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SCATTERING OF FLEXURAL GRAVITY WAVES BY THE EDGES OF A PAIR OF FLOATING ELASTIC PLATES IN PRESENCE OF SMALL UNDULATIONS ON THE OCEAN BED

A. Chakrabarti & A. T. Chwang

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ABSTRACT: The problem of scattering of surface water waves by the edges of a pair of semi-infinite thin elastic plates, with a very small gap in between, floating on water in an ocean of finite depth with its rigid bed possessing patches of small undulations, is studied. By utilizing the Wiener-Hopf technique, in conjunction with a perturbation analysis, an approximate analytical solution of the resulting boundary value problem is determined and analytical formulae are obtained for the reflection and transmission coefficients up to first order of accuracy in terms of the small parameter of undulations.

1. INTRODUCTION

Problems of scattering of surface water waves in the two-dimensional linearised theory have created varieties of challenges to applied mathematicians(see Stoker (1957), Newman (1965), Ursell (1947), Weitz and Keller (1950), Keller and Weitz (1953), Evans and Linton (1994), amongst others), willing to handle a class of mixed boundary value problems for the two-dimensional Laplace's equation under different types of mixed boundary conditions occurring in the modeling of realistic physical situations applicable to ocean engineering sciences.

In the present paper we have considered the problem of scattering of water waves involving an ocean of finite depth whose rigid bed has small patches of undulations, to take care of reality, whereas on the upper surface of the ocean two different boundary conditions are met on two sides of a line of discontinuity, constituting the edges of a pair of semi-infinite thin elastic plates, very near to each other, floating on the surface. Such discontinuous surface boundary conditions are realized in practical problems involving very large floating structures (VLFS) as well as in problems in which the top surface of the fluid is composed of two different distributions of ice particles (see Weitz and Keller (1950), Gabov et al (1989), Evans (1994)). A boundary value problem of similar nature, involving an ocean of infinite depth, was handled some time back by Chakrabarti (2000), by the use of singular integral equations of Carlemann type.

The method of solution of the presently considered boundary value problem involves the use of a suitably designed perturbation approach and the Wiener-Hopf technique (see Jones (1964), Noble (1958)). The wiener-Hopf technique has been recently employed by Tkacheva (2001) (see Balmforth and Craster (1999) also), for the problem of scattering by the edge of a floating elastic plate, involving an ocean of finite depth whose rigid bed is simply flat. By utilizing the kind of arguments similar to those of Tkacheva (2001), we have solved the present boundary value problem in two stages, after introducing a perturbation approach to determine the unknown velocity potential associated with the irrotational motion of the fluid under consideration, by expanding this potential in a regular perturbation series involving a small positive parameter ε, representing the smallness of the bed undulations, explained later on. In the first stage, we have presented the analytical solution of the zero order term of the perturbation series introduced and, in the second stage, we have obtained the analytical solution of the first order term, thus determining the complete solution of the problem at hand, up to the first order of accuracy, involving the small parameter $\varepsilon > 0$.

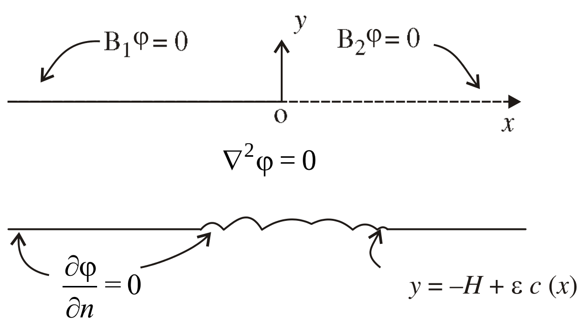
Analytical expressions are then derived for the determination of the reflection and transmission coefficients of the scattering problem under consideration here.

2. FORMULATION OF THE PROBLEM

We assume that water in the ocean of finite depth under consideration in the present paper is an ideal and incompressible fluid and that there exists a pair of thin semi-infinite elastic plates, with a very small gap in between, floating on the surface of the fluid. It is also assumed that the ocean bed has patches of small undulations in the shape of humps, as described below, through certain mathematical equations. A plane incident wave of small amplitude propagates normally to the edges of the floating plates and we wish to determine the reflected and the transmitted waves after the incident wave hits the edges and also understand the effect of the bed undulations in such scattering problems.

Introducing Cartesian coordinates (x, y), with the origin O representing the edges of the floating plates, the linearised version of the problem is as described below (see FIGURE–1).

Figure 1: Geometry of the Problem



To determine the velocity potential Re $\{\phi\,(x, y)\exp(-i\omega t)\}$ of the irrotational flow of the fluid, with t denoting time and ω denoting angular frequency, such that

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0, \qquad -\infty<x<\infty, \qquad -H+\varepsilon c(x)<y<0, \tag{2.1}$$

$$\frac{\partial\phi}{\partial n}=0, \qquad -\infty<x<\infty, \qquad y=-H+\varepsilon c(x) \tag{2.2}$$

$$B_1\varphi\equiv\left(\beta\frac{\partial^4}{\partial x^4}+1\right)\frac{\partial\phi}{\partial y}-\phi=0, \qquad -\infty<x<0, \qquad y=0, \tag{2.3}$$

$$B_2\varphi\equiv\left(\beta'\frac{\partial^4}{\partial x^4}+1\right)\frac{\partial\phi}{\partial y}-\phi=0, \qquad 0<x<\infty, \qquad y=0, \tag{2.4}$$

where β, β' (the parameters of the floating elastic plates [see Tkacheva (2001)]) and H are known positive constants, ε (> 0) is a known small parameter and $c(x)$ is a known differentiable function of compact support (i.e., $c(x)\to 0$, as $|x|\to\infty$), so that the equation $y = -H + \varepsilon c(x)$ represents the rigid bed of the ocean possessing small patches of undulations (humps), and $\partial/\partial n$ represents derivative in the direction of the outward normal to the bed.

We also need, an edge condition requiring that $\phi(x, 0) \approx O\,(x)$ as $x \to 0^{\pm}$, ensuring finiteness of energy near the edge as well as the conditions:

$$\frac{\partial^4\phi}{\partial x^3\partial y}=0=\frac{\partial^3\phi}{\partial x^2\partial y}, \text{ at } x=0^{\pm}, y=0, \tag{2.5}$$

for unique solution of the boundary value problem for which the following conditions at infinity are to be satisfied by the function $\phi(x, y)$:

$$\phi(x,y)\rightarrow\frac{\cosh(\gamma(y+H))}{\cosh(\gamma H)}\exp(i\gamma x)+R\frac{\cosh(\gamma(y+H))}{\cosh(\gamma H)}\exp(-i\gamma x), \text{ as } x\rightarrow-\infty, \tag{2.6}$$

and

$$\phi(x,y)\rightarrow T\frac{\cosh(\gamma'(y+H))}{\cosh(\gamma' H)}\exp(i\gamma' x), \text{ as } X\rightarrow\infty, \tag{2.7}$$

where γ and γ' are the positive roots of the two transcendental equations in α, as given by : $K_1(\alpha)\equiv(\beta\alpha^4+1)\alpha\tanh(\alpha H)-1=0$ and $K_2(\alpha)\equiv(\beta'\alpha^4+1)\alpha\tanh(\alpha H)-1=0$, so that γ and γ' represent the wave numbers of the incident flexural gravity wave ($\exp(i\gamma x-i\omega t)$) and the transmitted wave ($\exp(i\gamma' x-i\omega t)$) respectively and R and T represent the unknown reflection and transmission coefficients to be determined along with the unknown function ϕ.

3. ANALYTICAL SOLUTION

We solve the boundary value problem posed through the relations (2.1) to (2.7), approximately, for small values of the undulation parameter ε, in two stages, as described below.

Firstly, we express the boundary condition (2.2), to the first order of approximation , by writing the normal derivative in terms of the derivatives with respect to the coordinate axes in an usual manner and expand in powers of the small parameter ε (see Mandal and Chakrabarti (1989, 2000)) to obtain the condition as given by

$$\frac{\partial\phi}{\partial y}-\varepsilon\left[\frac{\partial}{\partial x}\left\{c(x)\frac{\partial\phi}{\partial x}\right\}\right]=0, \text{ on } y=-H,\ -\infty<x<\infty \tag{3.1}$$

Then, we assume that the following perturbation expansion holds good, for the unknown function $\phi(x)$ and the unknown complex constants R and T:

$$\phi=\phi_0+\varepsilon\phi_1+O(\varepsilon^2), \tag{3.2}$$

$$R=R_0+\varepsilon R_1+O(\varepsilon^2), \tag{3.3}$$

$$T=T_0+\varepsilon T_1+O(\varepsilon^2). \tag{3.4}$$

Substituting the above expansions in the relations (2.1), (2.2) to (2.7) and (3.1), we find that we can solve the boundary value problem of our concern above, approximately, to the first order of ε, by solving two independent boundary value problems for the two functions ϕ_0 and ϕ_1, both of which satisfy Laplace's equation (2.1) in the region $-\infty<x<\infty$, $-H<y<0$ and the two boundary conditions (2.3) and (2.4) on the boundary $y=0$, whereas the boundary conditions on the other boundary $y=-H$, are :

$$\frac{\partial\phi_0}{\partial y}=0 \text{ and } \frac{\partial\phi_1}{\partial y}=\frac{\partial}{\partial x}\left\{c(x)\frac{\partial\phi_0}{\partial x}\right\}=p(x). \tag{3.5}$$

Also both the functions ϕ_0 and ϕ_1 must satisfy the edge conditions (2.5).

The infinity conditions (2.6) and (2.7) give rise to the following conditions to be satisfied by the two functions ϕ_0 and ϕ_1 :

$$\phi_0 \to \frac{\cosh(\gamma(y+H))}{\cosh(\gamma H)}\exp(i\gamma x) + R_0 \frac{\cosh(\gamma(y+H))}{\cosh(\gamma H)}\exp(-i\gamma x), \text{ as } x \to -\infty, \tag{3.6}$$

$$\phi_1 \to R_1 \frac{\cosh(\gamma(y+H))}{\cosh(\gamma H)}\exp(-i\gamma x), \text{ as } x \to -\infty, \tag{3.7}$$

and

$$\phi_0 \to T_0 \frac{\cosh(\gamma'(y+H))}{\cosh(\gamma' H)}\exp(i\gamma' x), \text{ as } x \to \infty \tag{3.8}$$

$$\phi_1 \to T_1 \frac{\cosh(\gamma'(y+H))}{\cosh(\gamma' H)}\exp(i\gamma' x), \text{ as } x \to \infty\ . \tag{3.9}$$

We shall next utilize the Wiener-Hopf technique to solve the two independent boundary value problems for the two functions ϕ_0 and ϕ_1, as employed by Tkacheva (2001), to solve a similar boundary value problem for the two dimensional Laplace's equation.

Solution of the problem for ϕ_0

To start with, we assume that γ and γ' possess small imaginary parts (see Jones (1964)) which will be taken to be zero at the end of the analysis, as such an assumption will help transforming the boundary value problem here, as well as in the next part of this paper, into Wiener-Hopf problems valid in a certain strip of the complex α-plane, as explained below.

Next, we write

$$\phi_0 = \frac{\cosh(\gamma(y+H))}{\cosh(\gamma H)}\exp(i\gamma x) + \frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{V_+(\alpha)}{K_1(\alpha)} \frac{\cosh(\alpha(y+H))}{\cosh(\alpha H)}\left[U(\alpha) + \frac{1}{(\alpha+\gamma)}\right]\exp(-i\alpha x)d\alpha \tag{3.10}$$

where $K_1(\alpha)$ is as introduced before, $U(\alpha)$ is an unknown (to be determined) entire function and $V_+(\alpha)$ is an unknown (to be determined) analytic function in the upper half of the complex α-plane, $\mathrm{Im}(\alpha) > c_1$, where c_1 is a suitably chosen real number so that the Fourier transform of $\phi_0\,(x, y)$ represents an analytic function in a strip $c_1 < \mathrm{Im}\,(\alpha) < c_2$, c_2 being another appropriately chosen real number. The details (see Tkacheva (2001)) of the choices of the constants c_1 and c_2 depend on the zeros of the functions $K_1(\alpha)$ and $K_2(\alpha)$, and we use the convention that the zeros γ and γ' lie in the upper half plane $\mathrm{Im}\,(\alpha) > c_1$ where also lie all complex zeros with positive imaginary parts, whereas the zeros $-\gamma$ and $-\gamma'$, as well as the complex zeros with negative imaginary parts all lie in the lower half plane $\mathrm{Im}\,(\alpha) < c_2$.

It is important to note that , because of the requirement of finite energy in the neighbourhood of the edge at $x = 0$, we must have that the integrand in the relation (3.10), at the point, $x = 0 = y$ must be $O\left(\frac{1}{\alpha^2}\right)$, as $\alpha \to \infty$.

We observe that the above choice (3.10) of the function ϕ_0 satisfies the boundary condition (2.3) on the boundary $y = 0$ automatically, and, it will also satisfy the other boundary condition (2.4), if we select the function $V_+(\alpha)$, such that

$$\frac{V_+(\alpha)}{K_1(\alpha)} = \frac{Q_-(\alpha)}{K_2(\alpha)} \text{ and } \frac{Q_-(-\gamma)}{K_2(-\gamma)} = 1, \tag{3.11}$$

where $Q_-(\alpha)$ is another unknown (to be determined) analytic function in the lower half plane $\mathrm{Im}(\alpha) < c_2$, and $K_2(\alpha)$ is as introduced before.

The first of the two relations in (3.11) represents a simple Wiener-Hopf problem, whose solution is given by the following relations:

$$\frac{V_+(\alpha)}{K_+(\alpha)} = K_-(\alpha)Q_-(\alpha) = E(\alpha)\text{, (where } E(\alpha) \text{ is an entire function)} \tag{3.12}$$

obtained after utilizing the Wiener-Hopf factorization of the function $K(\alpha) = \frac{K_1(\alpha)}{K_2(\alpha)}$, in the form $K(\alpha) = K_-(\alpha)$ $K_+(\alpha)$, in usual notations.

Then, by utilizing the behavior of the integrand in (3.10), as $\alpha \to \infty$ (see above), we find that we must have the following results:

$U(\alpha)$ = A polynomial of degree three = $a\alpha^3 + b\alpha^2 + c\alpha + d$, (say), (3.13)

where a, b, c and d are four unknown constants to be determined and

$$E(\alpha) = \text{A constant} = Q_-(-\gamma)K_-(-\gamma) = K_2(-\gamma)K_-(-\gamma) = \frac{(\beta' - \beta)\gamma^4}{(\beta\gamma^4 + 1)} K_+(\gamma)\,, \tag{3.14}$$

by using the second relation in (3.12) and the identity $K_2(-\gamma) = \frac{(\beta' - \beta)\gamma^4}{(\beta\gamma^4 + 1)}$.

Thus, using the relations (3.12) ,(3.13) and (3.14) in the representation (3.10), we determine the solution $\phi_0(x, y)$ completely, as given by the following formula:

$$\phi_0(x,y) = \frac{\cosh(\gamma(y+H))}{\cosh(\gamma H)} \exp(i\gamma x)$$

$$+ \frac{(\beta' - \beta)\gamma^4}{(\beta\gamma^4 + 1)} \cdot \frac{K_+(\gamma)}{2\pi i} \int_{-\infty}^{\infty} \frac{K_+(\alpha)}{K_1(\alpha)} \left(a\alpha^3 + b\alpha^2 + c\alpha + d + \frac{1}{\alpha + \gamma} \right) \frac{\cosh(\alpha(y+H))}{\cosh(\alpha H)} \exp(-i\alpha x) d\alpha. \tag{3.15}$$

The four unknown constants a, b, c and d can be determined by utilizing the four edge conditions (2.5), satisfied by the function ϕ_0.

Then, the zero order reflection and transmission coefficients R_0 and T_0, as defined by the relations (3.6) and (3.8) can be easily determined by using the solution formula (3.15), by closing the contour in the upper half plane for $x < 0$ and in the lower half plane for $x > 0$, respectively, along with the identity:

$$\frac{K_+(\alpha)}{K_1(\alpha)} = \frac{1}{K_-(\alpha)K_2(\alpha)}.$$

We obtain the following results:

$$R_0 = \frac{(\beta' - \beta)\gamma^4 \{K_+(\gamma)\}^2}{K_1'(\gamma)(\beta\gamma^4 + 1)} \left(a\gamma^3 + b\gamma^2 + c\gamma + d + \frac{1}{2\gamma} \right) \tag{3.16}$$

and

$$T_0 = \frac{(\beta - \beta')\gamma^4 K_+(\gamma) \left(d - c\gamma' + b\gamma'^2 - a\gamma'^3 + \frac{1}{\gamma - \gamma'} \right)}{K_+(\gamma') K_2'(-\gamma')(\beta\gamma^4 + 1)}\,. \tag{3.17}$$

The zero order problem can thus be solved completely, after determining the four constants a, b, c and d, as explained above.

Thus, everything will be computable, for known values of the parameters β and β', except the four constants a, b, c, d, which will have to be determined by solving four linear algebraic equations for these unknowns, obtained by using (3.15) and (2.5).

Solution of the Problem for ϕ_1

The solution of the boundary value problem for the function $\phi_1(x, y)$, satisfying the conditions (2.3), (2.4), (3.5), (3.7) and (3.9) can be converted into a two-part Wiener-Hopf problem, by using Jones's method (see Noble (1958) and Tkacheva (2001)), in a straightforward manner.

Leaving aside the details, we find that the resulting Wiener-Hopf functional relation to be solved is:

$$F_+(\alpha) - K(\alpha)G_-(\alpha) = (\beta' - \beta)\alpha^4 \frac{P(\alpha)}{\cosh(\alpha H)K_2(\alpha)}, \tag{3.18}$$

(for α belonging to the strip $c_1 < \text{Im}(\alpha) < c_2$, introduced before)

where the Wiener-Hopf unknowns $F_+(\alpha)$ and $G_-(\alpha)$ are the half-range Fourier transforms which are analytic functions in the upper and lower half of the complex α-plane, as defined by the relations:

$$F_+(\alpha) = \int_0^\infty f(x)e^{i\alpha x}dx \quad \text{and} \quad G_-(\alpha) = \int_{-\infty}^0 g(x)e^{i\alpha x}dx, \tag{3.19}$$

where the unknown functions $f(x)$ and $g(x)$ are defined as:

$$f(x) = \left(\beta\frac{\partial^4}{\partial x^4} + 1\right)\frac{\partial \phi_1}{\partial y} - \phi_1, \text{on } y = 0, x > 0 \ \text{ and } \ g(x) = \left(\beta'\frac{\partial^4}{\partial x^4} + 1\right)\frac{\partial \phi_1}{\partial y} - \phi_1, \text{on } y = 0, x < 0, \tag{3.20}$$

$$P(\alpha) = \int_{-\infty}^\infty p(x)e^{i\alpha x}dx, \tag{3.21}$$

and $K(\alpha) = \dfrac{K_1(\alpha)}{K_2(\alpha)}$, as introduced before.

The function $\phi_1(x, y)$ can be obtained by using the Fourier inversion formula, as given by

$$\phi_1(x, y) = \frac{1}{2\pi}\int_{-\infty}^\infty \Phi(\alpha, y)e^{-i\alpha x}d\alpha, \tag{3.22}$$

applied to the transformed function

$$\Phi(\alpha, y) \equiv \Phi_+(\alpha, y) + \Phi_-(\alpha, y) = \int_0^\infty \phi_1(x, y)e^{i\alpha x}dx + \int_{-\infty}^0 \phi_1(x, y)e^{i\alpha x}dx, \tag{3.23}$$

where

$$\Phi(\alpha, y) = C_1(\alpha)\cosh(\alpha(y + H)) + C_2(\alpha)\sinh(\alpha(y + H)), \tag{3.24}$$

with

$$C_2(\alpha) = \frac{P(\alpha)}{\alpha} \tag{3.25}$$

and

$$C_1(\alpha)=\frac{F_+(\alpha)}{K_1(\alpha)\cosh(\alpha H)}-\frac{P(\alpha)((\beta\alpha^4+1)\alpha-\tanh(\alpha H))}{\alpha K_1(\alpha)},$$

or (3.26)

$$C_1(\alpha)=\frac{G_-(\alpha)}{K_2(\alpha)\cosh(\alpha H)}-\frac{P(\alpha)((\beta'\alpha^4+1)\alpha-\tanh(\alpha H))}{\alpha K_2(\alpha)}.$$

The Wiener-Hopf problem (3.18) can be solved in an usual manner, by using the factorization of the function $K(\alpha) = K_+(\alpha)\, K_-(\alpha)$, used before, and we obtain the following results:

$$F_+(\alpha) = K_+(\alpha)\,[X_+(\alpha) + P_3(\alpha)] \text{ and } G_-(\alpha)=-\frac{[X_-(\alpha)-P_3(\alpha)]}{K_-(\alpha)}, \quad (3.27)$$

where the Wiener-Hopf split functions $X_\pm(\alpha)$ are given by the following formulae:

$$X_\pm(\alpha)=\pm\frac{1}{2\pi i}(\beta'-\beta)\int_{-\infty+id}^{\infty+id}\frac{\xi^4P(\xi)K_-(\xi)}{K_1(\xi)\cosh(\xi H)(\xi-\alpha)}\,d\xi,(c_1<d<c_2), \quad (3.28)$$

with $P_3(\alpha)$ representing a polynomial of degree 3, whose four coefficients can be determined fully by the aid of the four edge conditions (2.5).

The complete solution for the function $\phi_1(x, y)$ can thus be determined by the aid of the relations (3.22) to (3.28) and we obtain, in particular, the following results of importance:

$$\phi_1(x,0)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{[F_+(\alpha)\cosh(\alpha H)-(\beta\alpha^4+1)P(\alpha)]}{K_1(\alpha)\cosh(\alpha H)}e^{-i\alpha x}\,d\alpha \quad (3.29)$$

or,

$$\phi_1(x,0)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{[G_-(\alpha)\cosh(\alpha H)-(\beta'\alpha^4+1)P(\alpha)]}{K_2(\alpha)\cosh(\alpha H)}e^{-i\alpha x}\,d\alpha. \quad (3.30)$$

The above results (3.29) and (3.30) can be used effectively to determine the first order reflection and transmission coefficients by evaluating the integrals on the right side by closing the contour in the upper or lower half plane according as $x < 0$, or $x > 0$.

We find the following formulae, which are in terms of known functions described above:

$$R_1=i\frac{[F_+(\gamma)\cosh(\gamma H)-(\beta\gamma^4+1)P(\gamma)]}{K_1'(\gamma)\cosh(\gamma H)}$$

and (3.31)

$$T_1=i\frac{\left[G_-(-\gamma')\cosh(\gamma' H)-(\beta'\gamma'^4+1)P(-\gamma')\right]}{K_2'(-\gamma')\cosh(\gamma' H)}.$$

It is possible to cast the above expressions in (3.31) in terms of the zeros of the functions $K_1(\alpha)$ and $K_2(\alpha)$ (see Evans and Linton (1994) and Tkacheva (2001)), by using the relations (3.28) and the residue calculus which helps in obtaining numerical results for specific choices of the undulation function $c(x)$.

This completes the description of obtaining the analytical solution of the boundary value problem considered in this paper, valid up to the first order accuracy in terms of the small undulation parameter ε. The numerical work can be carried out by using the various expressions and the zeros of $K_1(\alpha)$ and $K_2(\alpha)$, which is deferred to a later paper.

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REFERENCES

[1] Balmforth, N. J. & Craster, R. V., Ocean Waves and Ice Sheets, *J.Fluid Mech.* **395,** (1999), 89–124.

[2] Chakrabarti, A., On the Solution of the Problem of Scattering of Surface Water Waves by a Sharp Discontinuity in the Surface Boundary Conditions, *Anziam J.*, **42,** (2000), 277–286.

[3] Evans, D. V., The Solution of a Class of Boundary Value Problems with Smoothly Varying Boundary Conditions, *Q. J. Mech. Appl. Math.* **38**, 521–536.

[4] Evans, D. V. & Linton, C. M., On the Step Approximation for Water Wave Problems, *J. Fluid Mech.*, **278**, (1994), 229–249.

[5] Gabov, S. A., Svesnikov, A. G. & Shatov, A. K., Dispersion of Internal Waves by an Obstacle Floating on the Boundary Separating two Liquids, Prikl. *Mat. Mech.*, **53,** (1989), 727–730 (Russian).

[6] Jones, D. S. (1964), *The Theory of Electrmagne*tism, Pergamon Press, Oxford.

[7] Keller & Weitz, Reflection and Transmission Coefficients of Water Waves Entering or Leaving an Ice-field, *Comm.Pure Appl. Math.*, **6**, (1953), 415–417.

[8] Mandal, B. N. & Chakrabarti, A., A Note on Diffraction of Water Waves by a Nearly Vertical Barrier, *IMA J. Appl. Math.*, **43,** (1989), 157–165.

[9] Mandal, B. N. & Chakrabarti, A. (2000), *Water Wave Scattering by Barriers*, WIT Press, Southampton- Boston.

[10] Newman, J. N. (1965), Propagation of water waves past long two- dimensional obstacles , J.Fluid Mech. 23, 23-29.

[11] Noble, B. (1958), Methods Based on the Wiener-Hopf Technique for the Solution of Partial Differential Equations, Pergamon Press, London-New York-Paris.

[12] Stoker, J. J. (1957), *Water Waves*, Wiley Interscience, New York.

[13] Ursell, F., The Effect of a Fixed Vertical Barrier on Surface Water Waves in Deep Water, *Proc. Cambridge Philos. Soc.,* **43**, (1947), 374–382.

[14] Tkacheva, , L. A., Scattering of Surface Waves by the Edge of a Floating Elastic Plate, *J. Appl. Mech., Tech., Phys.*, **42**, (2001), 638–646.

[15] Weitz & Keller, Reflection of Water Waves from Floating Ice in Water of Finite Depth, *Comm. Pure Appl. Math.* **3**, (1950), 305–318.

A. Chakrabarti
Department of Mathematics
Indian Institute of Science
Bangalore–560012, India
E-mail : alok@math.iisc.ernet.in

A. T. Chwang
Department of Mechanical Engineering
The University of Hong Kong
Pokfulam Road, Hong Kong