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Reaching Collective Opinion[1]

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Abstract: *Some Multi-agent problems may be solved with use of Collective Behavior models. The latter approach is based on the general idea that reasonably behaving model (RBM) of a single agent may be used to build a Multi-agent system by combining a number of RBMs. The basic principles of RBM were borrowed from observation of behavior of primitive animals or insects and were introduced in the school of Prof. M.L.Tsetlin, Moscow State University. In 1961 M.L.Tsetlin defined the concept of an automaton with expedient behavior and demonstrated an example of such an automaton. The concept of Collective Behavior of expediently behaving automata was introduced by the present author in 1961-1963.*

As an illustration to the Collective Behavior modeling the present paper shows that a stable collective opinion may be reached via a simple local interaction of expedient agents by way of local exchange of individual opinions. The notions of expediency, memory, stability and optimization, which are typical for Collective Behavior, are supplemented in this example with important consideration of ergodicity property.

1. INTRODUCTION

Historically MAS came to AI scientific community many years after the Artificial Intelligence was proposed in 1956. Yet, prior to the MAS a popular science has been initiated in Russia with works by the famous mathematician and engineer Prof. Michail Tsetlin, Moscow State University, who introduced the idea of learning automata with an expedient behavior. Using an internal memory this learning agents are able to learn from the past experience of punishments and rewards the best action to take in future. In 1961 M.L.Tsetlin introduced the model, which he called "automaton with liner tactics" and proved its asymptotic optimality when its 'memory' tends to infinity [1, 2].

Below we give an example of such an automaton having only two actions. The upper graph shows transition between inner states under the punishment, the bottom one shows the transitions under the reward.

This model and a number of other models that followed it were based on the observations of simple animal behavior and on other reasonable knowledge from biological and technical systems. (The construction shown in fig. 1 was naturally generalized to the case of many actions.)

The concept of Collective Behavior was introduced in the publication [3], where two learning agents with the linear tactics have been considered. The two automata obtained punishments simultaneously when at least one

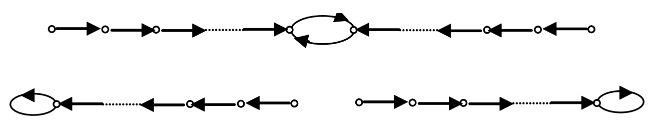

Figure 1: Learning Automaton with Linear Tactics of Behavior

of them was not doing what it was expected from both agents. Initially rather cumbersome mathematical analysis has shown that there are certain restrictions: the pair behaves itself expediently if the following restrictions on the probabilistic properties of the domain are fulfilled:

$$p_0 + p_1 < 1,$$

Here p_0 and p_1 are the probabilities of punishment for the desirable action (1, 1) and for the rest action combinations, correspondingly. Actually, this result was obtained in 1961 in application to some electronic learning machine built by the present author in Moscow State University.

Since that time quite a number of papers and books have been published by various authors, where the importance of Collective Behavior and Games of Automata was stressed. See the book [4] for necessary references to the original papers and for examples.

Collective Behavior being absolutely new concept at that time turned out to be rather useful in applications. The most remarkable one to the present author was our discovery of a Collective of Radio Stations in 1967 [5], see some other references also in [6]. This collective consisted in n pairs of radio stations working simultaneously and sharing common frequency range. Due to the sharing a number of problems have aroused in the collective, the most essential one is the need for an individual power control as each pair produces some noise to the rest of the collective when it transmits a useful signal to its correspondent.

The behavior of the Collective of Radio Stations was shown to be similar to that of people simultaneously speaking to corresponding partners in the same room. However, the mathematical model created for the Collective of Radio stations let us study various phenomena rigorously enough. Some unusual results were even published in a Russian professional math journal – *Mathematical Notes*. Eventually an optimal power control strategy has been developed, leading to an expedient behavior of individual agents in this collective. Several criteria of optimality have been introduced and studied, including *the Elasticity Criteria* (see [6]) approved by IEEE for the Mobile Communication only a few years ago.

The present author is probably just lucky to see how his rather abstract model of Collective of Radio Stations was implemented (with a delay of 30-40 years!) in several engineering locations in the World, thus opening a new epoch of cellular or mobile communication, as our Collective of Radio Stations has been called in these locations.

The Collective Behavior found application also for the networks of communication, where the information packets collide with each other, leading sometimes to the deadlocks observed in communication networks similar to ARPA net (see publications by A.V.Boutrimenko, and that by V.I.Varshavsky with his colleagues).

One may say that Collective Behavior is the *bottom up* approach to the modeling of complex system. It leads to a distributed systems organization, which is opposite to the centralized or decentralized system organizations. With this approach a number of strict mathematical models have been developed including various Games of Automata (see [4] for references and some new results). The possibility to study large system mathematically and obtaining results which are undoubtedly true is the most important feature of this approach. Thus, in case of Collective of Radio Stations the shear possibility of obtaining the stable power control despite the actual interferences in the common communication channel was mathematically proved in our studies. The results obtained were far from obvious. Before any actual implementation they demonstrated that it is indeed possible to arrange normal communication of n pairs of agents, that people enjoy so much to-day.

The present paper contains another illustration to the Collective Behavior paradigm by applying it to the task of opinion formation in a collective of agents, resulting from an exchange of opinions through the local interactions among the agents. Our analysis involves both a mathematical study and a computer simulation to obtain supplementary data. More examples of theory and applications of Collective Behavior paradigm may be found in the book [4], which might supplement traditional for MAS literature such as [7, 8].

2. MODEL OF OPINION FORMATION PROCESS

A simple mathematical model of the process of opinion formation for collective of agents, which starts after entering some "initial" information, will be considered in what follows. The collective opinion in such a group of agents is the result of exchange of information among the agents, who demonstrate an expedient behavior in some sense.

It is assumed that in the moments t = 1, 2,... the collective of agents is randomly disjoint into a number of groups and the information exchange occurs within each of the groups. Such a situation of iterative formation of opinions based on the opinions exhibited by other members of a group is typical for the Delphi method and is typical also for small group decision-making.

Let us consider the case when each member is a simple agent, the state of which, or its "opinion" $\varphi(t)$, may take one of two values: 0 or 1 – a binary opinion. Then the behavior of the agent over time is given with the next equation

$$\varphi(t + 1) = F(\varphi(t), s(t), \xi(t)),\ \ t = 0, 1, 2, ...,$$

where $s(t)$ is an input variable, taking 0 (a reward) or 1 (a penalty), and where $\xi(t)$ is a sequence of independent random variables also taking two values 0 and 1 with the probabilities $1 - \varepsilon$, ε correspondingly. (Thus, the value $1/\varepsilon$ corresponds to the memory of the agent.)

The function F is formulated in the following way:

$$F(\varphi(t), 0, 0) = F(\varphi(t), 1, 1) = \varphi(t),$$

$$F(\varphi(t), 1, 0) = F(\varphi(t), 0, 1) = 1 - \varphi(t) \equiv \overline{\varphi}(t)$$

Above equations mean that the agent may reconsider its opinion depending on the opinions of other members of the group through a punishment/ reward scheme via variable $s(t)$. When $\xi(t) = 0$, then the agent changes its output only due to the punishment or rewards, hence $\xi(t) = 1$ may be seen as the stubbornness of the agent (exhibited with the probability ε). The latter property reminds the *trust inertia*, introduced in [9]. However, unlike the system discussed in [9], the value of 'trust resistance' remains constant in the present paper.

Let us consider a collective from *n* such agents, taking $\xi_i(t)$, $i = 1,..., n$ as independent random values for the members of the collective. Below it will be assumed that for each moment in time $t = 0, 1, 2,...$ the agents distributed over several groups, provided that the input punishment/reward value for each agent in the state $\varphi(t)$ depends upon the average fraction $X(t)$ of agents in this group, who are in the same state (or having the same "opinion"):

$$If\ \ X(t) > \frac{1}{2},\ then\ s(t) = 1,$$

$$If\ \ X(t) \leq \frac{1}{2},\ then\ s(t) = 0.$$

Thus, in each group we have majority voting system. Below for simplicity we will consider the case when only one randomly picked up agent "is voting" at a given moment. Besides, we will suppose that exactly 3 agents are in each group and the distribution of agents over groups is made randomly. Finally, the value ε is the same for each agent.

The above restrictions are to simplify the study, yet they are not essential and may be relaxed.

The interaction of agents has a lot in common with the problem of "voting with error", which was considered in some publications [N.Vasiljev *et al.*, 1969], where it was assumed that all the agents are located in an infinite line (or in an infinite plane), and thus all their neighbors are strictly fixed. The purpose of their study was to discover some cases when there was a breach of ergodicity.[2]

In our case it is easy to show that the group interaction described above leads certainly to an ergodic system. However, we will consider our system over time span, which is small enough and the system state will depend upon initial situation.

Let us consider the function $\mu(t)$ to be the fraction of agents, which are in the state 1 at the moment *t*. It is possible to obtain an approximate equation, describing the process:

$$\mu(t+1) = -\frac{2}{3}(1-2\varepsilon)\,\mu^3(t) + (1-2\varepsilon)\mu^2(t) + \frac{2}{3}\mu(t) + \frac{\varepsilon}{3}.$$

Stationary points μ then will satisfy to the following equation

$$2(1-2\varepsilon)\ \mu^3 + 3(1-2\varepsilon)\ \mu^2 + \mu - \varepsilon = 0.$$

Under $\varepsilon < \frac{1}{6}$ we have here exactly 3 stationary points

$$\mu_{1,3} = -\frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{1-6\varepsilon}{1-2\varepsilon}},\quad \mu_2 = \frac{1}{2}.$$

It is easy to see that under $\varepsilon < \frac{1}{6}$ the points μ_1 and μ_3 are stable, and the stability region for the root μ_1 is $\left[0, \frac{1}{2}\right)$ and the stability interval for the root μ_3 is $\left(\frac{1}{2}, 1\right]$.

3. DISCUSSION

The result obtained "contradicts" to the fact that original system is an ergodic one as the ergodicity means that the final distribution is unique and does not depend upon initial system state. This contradiction demonstrates that the used approximation may be valid only over a "small enough" time interval. During this small time interval one may observe the formation and conservation of the average fraction of "ones" in the collective, which does depend upon initial conditions but only qualitatively.

In order to estimate the real time interval of being in quasi-stationary states and for evaluation of validity of the whole approach a computer simulation was performed. The simulation was made for $n = 1024$, $n = 510$, $n = 225$ agents. The modeling showed that our approximate analysis was rather precise.

Example. For $n = 510$, $\varepsilon = 0{,}05$ and for initial state $M_{init} = 200$ (i.e. the 200 agents initially were in the state $\varphi(0) = 1$) our systems arrives to the quasi-stationary point after 26 units of time. After it the system was observed during 40 000 units of time and all this time the system was in the vicinity of the stationary point. Thus, the "small" time interval, i.e. when our mathematical study is considered to be valid, may be "large" enough in practice.

The dispersion becomes larger if $\varepsilon = 0.1$. However the time of formation of the average fraction of "ones" is approximately the same. It is interesting to note that the results are different from those observed in the case when the agents are fixed in a line.

One may say that under small values ε such a system has a memory, as the state of the collective is defined with the initial conditions. (The system memory is defined here in the same sense as the memory of a single agent, who remembers its initial state during the time interval of the order $1/\varepsilon$, provided that no punishment comes to its input.)

Thus, the collective of agents "remembers" its initial state *much longer than any individual agent*. In fact we may see the phenomenon of increase of reliability in a manner close to the approach to reliability used by John von Neumann in 1956. In our case we may speak on the reliability of the keeping firm opinion in the society of agents due to the voting.

Similar analysis may be applied for case of inhomogeneous collective of agents in the following way.

Let the interaction of agents will be the same as above, but the collective of agents consists now from m subcollectives of agents $n_1, n_2, ... n_m$, and each subcollective is characterized by its own parameter ε (the error probability), and assume that $n_1, n_2, ..., n_m >> 1$.

At this point we will consider how the properties of separate parts influence the property of the system as a whole. Let $\alpha_i = n_i / n$, $i = 1, ..., m$, then in above equation for μ the parameter ε should be replaced with $\bar{\varepsilon} = \sum_{i=1}^{m} \alpha_i \varepsilon_i$, where ε_i – is the "error" probability for the i-th subcollective.

Now it is obvious that one has 3 stationary points under the condition $\bar{\varepsilon} < 1/6$.

Let us have only two parts, $\alpha_1 < 7/8$ (the first subcollective is essentially bigger) and $\varepsilon_1 < 1/7$. Then for the fulfillment of the condition $\bar{\varepsilon} < 1/6$ it is sufficient to have $\varepsilon_2 < 1/3$.

In Figures 2-5 we show some experimental trajectories of $\mu(t)$– the averaging has been made over 100 time units to make the pictures more readable, i.e. t = 100, 200,.... (The first two figures are given for the homogeneous collective, the rest two are for inhomogeneous collectives of agents.)

In [4] the further details are given and the original equation for may be found. This equation was obtained under some heuristic consideration; hence the computer simulation is important to justify the whole approach.

We think that our results are applicable to the *voting* in a very general sense. For instance it could be used in case of opinion formation during discussion among people, when they attempt to convince the partners in some idea. In some highly intuitive sense even bringing a child in a family is also a kind of voting, probably unconscious, in favor of one of the parents. (For instance,

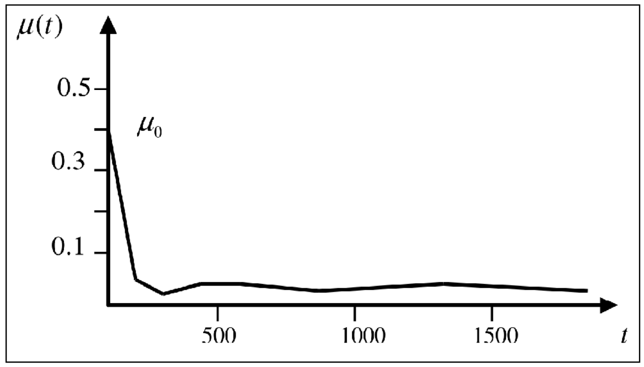

Figure 2: Collective Opinion for Initial state $\mu_0 = 0.4$ ($\varepsilon = 0.05$; $n = 510$)

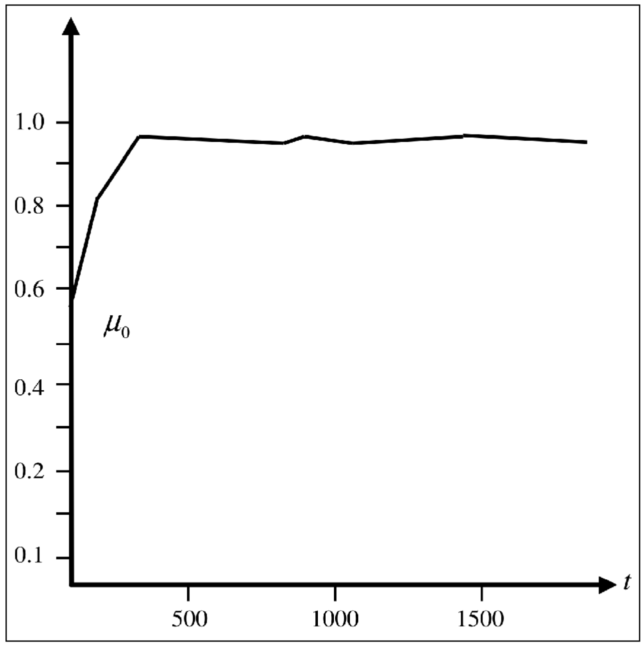

Figure 3: Another Example of Collective Opinion Formation for the Initial State $\mu_0 = 0.5$ ($\varepsilon = 0.05$; n = 1024)

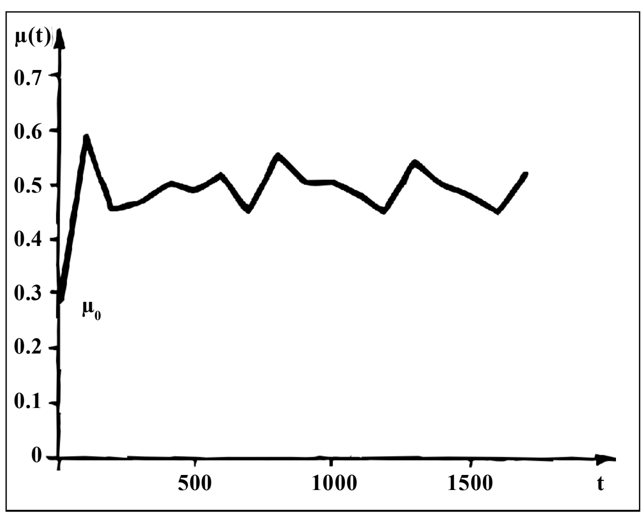

Figure 4: An Example for a Mixed Collective for $\mu_0 = 0.3$ ($\alpha = 0.7$, $\varepsilon_1 = 0.05$, $\varepsilon_2 = 0.5$ and $n = 1024$)

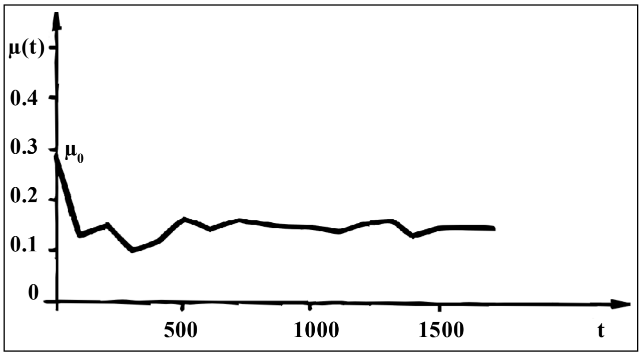

Figure 5: Another Example for a Mixed Collective for $\mu_0 = 0.3$ ($\alpha = 0.7$, $\varepsilon_1 = 0.05$, $\varepsilon_2 = 0.2$ and $n = 1024$)

it is known that the majority of children inherit their characters from one of the parents.)

Understood so widely the above results may explain how certain social entities, such as national habits, languages, cultures and etc., are preserved in the human society for such a long time, despite the constant mutual exchange with opinions on these matters and hence constant modification of points of view.

4. CONCLUSION

Multi Agent Systems as a definite branch of AI was finally established by the year 1995, when there was held the *1st International Conference on Multiagent Systems* in Menlo Park, California. It seems that there is again the 'time delay constancy' here. Indeed, it turns out that approximately 30 years should have past till the wide recognition of importance of collective behavior for systems. A certain difference between Multi Agent Systems and Collective Behavior, which does exist, is due to the fast development of Computer Science and Artificial Intelligence during this period. Many authors agree that the agents in MAS are usually software pieces. Yet, in Collective Behavior field the agents are mostly finite or probabilistic automata or entities from control theory. We believe that this difference is to the advantage of both directions in our common science as it was stressed in [10], and we hope the present publication demonstrates it.

5. ACKNOWLEDGEMENTS

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NOTES

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2. A young member of my scientific group G. Kurdumov has proved that the problem of ergodicity for an infinite homogeneous system mentioned here is an NP-complete problem.

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