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MATHEMATICAL MODELING ON STIMULATED RAMAN SCATTERING INSTABILITY OF LASER BEAM PROPAGATING THROUGH COLLISIONAL PLASMA IN A SELF-FOCUSED FILAMENT INCLUDING THE EFFECT OF THERMAL CONDUCTION

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ABSTRACT: *Collisional plasma causes self-focusing of laser beam via Ohmic heating and density distribution. The self focusing, leads to enhancement of wave intensity, elevation of electron temperature and reduction of local electron density, leading to diminished attenuation rate. Thermal conduction plays an important role in temperature equilibrium when the electron mean free path λ_m is greater than the beam radius ($\lambda_m \geq r_o$). For $\lambda_1 m$, (> r_o, thermal conduction suppresses any non-uniformities in electron temperature, and nonlinearity is dominated by ponderomotive force. Stimulated Raman scattering instability is treated for laser beam propagating through collisional plasma in a self-focused filament where nonlinear refraction due to the redistribution of the electron density caused by nonuniform Ohmic heating of electron is balanced by diffraction divergence. Thermal conduction could play a dominant role in determining the energy dissipation of electrons. Inside a filament, the laser undergoes stimulated Raman backscattering (B-SRS). The filament supports radially localized Langmuir waves. Since the temperature inside a filament is higher and density lower than those outside, the collisional damping rates of the decay waves are lowered hence the threshold power for B-SRS is reduced.*

Keywords: *Laser beam, stimulated Raman scattering, self-focused filament, Langmuir wave, thermal conduction. PACS Nos.46.65.Dr, 42.65.Jx, 52.38Bv*

1. INTRODUCTION

It has been shown in recent years[1-2] that a high amplitude electromagnetic beam propagating in plasma is unstable to small-amplitude perturbations. This instability causes the breaking of the beam into filaments and is known as filamentation instability. On the time scale $t > \tau_h$ (which is more relevant to laser-plasma interactions), where τ_h is the heating time of electrons, the nonlinearity arises through nonuniform heating and redistribution of electrons[1]. The understanding of filamentation of laser light may be important to the success of laser fusion. In the long scale length plasmas envisioned for reactor targets, local intensity hot spots caused by self-focusing or laser light filamentation can drive the plasma above parametric instability thresholds. These instabilities tend to be saturated by the creation of super thermal electrons4. The hot electrons can penetrate deeply into the pellet, heating the interior, making high compressions difficult. Directly driven targets require very uniform driving pressures. Filamentation could spoil this uniformity, making large compressions difficult. The laser light absorption, penetration, and conversion to *X* rays could also be affected by self-focusing and filamentation. The earlier investigations of filamentation of laser beams on a long time scale are restricted to large-scale perturbations where the thermal conduction effects may be neglected[1,2]. But in the cases of real interest one is much more concerned about the growth of small-scale perturbations where thermal conduction could play a dominant role in determining the energy dissipation of electrons. The relative size of perturbations depends on the ratio m_i/m, since beam radius r_o is generally of the same order as electron mean free path λ_m. In this paper we have studied the filamentation of laser beams in plasmas where both collisional and thermal-conduction losses are present simultaneously. At short wavelengths collisional effects considerably influence laser plasma interaction. The nonlinear process of stimulated Raman scattering is seen to require laser power greater than a threshold value, determined by collisions. In several experiments the observed values of threshold power are far below the values predicated theoretically[5-18] Simon et al.[12] invoked a two state process to explain some of these results. The hot electrons produced via resonance absorption drive a Langmuir wave in the under dense region, via, two stream instability. The Langmuir wave couples with the pump to produce sidebands. Barr et al.[6] have recently examined the SRS in the presence of a static sinusoidal density modulation transverse to the axis of the pump laser radiation. Numerical solutions reveal that the tendency of Langmuir wave localization of the pump and consequent enhancement of its power density at the filament bottom tend to enhance the growth rate. For parameters of interest the later tendency may win over the former giving rise to an overall enhancement in the growth rate over its value in the uniform case. However, this calculation is numerical and the assumed density and intensity modulation are not self consistent. Afshar-rad et al.[14] have studied the evidence of stimulated Raman scattering occurring in laser filaments in long scale length plasmas.

In this paper we study B-SRS in a cylindrical filament where nonlinear refraction due to the redistribution of the electron density caused by nonuniform Ohmic heating in presence of thermal conduction loss of the electron is balanced by diffraction divergence. The filament supports radially localized Langmuir waves. The backscattered electromagnetic wave propagates in the density depleted channel primarily in the same mode as the pump wave; its width being comparable to the diameter of the filament. The coupled mode equations are solved by the first-order perturbation theory and we have only ponderomotive nonlinearity.

In section 2 we examine the self-consistent equilibrium of filaments. The coupled mode equations inside the filaments are derived in section 3 for the Langmuir wave and the backscatter radiation. Employing perturbation theory a nonlinear dispersion relation derived and an expression for the growth rate is obtained. The results are discussed in section 4.

2. NATURE OF FILAMENT

Let us consider the propagation of a plane uniform laser beam in collisional plasma along the z-axis,

$$\vec{E} = \vec{A}_0(r,z)\exp[-i(\omega t - k_o z)] \tag{1}$$

$$k_o = (\omega/c)\left(1-\frac{\omega_{po}^2}{\omega^2}\right)^{\frac{1}{2}}, \tag{2}$$

$$\omega_{po}^2 = 4\pi n_0 e^2/m \tag{3}$$

and, ω, ω_{po}, c, $-e$, m and n_o are the frequency of the main beam, the unperturbed plasma frequency of the medium, the velocity of light, the electron charge, the electron mass and the unperturbed concentration of the plasma respectively. In the presence of the field (1), the electrons acquire drift velocity in accordance with the momentum balance equation

$$\frac{\partial \vec{v}}{\partial t} = e\vec{E} - m v_{ei}\vec{v}, \tag{4}$$

where v_{ei} is the electron collision frequency. Expressing the variation of $\vec{v}$ as exp $[-i\ (\mathrm{w}t-kz)]$, we obtain, in the limit $\omega^2 > v_{ei}^2$,

$$\vec{v} = \frac{e\vec{E}}{\mathrm{i}m\omega}\left(1-\frac{iv_{ei}}{\omega}\right). \tag{5}$$

Besides this, the electrons absorb energy absorb energy from the wave at the rate of $-e\vec{E}.\vec{v}$. Whose time average is

$$-\frac{1}{2}e\vec{E}^{*}.\vec{v} = \frac{e^2 A_o A_o^*}{2m\omega^2}. \tag{6}$$

In the steady state the rate of energy gain must balance with the rate of energy loss through collisions and thermal conduction. Hence

$$\nabla.\left(\frac{\chi}{n}\nabla T_e\right) + \frac{3}{2}\delta\nu_{ei}\left(T_e - T_o\right) = \frac{e^2\nu_{ei}A_o A_o^*}{2m\omega^2} \tag{7}$$

where $$\frac{\chi}{n} = \frac{v_{th}^2}{\nu_{ei}} \tag{8}$$

$$\delta = 2m/m_i$$

is the fraction of excess energy lost per electron-ion energy exchange collision, T_e is the nonlinear field–dependent electron temperature and $v_{\mathrm{th}} = \left(2T_o/m\right)^{\frac{1}{2}}$ is the electron thermal speed. For $\nu_{ei}r_o^2/v_{th}^2 < (\delta v)^{-1}$ thermal conduction is important, and we solve the energy–balance equation in the perturbation approximation. For a beam of finite extent we express

$$T_e = T_o + \Delta T_e,$$

where $\Delta T_e << T_o$. Then Eq. (7) can be recast as

$$\nabla^2(\nabla T_e) - \frac{3}{2}\frac{\delta\nu^2}{v_{th}^2}(\Delta T_e) = \frac{e^2\nu_{ei}^2}{2m\omega^2 v_{th}^2}|A_o|^2 \quad (9)$$

Now we perturb the beam by a perturbation

$$A_1(x, z) \exp[-i(\omega t - kz)], \quad (10)$$

where $A_1(x, z)$ is not necessarily a slowing varying function of space variables. The total electric vector of the laser may now be written as

$$\vec{E} = \left[\vec{A}o + \vec{A}1(x,z)e^{-i(\omega t - kz)}\right], \quad (11)$$

where A_o is the amplitude in the absence of fluctuations (polarized in the y direction) and A_1 is the amplitude of the fluctuations, which is a spatially slowly varying function. The combined effect of these two fields is to heat the electrons and exert a pressure-gradient force, causing redistribution of plasma via ambipolar diffusion. The nonlinear field-dependent electron temperature T_e in the steady state may be obtained by solving Eq. (9) only the x dependence of A_1 is known. Taking $A_1 \propto e^{iq.x}$ with $q_{||} << q_\perp$, where $q = q_\perp + q_{||}$ is the scale length of the perturbation (the subscripts || and $\perp$ referring to components parallel and perpendicular to the z direction), T_e may be written as

$$T_e - T_0 = \frac{e^2\left[A_0.(A_1 + A_1^*) + A_0^2\right]}{3m\omega^2\delta'} \quad (12)$$

Where $\delta' = \delta + \frac{2}{3}\frac{q^2 v_{th}^2}{3v_{ei}^2}$

As a result of non-uniformity in heating, the plasma is redistributed so that

$$n(T_e + T_0) = n_0(T_{e0} + T_0), \quad (13)$$

Where $T_e = T_0 + \frac{e^2 A_0^2}{3m\omega^2\delta'}$ (14)

Using Eq. (12), (13) and (15) in (14), the modified electron density may be written as

$$n = n_0\left[1 - \frac{e^2 A_0.(A_1 + A_1^*)}{3T_0 m\omega^2\delta'\left(2 + e^2A_0^2/3m\omega^2 T_0\delta'\right)}\right]. \quad (15)$$

The dielectric constant of the plasma may be written as

$$\in = \in_0 + (_\perp 2\, A_0 . (A_1 + A_1^*)), \quad (16)$$

where $_\perp 2 = \frac{\omega_{po}^2}{\omega^2}\frac{\alpha P}{2 + \alpha P A_0^2}$

$$P = \frac{1}{1 + 2q^2 v_{th}^2/3v_{ei}^2\delta},$$

$$\alpha = \frac{e^2}{3m\omega^2 T_0\delta}.$$

Substituting E, from Eq. (1) into the wave equation and using $\nabla . (\in E) = 0$ and linearizing in A_1, we obtain the following equation for A_1:

$$2ik_o\frac{\partial A_1}{\partial z} + \frac{\partial^2 A_1}{\partial r^2} + \frac{1}{r}\frac{\partial A_1}{\partial r} + \frac{\omega_p^2}{c^2}\frac{\alpha A_o^2 p\ (A_1 + A_1^*)}{(1 + \alpha\ \ pA_o^2)} = 0 \quad (17)$$

where $r = (x^2 + y^2)^{1/2}$ refers to a cylindrical polar co-ordinate. Expressing $A_1 = A_{ir} + iA_{1i}$ and separating real and imaginary parts,

$$2k_0\frac{\partial A_{\ell r}}{\partial z} + \frac{\partial A_{\ell i}}{\partial r^2} + \frac{1}{r}\frac{\partial A_{\ell i}}{\partial r} = 0 \quad (18)$$

2 $\quad k \quad$ 0

$$\frac{\partial A_{1i}}{\partial z} + \frac{\partial^2 A_{1r}}{\partial r^2} + \frac{1}{r}\ \frac{\partial A_{1r}}{\partial r} + \frac{2\omega_p^2}{c^2}\frac{p\alpha\ \ A_o^2}{p(1 + \alpha A_o^2)}\ \ A_{1r} = 0$$

For A_{1r}, $A_{1i} \sim J_o(q_\perp r)\ e^{\Gamma z}$ Eq. (18) straight way yields the spatial growth rate

$$\Gamma = \frac{q_\perp}{2k_o}\left[-q_\perp^2 + 2k_o^2\frac{\in_2\ A_o^2}{\in_o}\right]^{1/2} \quad (19)$$

The spatial growth maximizes to

$$\Gamma_{\max} = \frac{\omega_p^2}{2k_o^2c^2}\frac{p\alpha A_o^2}{1 + p\alpha A_o^2}$$

at $$q_{\text{opt}} = \frac{\omega p}{c}\left(\frac{p\alpha A_o^2}{1 + p\alpha A_o^2}\right)^{1/2} \quad (20)$$

where $\alpha A_0^2 = \frac{1}{6}\frac{V_o^2}{c_s^2}, V_o = \frac{e|A_o|}{m\omega_o}, c_s = \left(\frac{2T_o}{m_i}\right)^{1/2}$

and m_i is the mass of ion. The first zero of J_o occurs at $q_\perp r = 2.4$. The amount of power tends to localize in maximally growing filament can be expressed as

$$P' = \frac{c}{8\pi}\pi r^2 A_o^2$$

$$= 4.3\frac{c^3 c_s^2 m^2 w_o^2}{e^2\omega_\pi^2}\left(1 + p\alpha A_o^2\right) \quad (21)$$

Following Sodha et. al.[14] the temperature and density profile in the filament can be written as

$$T_e = T_0\left[1 + 2\alpha p A_o^2\right]\left[1 + 2\alpha_1 p E_o^2(r)\right],$$

$$n_o' = \frac{n_o}{\left[1 + \alpha_1 p E_o^2(r)\right]},$$

$$v = v_o\left(\frac{T_e}{T_o}\right)^{-3/2}\left(\frac{n_o'}{n_o}\right), \quad (22)$$

and $\alpha_1 = \frac{\alpha}{1 + 2\alpha p A_o^2},$

where $\vec{E}_o(r)$ is the total electric field of filament at r and ν_o is collision frequency corresponding to n_o and T_o, Expressing $\vec{E}_o(r)$ for cylindrically symmetric beam, as = $\vec{E}_o = \vec{A}(r,z)\exp\{-i(\omega_o t - k_o z)\}$ and neglecting $\frac{\partial^2 A}{\partial z^2}$ which implies that the characteristic distance (in the z directions) of the intensity variation is much greater than the wavelength, the wave equation reduces to

$$2ik_o\frac{\partial A}{\partial z} + \nabla_\perp^2 A + \frac{\omega_p^2}{c^2}\left(1 - \frac{n_o^/}{n_o}\right)A = 0, \quad (23)$$

where $\nabla_\perp^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$,

and $A^2 = E_{oo}^2 \,|_{r=o,\, z=0}\, e^{-r^2/r_o^2}$

Employing paraxial ray approximation, the radius of nonlinear steady state self-trapped cylindrical filament propagating through a homogeneous plasma can be obtained from Eq. (23) balancing diffraction and self-focusing terms,

$$R_d^2 = R_n^2 \quad (24)$$

where $R_d = k_o r_o^2$

and $R_n = \frac{\omega_o^2}{\omega_p^2} r_o^2 \frac{\left(1+\alpha_1 p E_{oo}^2\right)^2}{\alpha_1 p E_{oo}^2}$

Equation (24) determines the radius r_o of a self-trapped filament,

$$r_o = \frac{c}{\omega_p}\frac{\left(1+\alpha_1 p E_{oo}^2\right)}{\omega_p\left(\alpha_1 p E_{oo}^2\right)^{1/2}} \quad (25)$$

where E_o is the amplitude of the filament of radius r_o, in the nonlinear state, on the axis. The corresponding power in nonlinear steady state is

$$P = \frac{c}{8\pi}\pi r_o^2 E_{oo}^2 = \frac{c^3}{8\omega_p^2}\frac{\left(1+\alpha_1\, p E_{oo}^2\right)^2}{\alpha_1 p}$$

Equating the power contained in the filament p to $p^/$ one obtains

$$\alpha_1 p E_{oo}^2 = \left[2.4\frac{\left(1+\alpha p A_o^2\right)^{1/2}}{\left(1+2\alpha p A_o^2\right)} - 1\right] \quad (26)$$

Thus the radius and field intensity in a self-trapped filament are dependent of the initial power density of the incident beam. The density, temperature and collision frequency variation near the axis of the filament can be obtained by expanding $n_o^/$, T_e and ν around $r \cong 0$

$$n_o^/ = n_o^o\left(1+\frac{r^2}{a^2}\right), \quad (27)$$

$$T_e = T_o^o\left(1-\frac{r^2}{b^2}\right) \quad (28)$$

$$\nu = \nu_o^o\left(1+\frac{r^2}{d^2}\right), \quad (29)$$

where

$$a^2 = \frac{r_o^2\left(1+\alpha_1 p E_{oo}^2\right)}{\alpha_1 p E_{oo}^2} \quad (30)$$

$$b_2 = \frac{r_o^2\left(1+2\alpha p E_{oo}^2\right)}{2\alpha_1 p E_{oo}^2} \quad (31)$$

$$d_2 = \frac{2a^2 b^2}{3a^2+2b^2} \quad (32)$$

$$n_o^o = \frac{n_o}{\left(1+\alpha_1 p E_{oo}^2\right)}, \quad (33)$$

$$T_o^o = T_o\left(1+2\alpha p A_o^2\right)\left(1+2\alpha_1 p E_{oo}^2\right)$$

$$\nu_o^o = \nu_o^o\left(\frac{n_o^o}{n_o}\right)\left(\frac{T_o^o}{T_o}\right)^{-3/2} \quad (35)$$

3. COUPLED MODE EQUATIONS FOR B-SRS

Next we consider the instability arising through the coupling of the pump wave (laser filament) obtained in the previous section with two small amplitude lower frequency waves in the filament: an electromagnetic wave with frequency ω_1 and axial wave number $\vec{k}_1$ and a plasma wave with frequency ω and axial wave number $\vec{k}$, interacting with the pump wave with frequency ω_o and axial wave number $\vec{k}_o$ (cf. Fig. 1).

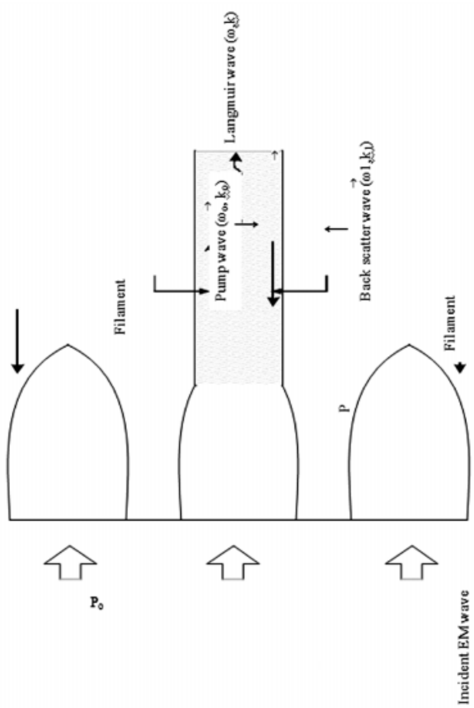

Figure 1: Schematic of Backward Stimulated Raman Scattering in a Self-Trapped Laser Filament

Consider the propagation of a laser filament. The density, temperature and collision frequency profiles are given by Eqs. (27), (28) and (29) :

$$\vec{E}_o = \vec{E}_o(r)e^{-i(\omega_o t - k_o z)} \tag{36}$$

and $$\vec{B}_o \cong \frac{c\vec{k}_o \times \vec{E}}{\omega_o}.$$

It produces an oscillatory electron velocity and $\vec{v}_o = \dfrac{e\vec{E}_o}{im\omega_o}$ excites a pair of waves, a low frequency Langmuir wave with scalar potential

$$\phi = \phi\,(r)\, e^{-i(\omega t - kz)} \tag{37}$$

and a backscatter electromagnetic wave with electric and magnetic fields

$$\vec{E}_1 = \vec{E}_1(r)e^{-i(\omega_1 t - k_1 z)} \tag{38}$$

and $$\vec{B}_1 = \frac{c\vec{k}_1 \times \vec{E}_1}{\omega_1}$$

where $\vec{k}_1 = \vec{k} - \vec{k}_o$

and $\omega_1 = \omega - \omega_o$.

The linear response of electrons to the side band is

$$\vec{v}_1 = \frac{e\vec{E}_1}{im\omega_1} \tag{39}$$

The pump and backscatter waves exert a low frequency (frequency) ponderomotive force on electrons,

$$\vec{F}_p = e\nabla\phi_p = -\frac{m}{2}\left[\vec{v}_o.\nabla\vec{v}_1 + \vec{v}_1.\nabla\vec{v}_o\right] - \frac{e}{2c}\left[\left(\vec{v}_o \times \vec{B}_0\right)\right] \tag{40}$$

Solving Eq. (40) the ponderomotive potential turns out to be

$$\phi_p = \frac{e\vec{E}_o \cdot \vec{E}_1}{2m\omega_o\omega_1} \tag{41}$$

driving the Langmuir wave

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \left[\left\{\frac{\omega^2 - \omega_{po}^2 - k^2 v_{tho}^2 + i\omega\nu_o^o}{v_{tho}^2}\right\} - \left\{r^2\frac{\omega_{po}^2}{a^2 v_{tho}^2}\left(1 - \frac{i\nu_o^o\omega}{\omega_{po}^o}\ \frac{a^2}{c_1^2}\right)\right\}\right]\phi = -\frac{\omega^2\,|\,v_o}{2\omega_o v_{th}^2}E_1 \tag{42}$$

where $v_{tho} = \left(\dfrac{2T_o^o}{m}\right)^{1/2}$.

$$v_o = v_{osc}\exp\left(-\frac{r^2}{2a^2}\right).$$

$$v_{osc} = \frac{eF_{oo}}{m\omega_o},\ c_1^2 = \frac{b^2 d^2}{b^2 + d^2}$$

$$\omega_{po} = \left(\frac{4\pi n_o^o e^2}{m}\right)^{1/2}$$

and we have assumed only collisional damping. The ponderomotive and self-consistent low frequency force $e\nabla(\phi + \phi_p)$ on the electrons drive density oscillation $n(\omega, k)$.

$$n(\omega, k) = \frac{k^2}{4\pi e}\chi_e(\phi + \phi_p) \tag{43}$$

where χ_e is the electron susceptibility.

Using n in the poisson equation, $\nabla^2\phi = 4\pi en$, we get

$$\in\phi = \chi_e\phi_p, \tag{44}$$

where $\in = 1 + \chi_e$.

The current density at the side band frequency can be written as

$$\vec{J}_1 = -n_o^o e\vec{v}_1 - \frac{1}{2}ne\vec{v}_o$$

$$= \left[\left\{-\frac{n_o^o e^2\vec{E}_1}{im\omega_1}\right\} - \left\{\frac{k^2}{4\pi e}\ \frac{e^2\vec{E}_o\phi}{2im\omega_o}\right\}\right] \tag{45}$$

Using Eq. (45) in the wave equation we get

$$\frac{\partial^2 E_1}{\partial r^2} + \frac{1}{r}\frac{\partial E_1}{\partial r} + \left(\frac{\omega_1^2 - \omega_{po}^2 - k_1^2c^2}{c^2} - \frac{\omega_{po}^2}{a^2}\frac{r^2}{c^2}\right)E_1 = \frac{k^2\omega_o|v_o|}{2c^2}\ \phi \tag{46}$$

It is considered that the sideband wave is not affected by Landau damping. However, it may suffer damping due to collisions. In this case Eq.(46) is modified to

$$\frac{\partial^2 E_1}{\partial r^2} + \frac{1}{r}\frac{\partial E_1}{\partial r} + \left[\left\{\frac{\omega_1^2 - \omega_{po}^2\left(1 - i\frac{\nu_o^o}{\omega_1}\right) - k_\ell^2 c^2}{c^2}\right\} - \left\{\frac{\omega_{po}^2}{a^2}\frac{r^2}{c^2}\left(1 - \frac{i\nu_o^o a^2}{\omega_1 d^2}\right)\right\}\right] E_1 = -\frac{k^2\omega_o|v_o|}{2c^2}\phi \tag{47}$$

The electromagnetic and plasma normal modes satisfy Eqs. (42) and (47) in the absence of nonlinear coupling, i.e.

$$\frac{\partial^2 E_1}{\partial r^2} + \frac{1}{r}\frac{\partial E_1}{\partial r} + k_{1\perp}^2 E_1 - \alpha_1^{-4} r^2 E_1 = 0$$

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+k_{1\perp}^2\phi-\alpha^{-4}r^2\phi = 0, \quad (48)$$

where $$k_\perp^2 = \frac{\omega^2-\omega_{po}^2-k^2v_{tho}^2+i\omega\nu_o^o}{v_{tho}^2}$$

$$k_{1\perp}^2 = \frac{\omega^2-\omega_{po}^2-k_1^2c^2+i\omega_{po}^2\frac{\nu_o^o}{\omega_1}}{c^2}$$

$$\alpha^{-4} = \frac{\omega_{po}^2}{a^2v_{tho}^2}\left(1-\frac{i\nu_o^o\omega}{\omega_{po}^2}\ \frac{a^2}{c_1^2}\right)$$

and $$\alpha_1^{-4} = \frac{\omega_{po}^2}{a^2c^2}\left(1-\frac{i\nu_o}{\omega_1}\frac{a^2}{d^2}\right)$$

Eqs. (48) have well-behaved solutions when

$$k_\perp^2 = 2(\ell+1)\frac{\omega_{po}}{a\ v_{tho}}\left(1-\frac{i\nu_o^o\omega}{\omega_{po}^2}\ \frac{a^2}{c_1^2}\right)^{1/2} \quad (49)$$

and $$k_{1\perp}^2 = 2(m+1)\frac{\omega_{po}}{ac}\left(1-\frac{i\nu_o^o}{\omega_1}\frac{a^2}{d^2}\right)^{1/2} \quad (50)$$

where 1 = 0, 1, 2,.... and m = 0, 1, 2... and the solutions are[16]

$$\phi = \phi_1 = \Gamma_\ell L_\ell\left(\frac{r^2}{b_1^2}\right)\exp\left(-\frac{r^2}{2b_1^2}\right)\exp\left(\frac{ir^2}{4b_1^2}\frac{\nu_o^o\omega}{\omega_{po}^2}\frac{a^2}{c_1^2}\right) \quad (51)$$

$$E_1 = E_{1m} = \Gamma_m L_m\left(\frac{r^2}{b_2^2}\right)\exp\left(-\frac{r^2}{2b_2^2}\right)\exp\left(\frac{ir^2}{4b_2^2}\frac{\nu_o^o}{\omega_1}\frac{a^2}{d^2}\right)$$

where $$b_1 = \left(\frac{av_{tho}}{\omega_{po}}\right)^{1/2},$$

$$b_2 = \left(\frac{ca}{\omega_{po}}\right)^{1/2}$$

$$L_1(\xi) = e^\xi\frac{d^\ell}{d\xi^\ell}\left(\xi^\ell e^{-\xi}\right)$$

and Γ_1 and Γ_m are normalization constants. Where the boundary conditions that the eigenfunctions $\phi_1(r)$ and $E_{1m}(r)$ be continuous at the origin and vanish as $r\to\infty$. It is easily shown that the eigenfunctions are orthogonal, and we take them to be real and normalized.

$$\int_0^{oo}\phi_i(r)\phi_j(r)rdr = \int_0^{oo}E_{1i}(r)E_{1j}(r)rdr=\delta_{ij} \quad (52)$$

Since the pump field (hence v_o) scales as exp $\left(-\frac{r^2}{2a^2}\right)$, the most unstable backscatter mode would correspond to m = 0. In the presence of nonlinear coupling terms one could express ϕ in terms of an orthogonal set of wavefunctions ϕ_1, where as E_1 can be taken to be the dominant mode,

$$\phi = \sum_1 s_1\phi_1,$$

and $$E_1 = TE_{10}. \quad (53)$$

Using (53) in (42) and (47), multiplying the resulting equation by ϕ_1 and E_{10}, respectively and integrating over rdr one obtains

$$\left[\left\{\frac{\omega^2-\omega_{po}^2-k^2v_{tho}^2+i\omega\nu_o^o}{v_{tho}^2}\right\}-\left\{2(\ell+1)\frac{\omega_{po}}{av_{tho}}\left(1-\frac{1}{2}\ \frac{\nu_o^o\omega}{\omega_{po}^2}\ \frac{a^2}{c_1^2}\right)\right\}\right]s_1$$

$$= -\frac{\omega^2T}{2\omega_oV_{tho}^2}\int rdr|v_o|\phi_\ell E_{lo}\left(1+\frac{r^2}{b^2}\right), \quad (54)$$

$$\left[\left\{\frac{\omega_1^2-\omega_{po}^2\left(1-\frac{i\nu_o^o}{\omega_1}\right)-k_1^2c^2}{c^2}\right\}-\left\{\frac{2\omega_{po}}{ac}\left(1-\frac{1}{2}\ \frac{i\nu_o^o}{\omega_1}\right)\right\}\right]T$$

$$= -\frac{k^2\omega_o}{2c^2}\sum_\ell S_\ell\int rdr|v_o|\phi_\ell E_{lo} \quad (55)$$

leading to a nonlinear dispersion

$$\left[\left\{\omega_1^2-\omega_{po}^2\left(1-\frac{i\nu_o^o}{\omega_1}\right)\right\}-\left\{k_1^2+\frac{2\omega_{po}}{ac}\left(1-\frac{1}{2}\frac{i\nu_o^o}{\omega_1}\ \frac{a^2}{d^2}\right)\right\}c^2\right]$$

$$=4\Gamma_o^2\omega\ \omega_o\times\sum\frac{\left(I_\ell^2+\frac{I_\ell.I_\ell(1)}{b^2}\right)}{\left[\left(\omega^2-\omega_{po}^2+i\omega\nu_o^o\right)-\left\{k^2+\frac{(2\ell+1)\omega_{po}}{av_{tho}}\left(1-\frac{1}{2}\frac{\nu_o^o\omega a^2}{\omega_{po}^2c_1^2}\right)\right\}v_{tho}^2\right]} \quad (56)$$

where $$I_1 = \int_0^\infty rdr\phi_\ell E_{lo}\exp\left(-\frac{r^2}{2a^2}\right) \quad (57)$$

$$I_1(1) = \int_0^\infty r^3dr\phi_\ell E_{lo}\exp\left(-\frac{r^2}{2a^2}\right) \quad (58)$$

$\frac{1}{4}(k\nu_{osc})(\omega/\omega_o)^{1/2}$ is uniform medium growth rate and we have used $v\cong v_{osc}e^{-r^2/2a^2}$. Since ϕ_1 is localized in a narrow region around $r\le 1 << a$. $I(1)$ may be simplified to become

$$I_1 \cong \frac{\sqrt{2}}{b_2}\int_0^{\infty} r^3 dr \phi_\ell \qquad (59)$$

and $$I_1(1) \cong \frac{\sqrt{2}}{b_2}\int_0^{\infty} r^3 dr \phi_\ell \qquad (60)$$

The instability growth rates are readily found from Eq. (56), one needs retain only the resonant term of Eq. (56). Expressing $\omega = \omega + i\Gamma$, obtains

$$(\Gamma + \Gamma_s)(\Gamma + \Gamma_e) = \Gamma_{oo}^2, \qquad (61)$$

where Γ_e an Γ_s and Γ_{oo} can be expressed as

$$\Gamma_e = \frac{\nu_o^o}{2}\left(1 + \frac{v_{tho}}{a\omega_{po}}\frac{a^2}{c_1^2}\right), \qquad (62)$$

$$\Gamma_s = \frac{1}{2}\frac{\nu_o^o \omega_{po}^2}{\omega_o^2}\left(1 + \frac{a^2}{d^2}\frac{c}{a\omega_{po}}\right) \qquad (63)$$

and $$\Gamma_{oo} = \Gamma_o I_1\left(1 + \frac{I_\ell I_\ell(1)}{b^2 I_\ell^2}\right)^{1/2}. \qquad (64)$$

Damping of the unstable waves introduces threshold intensity for instability generation. The threshold condition due to damping then is

$$\Gamma_{\infty} \geq \sqrt{\Gamma_e \Gamma_s} \qquad (65)$$

Considering backscatter for $\frac{\omega_{pe}}{\omega_o} << \frac{1}{2}$ and assuming only collisional damping Eqs. (62), (63), (64) and (65) give threshold value of $\frac{v_{osc}^2}{c_s^2}$,

$$P_{th}^{//} = \left(\frac{v_{osc}}{c}\right)^2_{SRS-th} = \frac{1}{4}\left(\frac{\omega_{po}}{\omega_o}\right)^2 \frac{\nu_o^{o^2}}{\omega_o \omega_{po}}\frac{yz}{x} \qquad (66)$$

where $$y = \left(1 + \frac{v_{tho}}{a\omega_{po}}\frac{a^2}{c_1^2}\right)$$

$$z = \left(1 + \frac{c}{a\omega_{po}}\frac{a^2}{d^2}\right) \qquad (67)$$

and $$x = \frac{b_1^2}{b_2^2}\left(1 + \frac{2b_1^2}{b^2}\right)$$

This threshold intensity can be quite low. One may mention that the threshold condition for B-SRS, when background plasma and intensity of laser beam is uniform is written as

$$P_{th}^{/} = \left(\frac{V_o}{c}\right)^2_{SRS-th} = \frac{1}{4}\left(\frac{\omega_p^2}{\omega_o^2}\right)\frac{\nu_o^2}{\omega_o \omega_p}, \qquad (68)$$

Substituting Eqs. (62), (63) and (64) into Eq. (61), the maximum growth rate can be expressed as

$$\Gamma = \Gamma_o \frac{2b_1}{b_2}\left(1 + \frac{2b_1^2}{b^2}\right) \qquad (69)$$

It is much more worthwhile to compare this growth rate with the one ($\Gamma_{oo}^{/}$) when the lower wave is uniform. Since, $\Gamma_{oo}^{/} = \frac{1}{4}kV_o\left(\frac{\omega}{\omega_o}\right)^{1/2}$ one obtains

$$\frac{\Gamma}{\Gamma_{oo}^{/}} = \frac{v_{OSC}}{V_o}\; 2\left(\frac{v_{tho}}{c}\right)^{1/2}\left(1 + \frac{2b_1^2}{b^2}\right) \qquad (70)$$

4. RESULTS AND DISCUSSIONS

A uniform-laser beam propagating through collisional plasma is unstable to a transverse perturbations, and break up into filaments. An optimum value of $q_\perp$ of the perturbation is required for a maximum growth rate. A uniform plane wave does not cause redistribution of carriers. However, as a result of perturbations in the intensity distribution along the wave front, electrons do become redistributed. The process of B-SRS in a filament is aided by the enhancement of power density over its initial value but it is inhibited by thermal conduction and it is observed that the power density inside the filament is much greater than the initial power density of the laser beam. Hence, the enhanced intensity in laser filament reduces collisional damping of backscatter light wave, diminishing the threshold power for B-SRS. The onset of B-SRS is strongly correlated with intensity threshold of the filamentation instability.

References

M S Sodha, A K Ghatak and V K Tripathi, "Self Focusing of Laser Beam in Plasmas and Dielectrics" *Prog Opt*, 13 pp. 169- 265(1976).

P K Kaw, G Schmidt and T Wilcox, "Filamentation and Trapping of Electromagnetic Radiation in Plasmas" *Phys Fluids*, 16, pp. 1522-1525 (1973).

A J Palmer, "Stimulated Scattering and Self-Focusing in Laser-Produced Plasmas" *Physics of Fluids, Phys Fluids*, 14, pp. 2714-2718 (1971).

K Estabrook, W L Kruer and B F Lasinski, "Heating by Raman Backscatter and Forward Scatter" *Phys. Rev. Lett.*, 45, pp. 1399-1403 (1980).

Kruer W L., "The Physics of Laser-Plasma Interactions. Addison-Wesley Publishing Company", *California* (1987)

H C Barr, T J M Boyd and G A Coutts, "Stimulated Raman Scattering in the Presence of Filamentation in Under Dense Plasmas", *Phys. Rev. Lett.*, 56, pp. 2256-2259 (1986).

H A Baldis. "Stimulated Scattering in Laser Produced Plasmas", *Plasma Phys*, 25, pp. 1297-1310 (1983)

C L Shepard, J A Tarvin, R L Berger, Gar E Busch, R R Johnson and R J Schroeder, "Phys. Raman Scattering in Experiments with Planar Au Targets Irradiated with 0.53 µm Laser Light" *Phys Fluids*, 29(2), pp. 583-593 (1986).

B Amini and F F Chen, "The Stimulated Raman Threshold.I.experiment", *Phys Fluids*, 29, pp. 3864-3871 (1986).

W Seka, E A Williams, R S Craxton, L.M. Goldman, R W Short and K Tanaka, "Convective Stimulated Raman Scattering Instability in UV Laser Plasmas", *Phys Fluids*, 27, pp 2181-2186 (1984).

J A Tarvin, G E Busch, E F Gabl, R J Schroeder and C.L. Shepard, "Laser and Plasma Conditions at the Onset of Raman Scattering in an Under Dense Plasma" *Laser Part Beams*, 4, pp. 461-471 (1986).

A Simon and R W Short "New Model of Raman Spectra in Laser Produced Plasma" *Phys Rev. Lett*, 53, pp. 1912-1914 (1984).

R. Short, W Seka and R Bahr "Stimulated Raman Scattering in Self Focused Light Filaments in Laser Produced Plasmas", *Phys Fluids*, 30, pp. 3245-3251 (1987).

T Afshar-Rad, S E Coe, O Willi and M Desselberger, "Evidence of Stimulated Raman Scattering Occurring in Laser Filaments in Long-Scale-length Plasmas", *Phys. Fluids*, B4 (5), pp. 1301-1322 (1992).

A K Sharma and V.K. Tripathi, "Two-Plasmon Decay of Self-Trapped Laser Radiation in a Plasma", *IEEE Trans Plasma Sci.*, 15(3), pp. 257-260(1987)

M. R. Spiegel,: Mathematical Handbook. McGraw-Hill, New York (1968).

V Sajal, "Relativistic Forward Stimulated Raman Scattering of a Laser in a Plasma Channel" *Phys. Scr.*, 74, pp 484-488 (2006).