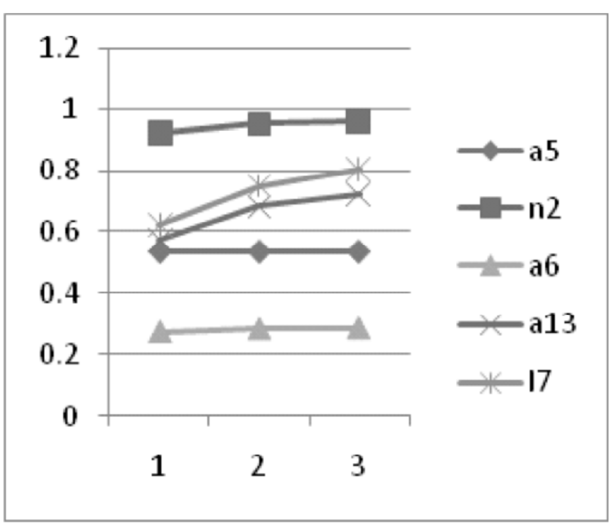

Figure 3: Sensitivity Analysis on the 4 criteria model top 5 with substantial changes to criterion weighting

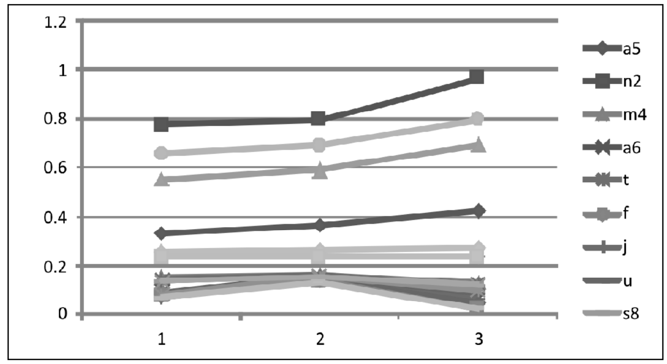

Figure 4: Sensitivity Analysis

Finding Break Points, if they exist

A break point is defined as the value of weight, w_j', that causes the ranking to be significantly change implying a change in the top alternative ranking. The method that we suggest is taking the largest weighted criterion and reduces it is slight increments which increases the weights of the other criteria and re-computing the rankings until another alternative is ranked number one.

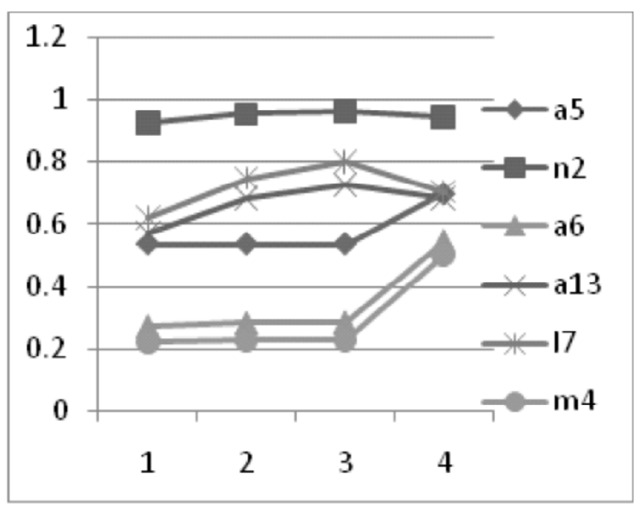

Figure 5: Looking for break points

In this examination, the top ranked node, *n2*, never changes as shown in Figure 5. We can get changes in the nodes ranked 2-4 through an increase change in the criterion weight for closeness centrality from 0.1611 to 0.4611, an increase of 0.3.

Table 6: Summary of Methods

H	I	J	K	L	M	N	O	P
						4 Criertion		8 Critertion
	Averaging		AHP			TOPSIS		TOPSIS
A5	1.75	n2	1.010388		n2	0.999922	n2	0.965425
N2	2.125	a5	0.884257		I7	0.999505	I7	0.79819
M4	3.285714286	I7	0.831784		a13	0.999382	a13	0.693837
A4	4	a13	0.798567		a5	0.999086	a5	0.419677
T	5.571428571	a6	0.739287		u	0.998556	s6	0.271775
F	6	u	0.737152		d2	0.998551	a7	0.237506
A6	6.25	j	0.733255		j	0.998517	m3	0.237393
A6	6.25	f	0.717209		a6	0.998511	m4	0.127915
A12	7	m4	0.703259		a23	0.998479	t	0.126212
U	7.25	t	0.69607		s6	0.998472	s8	0.121465
Z	8	d2	0.68371		b	0.998442	a22	0.121077
J	8.25	a23	0.681615		f	0.998429	s5	0.120921
A13	8.6	b	0.679652		a17	0.998383	d2	0.097651
S8	9.857142857	s8	0.64126		m4	0.998189	u	0.08694
M5	10	a22	0.632246		t	0.998188	j	0.064185
S6	11	a17	0.630918		s8	0.998172	a6	0.062026
A23	11.125	s5	0.62649		a22	0.998167	a23	0.047693
I7	11.16666667	m3	0.600884		s5	0.998166	f	0.02831
A22	11.8	s6	0.569897		m3	0.998157	b	0.024764
B	12.5	a7	0.556473		a7	0.998141	a17	0.02388

We have provided a several approaches to ranking influential nodes (players) in the Noordin Dark network as well as provided insights using sensitivity analysis. We compared the results. We believe that the incorporation of decision maker weights with the metrics of a social network is invaluable to analysis of *key* and *influential* players.

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