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GENERALIZED REYNOLDS' EQUATION APPLIED TO MICROPOLAR FLUID FILM LUBRICATION PROBLEMS

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***ABSTRACT*:** This paper considers the application aspect of the generalized one-dimensional Reynolds' type equation for Micropolar Fluid Film (MMPF) lubrication to the cases of squeeze film, journal bearing and slider bearing problems. We obtained analytical expressions for some important physical factors such as load capacity etc. in lubrication problems. In the case of one-dimensional MPFF lubrication, we observe that the arbitrary boundary condition taken by some authors does not influence the volume flow rate which is an important physical factor in lubrication problems.

Keywords: Micropolar Fluids, Lubrication, Reynolds' Equation

INTRODUCTION

Reynolds' lubricant was a Newtonian fluid. But the latest needs of technology necessitated the consideration of a class of non-Newtonian fluids as lubricants. One of the mathematical models of non-Newtonian fluids that have been considered extensively in recent times is that of the Micropolar fluids. Cowin [1] was one of the earliest to suggest the application of polar fluid theory to the problems of lubrication. Green [2] studied the problem of slider bearing lubrication, making use of a polar fluid film as a lubricant. He [3] established a variational theory for Micropolar Fluids in Lubrication Journal Bearing problem. Allen and Kline [4] reviewed the development of polar fluid theories. Several researchers have eventually studied the problem of lubrication with micropolar fluids as lubricants. But all these researchers have assumed the no spin boundary condition i.e. $v = 0$ on the rigid boundaries and the usual no slip condition $V = 0$ for velocity at rigid walls. Ariman and Cakmak [5] and Ariman et al [6] also used the no slip boundary condition. In Ariman and Cakmak [5], two separate boundary conditions were presented. One is the no slip boundary condition and the other one is that the micro rotation equals the fluid velocity at a solid boundary. Kirwan and Newman [7] felt that the arguments used to justify the restriction to boundary conditions for micro-rotation to the above two mentioned were not convincing. Consequently, they felt that the results previously published do not indicate the rich variety of phenomena that the theory allows for. Later on, Ariman et al [8] in their study of steady and pulsatile flow of blood, modeled after a micropolar fluid, considered the constant spin condition $v' = 0$ on the walls of the tube. It should be noted that the constant spin condition could be obtained as a special case of the general boundary condition given by Aero et al [9]. In their review on micro continuum fluid mechanics, Ariman et al [8] noted that the problem of identification of the best spin boundary condition for fluids with rigid micro elements remains unsolved even today. Aero et al [9] questioned the relevance of the formulated boundary conditions reflecting the nature of the interaction of the fluid with the solid surface, which must finally be determined by experiment. Not many experiments, however, were conducted in this field. Kolpashchikov et al [10] took up the task of determining the micropolar fluid material constants. Motivated by the work of Kolpashchikov et al [10], Chenchu Raju et al [11] reconsidered the MPFF lubrication and derived the generalized one-dimensional Reynolds' type equation.

In this paper, this equation has been applied to the cases of squeeze film, journal bearing and slider bearing problems. The work of Prakash and Sinha [12, 13], Allen and Kline [4] has been reconsidered in the light of the change in the spin boundary condition which is taken as

$$\vec{v}_b = \frac{\beta}{2}(curl\,\vec{V})_b$$

where β is an arbitrary parameter known as the parameter of boundary conditions and is restricted to $0 \le \beta \le 1$. When $\beta = 1$, the angular velocity of the fluid equals the spin at the boundary and it is observed in equations (20) and (21) of [11] interestingly that this will not show any polarity effects in the Reynolds' equation.

THE SQUEEZE FILM

The problem considered is that of a steady laminar flow of an incompressible micropolar fluid between two closely spaced plates in relative normal motion. The upper plate moves with a squeeze velocity V_o towards the stationary lower plate. No tangential velocity of the plates is allowed. The governing equations are

$$\frac{dp}{dx} = \frac{1}{2}(2\mu + \chi)\frac{\partial^2 u}{\partial y^2} + \chi\frac{\partial \nu}{\partial y} \tag{1}$$

$$\gamma\frac{\partial^2 \nu}{\partial y^2} - 2\chi\nu - \chi\frac{\partial u}{\partial y} = 0 \tag{2}$$

The boundary conditions are

$$\nu = -\frac{\beta}{2}\frac{\partial u}{\partial y},\ u = 0 \qquad \text{at } y = 0$$

$$\nu = -\frac{\beta}{2}\frac{\partial u}{\partial y}, \qquad u = 0 \quad \text{at } y = h \tag{3}$$

Taking $U_1 = U_2 = 0$ in (17) and (18) of [11]

$$u = \frac{N^2}{m}\frac{h}{2\mu}\frac{1}{\sinh mh}p_x f(y) + \frac{y}{2\mu}p_x(y-h) - \frac{\beta(1-N^2)}{1-\beta N^2}\frac{N^2}{m}\left\{\frac{h}{2\mu}\frac{1}{\sinh mh}p_x f(y\right\} \tag{4}$$

and

$$\nu = \frac{1}{4\mu}p_x(h-2y) + \frac{1-\beta}{1-\beta N^2}\frac{h}{4\mu}p_x\left\{\frac{(\cosh mh + 1)}{\sinh mh}\sinh my - \cosh my\right\} \tag{5}$$

For $\beta = 0$, these expressions coincide with equations (19) and (20) of Prakash and Sinha [12].

The equation of continuity gives

$$\frac{\partial}{\partial x}\int_0^h u\,dy = V_0$$

i.e.,

$$-\frac{h}{12\mu}\frac{d^2 p}{dx^2}\left(h^2 + 12l^2 - 6NlhCth\frac{mh}{2}\right) + \frac{\beta(1-N^2)}{1-\beta N^2}\frac{h}{2\mu}\frac{d^2 p}{dx^2}\left(2l^2 - NlhCth\frac{mh}{2}\right) = V_0$$

or

$$\frac{d^2 p}{dx^2} = -\frac{\mu V_0}{h^3 F(N,l,h,\beta)} \tag{6}$$

where

$$F(N,l,h,\beta) = \left(\frac{1}{12} + \frac{l^2}{h^2} - \frac{Nl}{2h}Cth\frac{mh}{2}\right) - \frac{\beta(1-N^2)}{1-\beta N^2}\left(\frac{l^2}{h^2} - \frac{Nl}{2h}Cth\frac{mh}{2}\right)$$

This is the modified form of the conventional one-dimensional Reynolds' equation which gives us the equations of Prakash and Sinha [12] for $\beta = 0$. In the case of circular plates, using polar co-ordinates and making use of axisymetry, the above equation transforms to

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dp}{dr}\right) = -\frac{\mu V_0}{h^3 F(N,l,h,\beta)} \tag{7}$$

(i) Circular Pates

The pressure is obtained by integrating the equation (7) with the boundary conditions

$$\frac{dp}{dr} = 0 \text{ at } r = 0$$

$$p = p_e \text{ at } r = a$$

where a is the radius of the circular plate. Thus

$$p - p_e = \frac{\mu V_0}{4h^3}\frac{(a^2 - r^2)}{F(N,l,h,\beta)} \tag{8}$$

The instantaneous load supporting capacity w is obtained by integrating the pressure over the surface

$$w = \int_0^a 2\pi r(p - p_e)dr$$

$$= \frac{\pi\mu V_o a^4}{8h^3 F(N,l,h,\beta)} \tag{9}$$

Assuming the load to be constant, the time taken in reducing the height h_0 to a prescribed film thickness h is obtained from the above equation by substituting $V_0 = -dh/dt$.

Then

$$t = -\frac{3\pi\mu a^4}{4w}\frac{1}{6}\int_{h_0}^{h}\frac{1}{h^3 F(N,l,h,\beta)}dh \tag{10}$$

For $\beta = 0$, equation (10) gives the corresponding result of Prakash and Sinha [12].

(ii) Rectangular Plates

The pressure distribution for rectangular plates (infinitely long) is obtained by integrating the equation (7) with the boundary conditions

$$p = p_e \text{ at } x = \pm a$$

where $2a$ is the length of the plate. Thus

$$p - p_e = \frac{\mu V_0 (a^2 - x^2)}{2h^3 F(N,l,h,\beta)} \tag{11}$$

The load is given by

$$w = B\int_{-a}^{a}(p - p_e)dx$$

$$= \frac{2\mu V_0 B a^3}{3h^3 F(N,l,h,\beta)} \tag{12}$$

and

$$t = -\frac{3\pi\mu a^3}{4w}\frac{1}{6}\int_{h_0}^{h}\frac{1}{h^3 F(N,l,h,\beta)}dh \tag{13}$$

The effect of the parameter of boundary values is reflected in $F(N,l,h,\beta)$.

JOURNAL BEARING

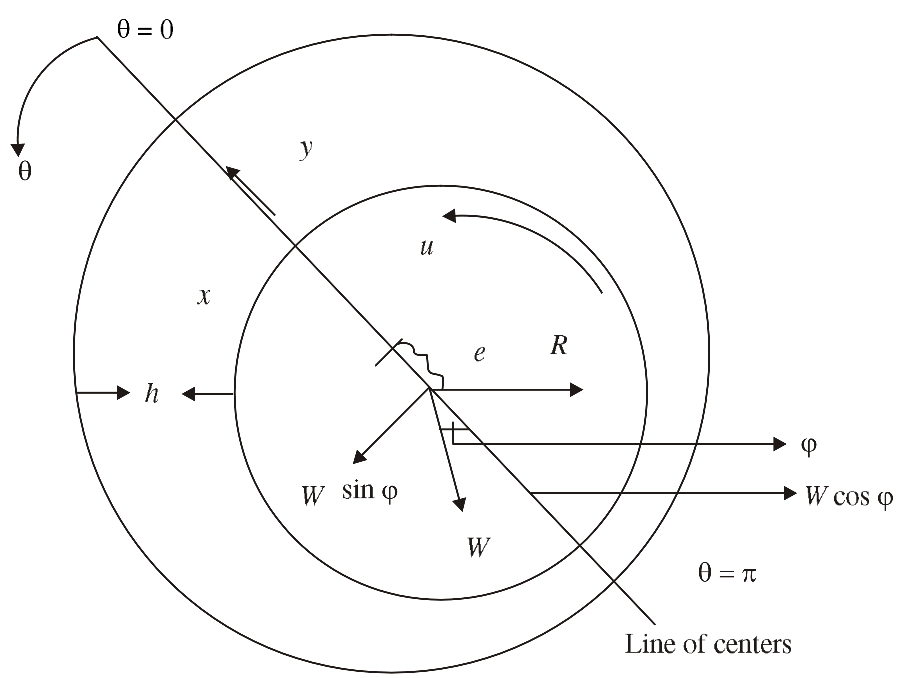
The problem considered is that of steady laminar flow of incompressible micropolar fluid between two eccentric cylinders in relative rotatory motion. The gap of the clearance c, between these cylindrical surfaces is small compared to the radius of the inner cylinder (journal). This is the traditional journal bearing geometry to lubrication theory. The bearing is assumed to be infinitely long in the axial direction. The curvature of the fluid film is neglected since the height of the fluid film is very small compared to the radius.

As a consequence the necessity of employing cylindrical polar co-ordinates is eliminated and one can use the Cartesian co-ordinate system with $x = R\,\theta$.

Following simplifications of Prakash and Sinha [13] based on dimensional analysis the problem can be stated as

$$\frac{\partial p}{\partial x} = \frac{1}{2}(2\mu+\chi)\frac{\partial^2 u}{\partial y^2} + \chi\frac{\partial \nu}{\partial y} \tag{14}$$

$$\gamma\frac{\partial^2 \nu}{\partial y^2} - 2\chi\nu - \chi\frac{\partial u}{\partial y} = 0 \tag{15}$$



Since $x = R\theta$, equation (14) becomes

$$\frac{1}{R}\frac{\partial p}{\partial \theta} = \frac{1}{2}(2\mu + \chi)\frac{\partial^2 u}{\partial y^2} + \chi \frac{\partial \nu}{\partial y} \tag{16}$$

The boundary conditions are

$$\nu = -\frac{\beta}{2}\frac{\partial u}{\partial y},\ u = U \text{ at } y = 0$$

$$\nu = -\frac{\beta}{2}\frac{\partial u}{\partial y},\ u = 0 \text{ at } y = h \tag{17}$$

where h is the film thickness given by

$$h = c\,(1 + e\cos\theta)$$

wherein c is the half of the difference between two radii and is the eccenentricity ratio.

(i) Velocity Distribution

From (17) and (18) of [11], we get, when U_1 is replaced by U and U_2 by 0,

$$u = U + \frac{N^2}{m}\frac{h}{2\mu}\frac{1}{\sinh mh}p_x f(y) + \frac{y}{2\mu}p_x(y-h)$$

$$+\frac{U}{D}\left\{y\sinh mh - \frac{N^2}{m}g(y)\right\}$$

$$+\frac{\beta(1-N^2)}{1-\beta N^2}\frac{N^2}{m}\left\{\frac{U}{D}g(y) - \frac{h}{2\mu}\frac{1}{\sinh mh}p_x f(y)\right\} \tag{18}$$

$$\nu = \frac{1}{4\mu}p_x(h-2y) + \frac{1-\beta}{1-\beta N^2}\left[\frac{h}{4\mu}p_x\left\{\frac{(\cosh mh+1)}{\sinh mh}\sinh my - \cosh my\right\}\right.$$

$$\left. +\frac{U}{D}\left\{\sinh mh\left(\cosh my - \frac{1-\beta N^2}{1-\beta}\right) - (\cosh mh - 1)\sinh my\right\}\right] \tag{19}$$

where

$$m = \frac{N}{l},\ D = \frac{2N^2}{m}\left(\frac{1-\beta}{1-\beta N^2}\right)(\cosh mh - 1) - h\sinh mh\,,$$

$$f(y) = \sinh mh \sinh my - (\cosh mh + 1)(\cosh my - 1),$$

and

$$g(y) = \sinh mh \sinh my - (\cosh mh - 1)\cosh my - 1)$$

where $0 < \beta < 1$. For $\beta = 0$, these expressions coincide with equations (22) and (23) of Prakash and Sinha [13].

(ii) Pressure Distribution and Flow Flux

The volume flow rate Q is given by

$$Q=\frac{Uh}{2}-\frac{h}{12\mu}p_x\left(h^2+12l^2-6Nlh\ \ Cth\frac{mh}{2}\right)+\frac{\beta(1-N^2)}{1-\beta N^2}\frac{h}{2\mu}p_x\left(2l^2-Nlh\ \ Cth\frac{mh}{2}\right)$$

$$=\frac{Uh}{2}-\frac{h}{12\mu R}\frac{\partial p}{\partial\theta}\left(h^2+12l^2-6Nlh\ \ Cth\frac{mh}{2}\right)+\frac{\beta(1-N^2)}{1-\beta N^2}\frac{h}{2\mu R}\frac{\partial p}{\partial\theta}\left(2l^2-Nlh\ \ Cth\frac{mh}{2}\right) \quad (20)$$

i.e.,

$$\frac{\partial p}{\partial\theta}\frac{h^3}{\mu R}F(N,l,h,\beta)=\frac{Uh}{2}-Q \quad (21)$$

For $\beta = 0$,

$$F(N, l, h, \beta) = F(N, l, h, 0) = F(N, l, h)$$

of Prakash and Sinha [13].

From equation (21),

$$\frac{\partial p}{\partial\theta}=\frac{\frac{Uh}{2}-Q}{\frac{h^3}{\mu R}F(N,l,h,\beta)}$$

for those values of N, l, h, β such that $F(N, l, h, b) \neq 0$.

$$\frac{\partial p}{\partial\theta}=\frac{\mu R}{2h^3F(N,l,h,\beta)}(Uh-2Q) \quad (22)$$

Integrating the above equation with respect to θ and using the boundary conditions

$$p = 0 \text{ at } \theta = 0$$

$$p = 0 \text{ at } \theta = \pi$$

the pressure distribution is obtained as

$$p(\theta)=\frac{1}{2}F_1(\theta)-QF_2(\theta) \quad (23)$$

where

$$Q=\frac{1}{2}\frac{F_1(\pi)}{F_2(\pi)},$$

$$F_1(\theta)=\int_0^{\theta}\frac{U\mu R}{h^2F(N,l,h,\beta)}d\theta \quad (24)$$

and

$$F_2(\theta)=\int_0^{\theta}\frac{\mu R}{h^3F(N,l,h,\beta)}d\theta \quad (25)$$

(iii) Load Capacity

The load components per unit length along and perpendicular to the line of centers are obtained in the usual way by integrating pressure over the bearing surface

$$w_{\pi/2} = \int_0^{\pi} p \sin\theta d\theta$$

Substituting for p and integrating by parts

$$w_{\pi/2} = \int_0^{\pi} \left[\frac{1}{2} F_1(\theta) - QF_2(\theta)\right] \sin\theta d\theta$$

i.e.,

$$w_{\pi/2} = \frac{1}{2}\int_0^{\pi} \frac{U\mu R(1+\cos\theta)}{h^2 F(N,l,h,\beta)} d\theta - Q\int_0^{\pi} \frac{\mu R(1+\cos\theta)}{h^3 F(N,l,h,\beta)} d\theta \tag{26}$$

Similarly the load component along the line of centre is

$$w_0 = -\int_0^{\pi} p \cos\theta d\theta$$

i.e.,

$$w_0 = \frac{1}{2}\int_0^{\pi} \frac{U\mu R \sin\theta}{h^2 F(N,l,h,\beta)} d\theta - Q\int_0^{\pi} \frac{\mu R \sin\theta}{h^3 F(N,l,h,\beta)} d\theta \tag{27}$$

(iv) Frictional Drag

The shear stress along the surface is

$$t_{21} = \frac{1}{2}(2\mu+\chi)\frac{\partial u}{\partial y} - \chi\nu = \frac{1}{2}(2\mu+\chi)u' + \frac{\beta}{2}\chi u'$$

i.e.,

$$t_{21} = \chi\left(\frac{1+\beta N^2}{2N^2}\right)u' \tag{28}$$

(prime refers to differentiation with respect to y)

The shear stress T_0 on the journal is

$$T_0 = \frac{\chi}{2}\left(\frac{1+\beta N^2}{N^2}\right)u'(0)$$

i.e.,

$$T_0 = \mu\left(\frac{1+\beta N^2}{1-\beta N^2}\right)\left(\frac{U_1 - U_2}{D}\sinh mh - \frac{h}{2\mu}p_x\right) \tag{29}$$

The frictional drag F, per unit length is obtained by integrating the shear stress over the moving surface.

Thus

$$F = \int_0^{2\pi} T_0 R d\theta$$

$$= \mu\left(\frac{1+\beta N^2}{1-\beta N^2}\right)\int_0^{2\pi}\left(\frac{U_1 - U_2}{D}\sinh mh - \frac{h}{2\mu}p_x\right)d\theta \tag{30}$$

$$\bar{F} = \frac{Fc}{\mu UR} \tag{31}$$

The above results show the influence of β on all the physical quantities of interest in the lubrication problem. When $\beta = 0$ all these results go back to those obtained incorporating the no spin boundary condition. $\beta = 1$ gives the 'Cauchy condition' where at the boundary the spin is equal to the angular velocity of the fluid and we have seen in (32) and (33) that this will not reflect any polarity effects in one-dimensional lubrication problems.

We finally study the effect of the changed boundary condition on the slider bearing lubrication, studied earlier by Allen and Kline [14] with the no spin condition.

SLIDER BEARING

Allen and Kline [4] was one of the early results published in the area of MPFF lubrication. The basic equations are same as those given by (1) and (2) together with the specific boundary conditions. Allen and Kline [4] however, followed a different notation. They used μ for the classical viscosity and μ_1 and γ for micropolar material constants. To see the results of the present study in the framework of Allen and Kline [4] the following change of notation must be carried out.

Present	*Allen and Kline*
μ	μ
χ	$2\mu_1$
γ	$2k^2\mu$
$N^2 = \dfrac{\chi}{2\mu+\chi}$	$\dfrac{1}{\mu}$
m	α

The geometry of the slider bearing is well known. In this section we restrict our attention to the specific geometry associated with a two-dimensional slider bearing lubrication analysis.

We assume that the guide surface is a plane wall of length L at $y = 0$ and moves with constant velocity U in the positive x-direction. The slide block is fixed and its surface is described by $y = h(x)$. The gap widths at inlet and exit are given, respectively, by

$$h(0) = h_1 \text{ and } h(L) = h_2.$$

The governing equations are

$$\frac{dp}{dx} = \frac{1}{2}(2\mu+\chi)\frac{\partial^2 u}{\partial y^2} + \chi\frac{\partial \nu}{\partial y} \tag{32}$$

$$\gamma\frac{\partial^2\nu}{\partial y^2}-2\chi\nu-\chi\frac{\partial u}{\partial y}=0 \tag{33}$$

The boundary conditions are

$$u=U, \nu=-\frac{\beta}{2}\frac{\partial u}{\partial y} \text{ at } y=0$$

$$u=0\ , \nu=-\frac{\beta}{2}\frac{\partial u}{\partial y} \text{ at } y=h \tag{34}$$

Replacing U_1 by U and U_2 by 0 in (29) and (30),

$$\begin{aligned}u=&\frac{y^2}{2\mu}p_x-\frac{h}{2\mu}p_x\left[\left(y-\frac{N^2}{m}\sinh my\right)+\frac{N^2}{m}\frac{\cosh mh+1}{\sinh mh}(\cosh my-1)\right]\\&+\frac{U}{D}\left[\sinh mh\left(y-\frac{N^2}{m}\sinh my\right)+\frac{N^2}{m}(\cosh mh-1)(\cosh my-1)\right]\\&+\frac{\beta\left(1-N^2\right)}{1-\beta N^2}\frac{N^2}{m}\left[\frac{h}{2\mu}p_x\left\{(\cosh my-1)\frac{\cosh mh+1}{\sinh mh}-\sinh my\right\}\right.\\&\left.+\frac{U}{D}\left\{\sinh mh\ \ \sinh my-(\cosh mh-1)(\cosh my-1)\right\}\right]+U\end{aligned} \tag{35}$$

and

$$\begin{aligned}\nu=&\frac{h}{2\mu}p_x\left(\frac{1}{2}-\frac{y}{h}\right)+\frac{1-\beta}{1-\beta N^2}\left[\frac{h}{4\mu}p_x\left\{\frac{\cosh mh+1}{\sinh mh}\sinh my-\cosh my\right\}\right.\\&\left.+\frac{U}{2D}\left\{\sinh mh\left(\cosh my-\frac{1-\beta N^2}{1-\beta}\right)-(\cosh mh-1)\sinh my\right\}\right]\end{aligned} \tag{36}$$

The volume flow rate Q is given by

$$\begin{aligned}Q=&\frac{Uh}{2}-\frac{h}{12\mu}p_x\left(h^2+12l^2-6Nlh\ \ \coth\frac{mh}{2}\right)\\&+\frac{\beta\left(1-N^2\right)}{1-\beta N^2}\frac{h}{12\mu}p_x\left(2l^2-Nlh\ \ \coth\frac{mh}{2}\right)\end{aligned} \tag{37}$$

For $\beta = 0$, these expressions coincide with those of Allen and Kline [4] proceeded with the further analysis with the first order term in mh. It is therefore interesting to note that when $\coth\frac{mh}{2}$ is replaced by $\frac{2}{mh}$ the influence of β is not felt, which suggests that first order approximation amounts to taking $\beta = 0$.

$$Q = \frac{Uh}{2} - \frac{h}{12\mu} p_x \left[h^2 + 12l^2 - 18l^2 \left(\frac{4 + m^2h^2}{6 + m^2h^2} \right) \right]$$

$$+ \frac{\beta\left(1 - N^2\right)}{1 - \beta N^2} \frac{h}{12\mu} p_x \left[2l^2 - 3l^2 \left(\frac{4 + m^2h^2}{6 + m^2h^2} \right) \right]$$

$$= \frac{Uh}{2} - \frac{h^3}{12\mu} p_x + \frac{h^3}{2\mu} p_x \frac{l^2 m^2 h^2}{6 + m^2h^2} \left[1 - \frac{\beta\left(1 - N^2\right)}{1 - \beta N^2} \right]$$

i.e.,

$$Q = \frac{Uh}{2} - \frac{h^3}{12\mu} p_x + \frac{h^3}{2\mu\bar{\mu}} p_x \frac{\eta^2}{6 + m^2h^2} \tag{38}$$

where

$$\eta^2 = \frac{1 - \beta}{1 - \beta N^2} .$$

Therefore

$$p_x = \left(\frac{Uh}{2} - Q \right) \frac{12\mu}{h^3} \left[1 + \frac{6\eta^2}{\bar{\mu}\left(6 + m^2h^2\right)} \right] \tag{39}$$

neglecting the higher powers of η^2.

The pressure must now be determined by the use of the equation (39) subject to the boundary conditions

$$p(0) = p(L) = p_o \tag{40}$$

Then integrating (39), we obtain

$$p(x) = p_o + 6U\mu \left[\int_0^x \frac{1}{h^2} dx + \frac{6\eta^2}{\bar{\mu}} \int_0^x \frac{1}{h^2\left(6 + m^2h^2\right)} dx \right]$$

$$-12Q\mu \left[\int_0^x \frac{1}{h^3} dx + \frac{6\eta^2}{\bar{\mu}} \int_0^x \frac{1}{h^3\left(6 + m^2h^2\right)} dx \right] \tag{41}$$

Since $h = \delta(a - x)$

$$p(x) = p_o + 6U\mu \left[\int_0^x \frac{1}{\delta^2 (a - x)^2} dx + \frac{6\eta^2}{\bar{\mu}} \int_0^x \frac{1}{\delta^2 (a - x)^2 \left\{ 6 + m^2\delta^2 (a - x)^2 \right\}} dx \right]$$

$$-12Q\mu \left[\left[\int_0^x \frac{1}{\delta^3 (a - x)^3} dx + \frac{6\eta^2}{\bar{\mu}} \int_0^x \frac{1}{\delta^3 (a - x)^3 \left\{ 6 + m^2\delta^2 (a - x)^2 \right\}} dx \right] \right] \tag{42}$$

After integration, using the given boundary conditions, and simplification we obtain

$$Q = \frac{\dfrac{U}{2}\left[\dfrac{L}{\delta^2 a(a-L)}\left(1+\dfrac{\eta^2}{\bar{\mu}}\right)+\dfrac{\eta^2 m^2}{6\bar{\mu}}\left\{\tan^{-1}\dfrac{m^2\delta^2(a-L)}{6}-\tan^{-1}\dfrac{m^2\delta^2 a}{6}\right\}\right]}{\dfrac{L(2a-L)}{2\delta^3 a^3(a-L)^2}\left(1+\dfrac{\eta^2}{\bar{\mu}}\right)-\dfrac{\eta^2 m^2}{6\bar{\mu}}\log\dfrac{(a-L)^2\left(m^2\delta^2 a^2+6\right)}{a\left\{m^2\delta^2(a-L)^2+6\right\}}} \tag{43}$$

Then from (42)

$$p_x - p_o = 6U\mu\left[\frac{x}{\delta^2 a(a-x)}\left(1+\frac{\eta^2}{\bar{\mu}}\right)+\frac{\eta^2 m^2}{6\bar{\mu}}\left\{\tan^{-1}\frac{m^2\delta^2(a-x)}{6}-\tan^{-1}\frac{m^2\delta^2 a}{6}\right\}\right]$$

$$-\frac{6U\mu\left[\dfrac{L}{\delta^2 a(a-L)}\left(1+\dfrac{\eta^2}{\bar{\mu}}\right)+\dfrac{\eta^2 m^2}{6\bar{\mu}}\left\{\tan^{-1}\dfrac{m^2\delta^2(a-L)}{6}-\tan^{-1}\dfrac{m^2\delta^2 a}{6}\right\}\right]}{\dfrac{L(2a-L)}{2\delta^3 a^3(a-L)^2}\left(1+\dfrac{\eta^2}{\bar{\mu}}\right)-\dfrac{\eta^2 m^2}{6\bar{\mu}}\log\dfrac{(a-L)^2\left(m^2\delta^2 a^2+6\right)}{a\left\{m^2\delta^2(a-L)^2+6\right\}}}\times$$

$$\frac{x(2a-x)}{2a^3\delta^3(a-x)^2}\left(1+\frac{\eta^2}{\bar{\mu}}\right)-\frac{\eta^2 m^2}{6\bar{\mu}}\log\frac{(a-x)^2\left(m^2\delta^2 a^2+6\right)}{a\left\{m^2\delta^2(a-x)^2+6\right\}} \tag{44}$$

We next compute the resultant pressure by integrating equation (44)

$$P = \int_0^L (p-p_o)\,dx \tag{45}$$

The integrals appearing in (45) can be evaluated numerically.

CONCLUSIONS

The modified Reynolds' equation [11] is applied in some basic lubrication problems namely those of the squeeze film, the journal bearing and the slider bearing. The boundary condition $\bar{\nu} = (\beta/2)\mathit{curl}\bar{V}$ on the spin is suggested by the work of Kolpashchikov et al [10] who have used it in explaining the structured character of water from the micropolar fluid dynamical point of view. They also have determined the micropolar material constants using this boundary condition. β is called the parameter of boundary values (in the case of one-dimensional MPFF lubrication). Earlier an arbitrary value for the spin at the boundary was considered in basic flow problems by Kirwan and Newman [7]. In the case of one-dimensional MPFF lubrication some interesting features are observed in the present work.

(i) that the arbitrary boundary condition taken by Kirwan and Newman [7] does not influence the volume flow rate which is an important physical factor in lubrication problems.

(ii) the parameter of boundary values β which strictly varies in [0,1]. However, we have seen [11] that the one-dimensional MPFF Reynolds' equation obtained in the case of incompressible Newtonian lubricants. Thus in one-dimensional MPFF lubrication β = 1. i.e., the condition that the spin at the boundary equals the angular velocity of the fluid does not reflect any polarity effects in one-dimensional MPFF lubrication. Thus

(iii) when the boundary condition $\overline{v} = (\beta/2)curl\ \overline{V}$ is used in one-dimensional MPFF lubrication, β is to be restricted to the half open [0,1).

(iv) $\beta = 0$ is discussed by several authors. Thus the results of the present paper would be useful for $\beta \in (0, 1)$

(v) theoretically for various values of $\beta \in (0, 1)$ the present paper obtains analytical expressions for some important physical factors such as load capacity etc. in lubrication problems.

(vi) the present paper would be useful in studying the effects of β in practical problems when the various expressions for important physical factors are numerically evaluated in the light of new results coming up with regards to the numerical evaluation of the micropolar material constants.

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