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UNSTEADY LAMINAR AXISYMMETRIC ETHANOL BOUNDARY LAYERS WITH VARIABLE VISCOSITY AND PRANDTL NUMBER

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ABSTRACT: This paper analyzes the influence of temperature-dependent viscosity and Prandtl number on the unsteady, laminar nonsimilar boundary-layer flow of ethanol over a axisymmetric body (sphere), where unsteadiness and (or) nonsimilarity are (is) due to free stream velocity, mass transfer and transverse curvature. The partial differential equations governing the flow have been solved numerically by using an implicit finite-difference scheme, along with quasilinearization technique. It is found that both skin friction and heat transfer strongly respond to the unsteady accelerating free stream velocity, variable viscosity and Prandtl number. The effect of temperature dependent viscosity and Prandtl number is to move the point of zero skin friction downstream, delaying boundary-layer separation. The heat transfer is found to depend appreciably on viscous dissipation, but the skin friction little affected by it. In general, the results pertaining to variable fluid properties differ significantly, from those of constant fluid properties.

Keywords: Temperature-dependent viscosity, Prandtl number, Skin friction, Heat transfer.

1. INTRODUCTION

A wide range of non-similar boundary layer flow and heat transfer problems of practical importance have attracted several investigators [1-3]. In these studies, non-similarities in the flow arise due to the free stream velocity, the curvature of the body, the surface mass transfer, or a combined effect of all these factors. Further, several studies also have been made on nonsimilar laminar boundary layer flows over heated bodies with temperature-dependent fluid properties [4-6].

It is well known that fluid viscosity and thermal conductivity are the main governing fluid properties in laminar boundary layer forced flow, and obviously their variations can be expected to affect separation of boundary layer from the solid surface. Further, these thermo-physical properties are temperature dependent, variations are most easily accomplished in the boundary layer by maintaining a temperature difference between the solid wall and the fluid. In practice, wall heating has been shown to be an efficient way to stabilize boundary layer flow and to delay the flow transition when fluid viscosity decreases by heating. For technological applications, surface heating is an effective means of controlling boundary layer separation since heating promotes stability through the interplay among the thermal boundary layer, the temperature dependent viscosity, and momentum balance in the crucial region near the wall.

In the present study, we investigate the effect of variable viscosity and Prandtl number on the unsteady, laminar incompressible boundary layer flow of ethanol over an axisymmetric body (sphere). It is remarked here that ethanol, produced from molasses in the sugarcane industry, is one of environmental friendly blended fuel. Further, ethanol is one of the most commonly used fluid found next to water, in all engineering applications, particularly in automobile/pharmaceutical industries.

2. NOMENCLATURE

f dimensionless stream function

β pressure gradient parameter

F dimensionless velocity

η transformed variable

G	dimensionless temperature
μ	dynamic viscosity
L	characteristic length
ψ	dimensional stream function
N	viscosity ratio
ξ	dimensionless stream wise co-ordinate
T	dimensional temperature
Pr	Prandtl number
Re_L	Reynolds number
R	radius of the sphere
u, v	velocity components along x- and y-directions, respectively
x, y	cartesian co-ordinates along and normal to surface, respectively
u_e	potential flow velocity
α_1, P	dimensionless parameters
Ec	viscous dissipation parameter (Eckert number)
A	surface mass transfer parameter
U	steady state velocity at the edge of the boundary layer
t, t^*	dimensional and dimensionless times, respectively

Subscripts

i	initial conditions
∞	conditions in the free stream
e, w	conditions at the edge of the boundary layer and on the surface, respectively
ξ	partial derivatives with respect to ξt,
t^*	partial derivatives with respect to t and t^*, respectively
f_w	surface mass transfer
u_∞	free stream velocity
x, y	partial derivatives with respect to x and y, respectively

Superscripts

$(')$	partial derivatives with respect to η

2. ANALYSIS

Let us consider the unsteady, laminar nonsimilar boundary - layer forced convection flow (of ethanol) with temperature-dependent viscosity and Prandtl number over an axisymmetric body(sphere) when the free stream velocity and mass transfer (suction or injection) vary with the axial distance (x) along the surface. The fluid is assumed to flow with moderate velocities, and the temperature difference between the wall and the free stream is small (< 40^0C). In the range of temperature considered (i.e. 0-40^0C), the variation of both density (ρ) and specific heat (c_p) of ethanol with temperature, is less than 1% (See Table 1) and hence they are taken as constants. However, since the thermal conductivity (k) and viscosity (μ) [and hence Prandtl number (Pr)] variation with temperature is quite significant, the viscosity and Prandtl number are assumed to vary as an inverse liner function of temperature [7, 8]:

$$\mu = 1/(b_1 + b_2 T) \tag{1}$$

$$\mathrm{Pr} = 1/(c_1 + c_2 T) \tag{2}$$

where

$$b_1 = 53.804, \quad b_2 = 1.584, \quad c_1 = 0.0428 \quad \text{and} \quad c_2 = 0.0006 \tag{3}$$

Table 1
Values of Thermo Physical Properties of Ethanol at Different Temperature [9]

Temperature (T) (^0C)	*Density (ρ) (g/cm^3)*	*Specific heat (c_p) ($J \times 10^7/kg\ K$)*	*Thermal conductivity (k) ($erg \times 10^5/cms\ K$)*	*Viscosity (μ) ($g \times 10^{-2}/cms$)*	*Prandtl number (Pr)*
0	0.8037	2.2416	0.1721	1.7730	23.0906
10	0.7981	2.2416	0.1703	1.4662	20.1064
20	0.7901	2.4283	0.1670	1.2003	17.4532
30	0.7821	2.5296	0.1611	1.0035	13.7531
40	0.7741	2.6242	0.1585	0.8340	12.6316

The numerical data, used for these correlations, are taken from Ref. [9]. The relations (1) and (2) are reasonably good approximations for liquids such as ethanol, particularly for small wall and ambient temperature differences.

As the fluid is incompressible, the contribution of heating due to compression is very small and it has been neglected. The effect of viscous dissipation is included in the analysis. It is assumed that the injected fluid possess the same physical properties as the boundary-layer fluid. Under the foregoing assumptions, the equations governing the above flow over a sphere are [10]:

$$(r^j u)_x + (r^j v)_y = 0 \tag{4}$$

$$u_t + uu_x + vu_y = (u_e)_t + u_e(u_e)_x + \rho^{-1}(\mu u_y)_y \tag{5}$$

$$T_t + uT_x + vT_y = \rho^{-1}\left(\frac{\mu}{\mathrm{Pr}} T_y\right)_y + \left(\frac{\mu}{\rho c_p}\right)(u_y)^2 \tag{6}$$

The initial and boundary conditions are:

$$u(x, y, 0) = u_i(x, y), \quad v(x, y, 0) = v_i(x, y), \quad T(x, y, 0) = T_i(x, y) \tag{7}$$

$$\left.\begin{array}{l} u(x, 0, t) = 0, \quad v(x, 0, t) = v_w(x, t), \quad T(x, 0, t) = T_w(x, t) \\ u(x, \infty, t) = u_e(x, t), \quad T(x, \infty, t) = T_\infty \end{array}\right\} \tag{8}$$

Applying the following transformations

$$\xi = \int_0^x \left(\frac{U}{u_\infty}\right)\left(\frac{r}{L}\right)^{2j} d\left(\frac{x}{L}\right), \quad t^* = \left(\frac{3}{2}\right)(\mathrm{Re}_L)\left(\frac{\mu_e}{\rho L^2}\right)t, \quad \eta = \left(\frac{U}{u_\infty}\right)\left(\frac{\mathrm{Re}_L}{2\xi}\right)^{1/2}\left(\frac{r}{L}\right)^j\left(\frac{y}{L}\right),$$

$$\psi(x, y, t) = u_\infty L \phi(t^*) f(\xi, \eta, t^*)\left(\frac{2\xi}{\mathrm{Re}_L}\right)^{1/3}, \quad \mathrm{Re}_L = \frac{\rho u_\infty L}{\mu},$$

$$u = \left(\frac{L}{r}\right)^j \frac{\partial \psi}{\partial y}, \quad v = -\left(\frac{L}{r}\right)^j \frac{\partial \psi}{\partial x}, \quad G = \frac{T - T_\infty}{T_w - T_\infty} \tag{9}$$

to Eqs. (4)-(6), we see that the continuity Eq. (4) is identically satisfied, and Eqs. (5) and (6) reduce, respectively, to:

$$(NF')' + \phi[fF' + \beta(1-F^2)] = P[F_{t^*} - \phi^{-1}\phi_{t^*}(1-F)] = 2\xi\phi(FF_\xi - f_\xi F') \tag{10}$$

$$(N\,\mathrm{Pr}^{-1}G')' + \phi f G' + NEc\left(\frac{u_e}{u_\infty}\right)^2 (F')^2 - PG_{t^*} = 2\xi\phi(FG_\xi - f_\xi G') \tag{11}$$

where

$$N = \left(\frac{\mu}{\mu_\infty}\right) = \frac{b_1 + b_2T_\infty}{b_1 + b_2T} = \frac{1}{1 + a_1G}, \qquad \mathrm{Pr} = \frac{1}{c_1 + c_2T} = \frac{1}{a_2 + a_3G}, \qquad a_1 = \left(\frac{b_2}{b_1 + b_2T_\infty}\right)\Delta T_w$$

$$a_2 = c_1 + c_2T_\infty, \qquad a_3 = c_2\Delta T_w, \qquad \Delta T_w = (T_w - T_\infty), \qquad \beta(\xi) = \frac{2\xi}{U}\left(\frac{dU}{d\xi}\right), \qquad \frac{u}{u_e} = f' = F$$

$$u_e = U\phi(t^*), \qquad v = -\left(\frac{r}{L}\right)^j 2\xi(\mathrm{Re}_L)^{-1/2}U\phi[f + 2\xi f_\xi + (\beta + \alpha_1 - 1)\,\eta F]$$

$$Ec = \frac{u_\infty^2}{[c_p(T_w - T_\infty)]}, \qquad P = 3\xi\left(\frac{L}{r}\right)^{2j}\left(\frac{u_\infty}{U}\right)^2, \qquad \alpha_1 = \left(\frac{2j\xi}{r}\right)\left(\frac{dr}{d\xi}\right), \qquad f = \int_0^\eta F d\eta + f_w$$

$$f_w = -(\xi)^{-1/2}\left(\frac{\mathrm{Re}_L}{2}\right)^{1/2}\phi^{-1}\int_0^x \left(\frac{v_w}{u_\infty}\right)\left(\frac{r}{L}\right) d\left(\frac{x}{L}\right) \tag{12}$$

The transformed boundary conditions are

$$\left.\begin{array}{llll} F(\xi,0) = 0, & G(\xi,0) = 1 & \text{at} & \eta = 0 \\ F(\xi,\infty) = 1, & G(\xi,\infty) = 0 & \text{at} & \eta \to \infty \end{array}\right\} \tag{13}$$

for $\xi \geq 0$ and $t^* \geq 0$.

Here it is assumed that the flow is, initially, steady and changes to unsteady state for $t^* > 0$. Therefore, the initial conditions for F and G at $t^* = 0$ are given by steady flow equations obtained by putting

$$\phi = 1, \qquad \phi_{t^*} = F_{t^*} = G_{t^*} = 0 \tag{14}$$

in Eqs. (10) and (11). Consequently, the initial conditions at $t^* = 0$ can be written as

$$\left.\begin{array}{r} (NF')' + fF' + \beta(1-F^2) = 2\xi(FF_\xi - f_\xi F') \\ (N\,\mathrm{Pr}^{-1}G')' + fG' + NEc\left(\dfrac{u_e}{u_\infty}\right)^2 (F')^2 = 2\xi(FG_\xi - f_\xi G') \end{array}\right\} \tag{15}$$

It may be remarked here that Eqs. (10) and (11) are exactly same as those of Eswara and Nath [11] who have studied the effect of variable viscosity and Prandtl number on unsteady water boundary layers over two-dimensional and axisymmetric bodies.

For an axisymmetric body (sphere), unsteadiness as well as nonsimilarity are both due to the external velocity at the edge of the boundary-layer, $u_e(\bar{x}, t)$ (where $\bar{x}$ is the dimensionless distance along the (surface) and the normal component of the velocity at the surface, $f_w(\bar{x}, t)$. The free stream velocity distribution in the case of a sphere, the pressure gradient parameter and the distance from the axis of body are given by Schlichting [8] as

$$\frac{u}{u_\infty} = \left(\frac{3}{2}\right) \sin \bar{x} \phi(t^*), \quad \frac{U}{u_\infty} = \left(\frac{3}{2}\right) \sin \bar{x}, \quad \bar{x} = \frac{x}{R}, \quad \frac{r}{R} = \sin \bar{x}, \quad j = 1, \quad L = R. \tag{16}$$

Consequently, the expression for ξ, β, f_w, and α can be written respectively, as

$$\xi = \frac{(1-\cos \bar{x})^2 (2+\cos \bar{x})}{2}, \quad \beta = \left(\frac{2}{3}\right) \frac{(2+\cos \bar{x})(\cos \bar{x})}{(1+\cos \bar{x})^2}, \quad P = \left(\frac{2}{3}\right) \frac{(2+\cos \bar{x})}{(1+\cos \bar{x})^2},$$
$$\alpha_1 = \beta, \quad f_w = A\phi^{-1} \frac{2}{\sqrt{(2+\cos \bar{x})}}, \quad A = \left(\frac{v_w}{v_\infty}\right) \sqrt{\left(\frac{\mathrm{Re}_L}{2}\right)}. \tag{17}$$

The local skin friction coefficient and the heat transfer coefficient in terms of Nusselt number, can be expressed, respectively, as

$$C_f (\mathrm{Re}_L)^{\frac{1}{2}} = \frac{2\tau_w}{\rho u_\infty^2} = \frac{2\left(\mu \dfrac{\partial u}{\partial y}\right)_w}{\rho u_\infty^2} = \left(\frac{9}{2}\right) \left(\frac{\sin \bar{x}\,(1+\cos \bar{x})}{\sqrt{(2+\cos(\bar{x}))}}\right) \phi(t^*)\, F'_w \tag{18}$$

$$Nu(\mathrm{Re}_L)^{-\frac{1}{2}} = -\frac{L\left(\dfrac{\partial T}{\partial y}\right)_w}{\Delta T_w} = -\left(\frac{3}{2}\right)\left(\frac{(1+\cos \bar{x})}{\sqrt{(2+\cos \bar{x})}}\right) G'_w \tag{19}$$

If the normal velocity at the wall v_w is taken as constant, then A is a constant since u_∞ and Re_L are constants. The mass transfer parameter $A > 0$ or $A < 0$, according as there is suction or injection.

3. RESULTS AND DISCUSSION

The set of partial differential equations (10) and (11) along with the boundary conditions (13) using the relations (14)-(17) has been solved numerically using an implicit finite difference scheme along with quasilinearization technique. Since the method is described in great detail in Ref [12], its description is omitted here for the sake of brevity. In order to obtained grid independent numerical results, the grid sizes $\Delta\eta$, $\Delta\bar{x}$ and Δt^* have been optimized. To achieve this, the computed values of the physical parameters with a step size $\Delta\eta$, (leeping $\Delta\bar{x}$, and Δt^* fixed) are compared with those obtained using reduced step sizes viz., $(\Delta\eta/2)$, $(\Delta\eta/2)$ and so on. The optimal values of the step sizes viz., $\Delta\eta = \Delta\bar{x} = 0.05$ and $\Delta t^* = 0.1$ have been used for computations, for $\bar{x} \le 1.5$. However, for $\bar{x} > 1.5$, finer step size for $\bar{x}$ has been used and in the neighborhood of the point of zero skin friction $\bar{x} = 0.0005$ used. The value of η_∞ (i.e. the edge of the boundary layer) has been taken as 6.0. for a typical data marching from $x = 0$ to $x = 1.8$ and $t^* = 1.5$, the CPU time necessary for a sphere, was 2.58 minutes for constant fluid properties and 3.33 minutes for variable fluid properties. In order to assess the accuracy of our method, we have compared the skin friction $[C_f(\mathrm{Re}_L)^{1/2}]$ and heat transfer $[Nu(\mathrm{Re}_L)^{-1/2}]$ coefficients [See Fig. 1(a) & Fig. 1(b)] with Eswara and Nath [11] who have studied the effect of variable viscosity and Prandtl number on unsteady nonsimilar two- dimensional and axisymmertic water boundary layer. Our results are found to be in good agreement with those of [11] for both the constant viscosity (Pr = 7.0) as well as variable viscosity ($\Delta T_w = 10.0$).

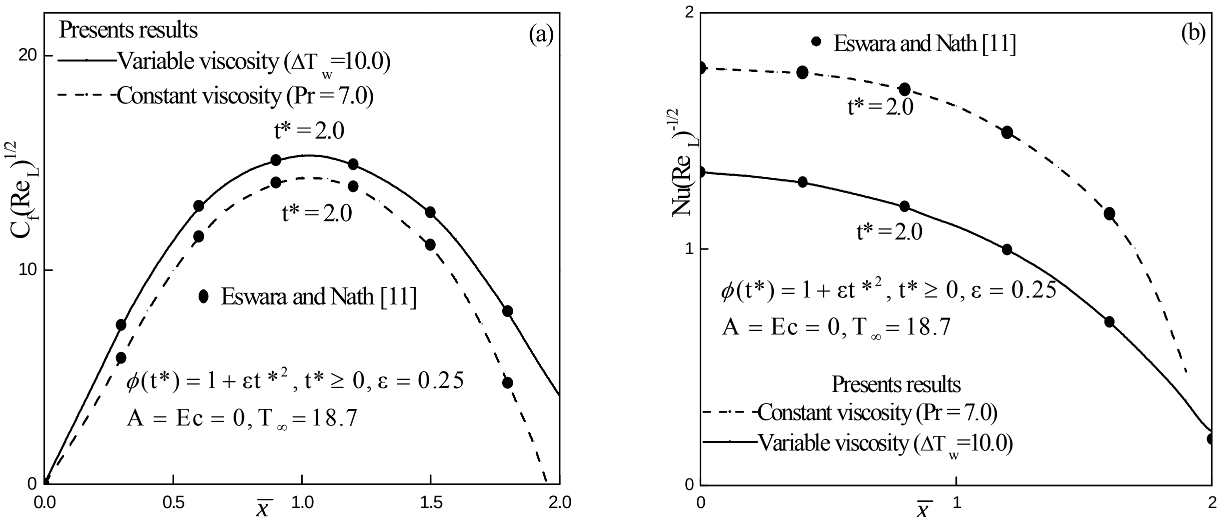

Figure 1: Comparison of Unsteady Skin Friction and Heat Transfer Coefficients with Those of [11] for an Axisymmetric Body (Sphere)

The nonsimilars solution of the problem under consideration, have been obtained for various sets of values of parameters but to reduce the number of figures, only some representative results are presented in Fig. 2-6 by taking $\phi(t^*) = 1 + \varepsilon t^{*2}$, $\varepsilon = 0.25$, which represents the accelerating free stream velocity distribution.

The variation of skin friction and heat transfer coefficients $[C_f(\mathrm{Re}_L)^{1/2}, Nu(\mathrm{Re}_L)^{-1/2}]$ with time (t^*) at different streamwise locations, in the presence of variable fluid properties [$T_\infty = 28.7$°C, $\Delta T_w = 10.0$], is shown in Fig. 2(a). It is found that $C_f(\mathrm{Re}_L)^{1/2}$ and $Nu(\mathrm{Re}_L)^{-1/2}$ increase with the increase of time. This behavior is same at all streamwise locations. In fact, the percentages of increase in $C_f(\mathrm{Re}_L)^{1/2}$, at $\bar{x} = 1.0$ is 89.57% , for an increase of t^* from 0 to 1.5 whereas in the case of $Nu(\mathrm{Re}_L)^{-1/2}$, at $\bar{x} = 1.5$, it is about 45.37%. The corresponding velocity (F) and temperature (G) profiles are displayed in Fig. 2(b). It is clear from these figures that both velocity and temperature profiles become steep with the increase of time (t^*). Thus, the effect of unsteadiness involved in the accelerating free stream is to decrease the thickness of both, momentum and thermal boundary layers.

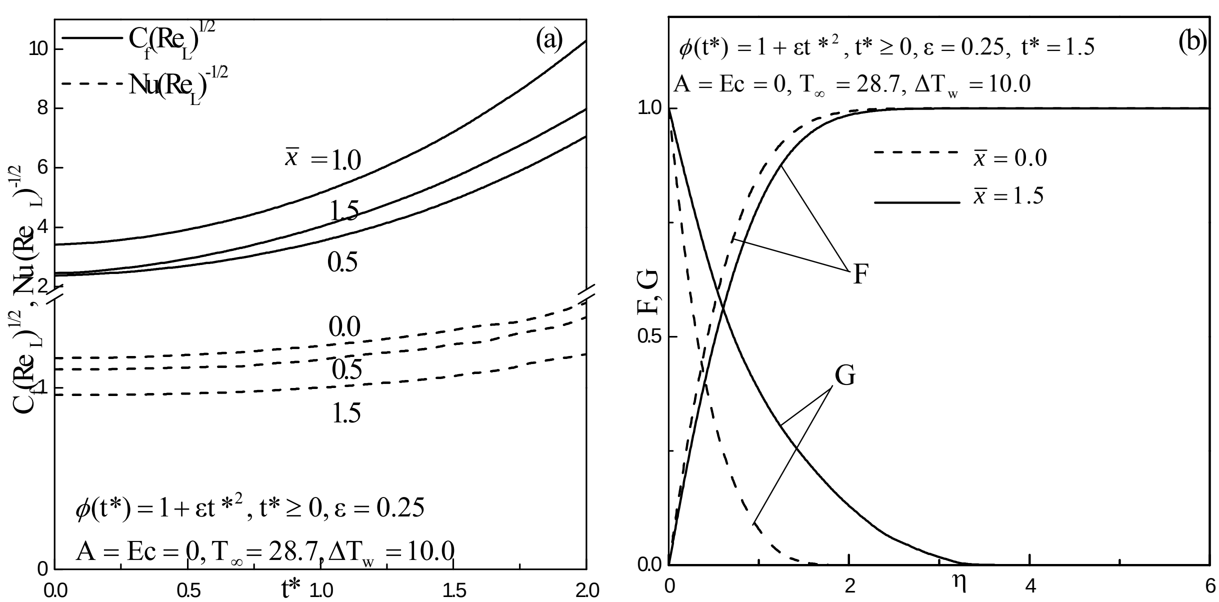

Figure 2: The Effect of Time on (a) Skin Friction and Heat Transfer Coefficients and (b) Velocity and Temperature Profiles

Figure 3 describe the effect of variable viscosity and Prandtl number on the skin friction [$C_f(\text{Re}_L)^{1/2}$] and heat transfer [$Nu\,(\text{Re}_L)^{-1/2}$] coefficients at different streamwise locations ($\bar{x}$). This figure also contains results for $C_f(\text{Re}_L)^{1/2}$ and $Nu\,(\text{Re}_L)^{-1/2}$ when $N = 1$ and Pr = 17.0 at room temperature $T = 20^{0}$C for constant viscosity and Prandtl number. It is observed from Fig. 3(a) that skin friction coefficient $C_f(\text{Re}_L)^{1/2}$ increases from zero to a maximum value in a certain range of $\bar{x}$ ($\bar{x}$ = 1.0) and then decreases as $\bar{x}$ further increases. It is also observed that the effect of variable fluid properties is to increase the skin friction coefficient and to decrease the heat transfer coefficient [See Fig. 3(b)]. When $t^* = 1.5$, skin friction coefficient $C_f(\text{Re}_L)^{1/2}$ for variable fluid properties differs from those of constant fluid properties by about 20.08% at $\bar{x}$ = 1.0. On the other hand, the percentage of difference between the constant and variable fluid properties, in the case of heat transfer $Nu\,(\text{Re}_L)^{-1/2}$ coefficient is 78.42% at $\bar{x}$ = 0.0 and 60.58% at $\bar{x}$ = 1.0. The quantitative analysis, presented below, reveals the fact that the effect of variable fluid properties is more pronounced on heat transfer as compared to skin friction. This is expected, because the effect of the variation of viscosity (with temperature) on momentum equation is introduced through $N\,[= \mu/\mu_\infty]$ only, whereas in the energy equation it is introduced through N as well as Pr and its derivative. Also, it is observed from this figure that in the case of variable fluid properties, the point of zero skin friction is moved down stream as compared to constant fluid properties. For the sphere, it may be noted that the point of skin friction predicted by our method is $\bar{x}$ = 1.801 for constant fluid properties while it is $\bar{x}$ = 1.912 for variable fluid properties.

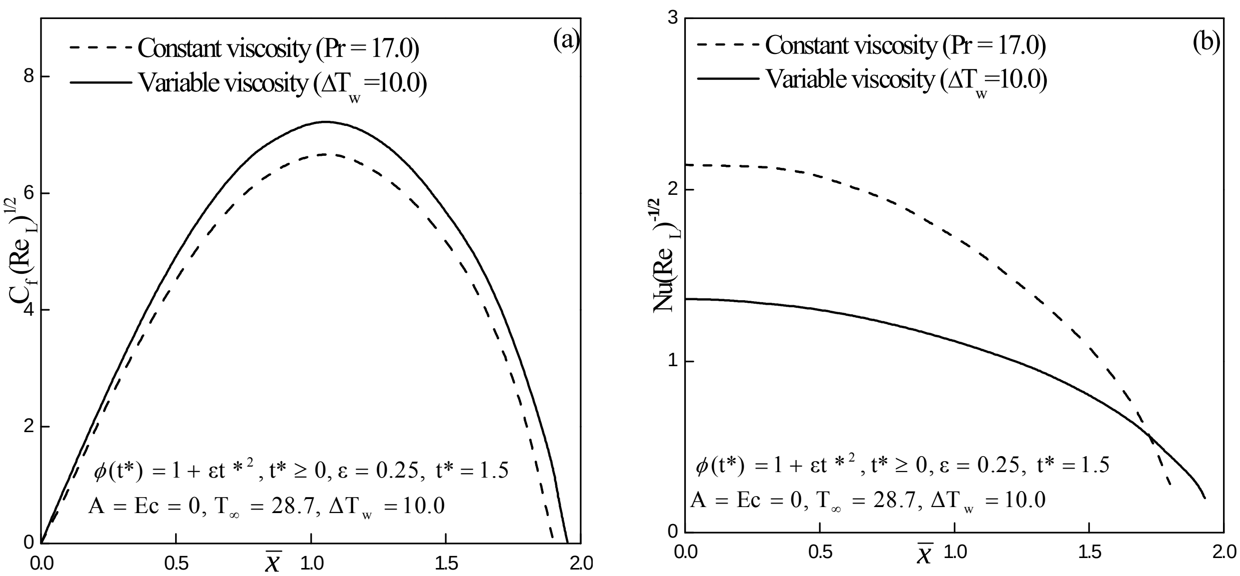

Figure 3: Comparison of Variable Fluid Property Results with Constant Fluid Properties Results. (a) Skin Friction Coefficient (b) Heat Transfer Coefficient

To see the effect of difference in the temperature (ΔT_w) between the wall and fluid, which is actually causes the variation of viscosity and Prandtl number across the boundary layer, the skin friction and heat transfer coefficients have been plotted against ΔT_w [See in Fig. 4]. Since $T_\infty = 28.7^{\circ}$C, the maximum value of ΔT_w taken is 30°C so as to keep the temperature within the allowed value (< 40°C). It is observed from this figure that $C_f\,(\text{Re}_L)^{1/2}$ increases with increase of ΔT_w whereas $Nu\,(\text{Re}_L)^{-1/2}$ is found to decrease with ΔT_w.

Figure 5 displays the effect of viscous dissipation parameter (Ec) on skin friction and heat transfer [$C_f(\text{Re}_L)^{1/2}$, $Nu\,(\text{Re}_L)^{-1/2}$] in the presence of variable fluid properties [$T_\infty = 28.7^{\circ}$C, $\Delta T_w = 10.0$]. It is observed that both skin friction [$C_f(\text{Re}_L)^{1/2}$] and heat transfer [$Nu\,(\text{Re}_L)^{-1/2}$] decrease with the increases of Ec and the effect of viscous dissipation is more pronounced on the heat transfer than on skin friction. Though the viscous dissipation parameter Ec appears only in energy equation, its effect becomes prominent on the skin friction since, the momentum equation is coupled with energy equation, due to the temperature-dependent viscosity and Prandtl number. It is also found that $C_f(\text{Re}_L)^{1/2}$ increases with the increase of time, irrespective of the value of Ec. On the other hand,

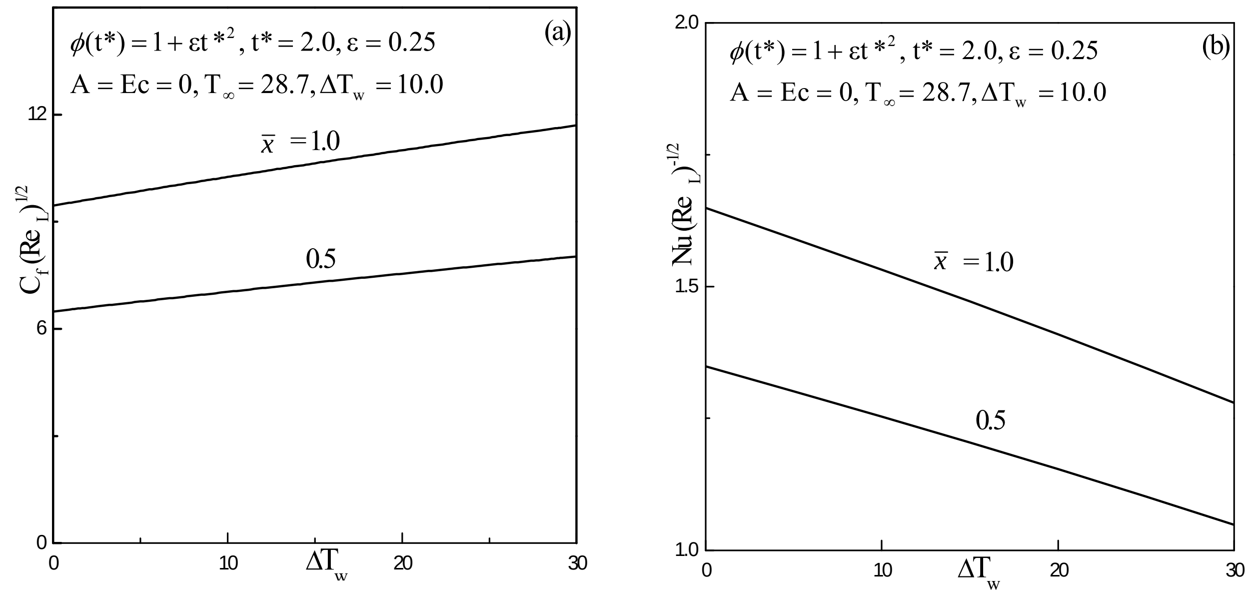

Figure 4: Variation of (a) Skin Friction and (b) Heat Transfer Coefficients with ΔT_w

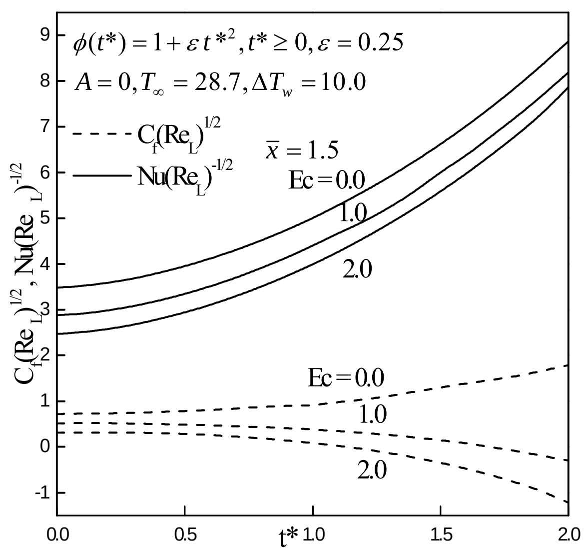

Figure 5: Effect of Viscous Dissipation on Skin Friction and Heat Transfer Coefficients

$Nu\,(Re_L)^{-1/2}$ reduces with the increase of Ec ($Ec \neq 0$) and time t^*. In fact, it is found that the percentage decrease of $Nu\,(Re_L)^{-1/2}$ for an increase in Ec from 0 to 2.0, for $t^* = 1.0$ and $\bar{x} = 1.5$, is 60.35% as compared to 19.1% of $C_f\,(Re_L)^{1/2}$ for the same data. This means that heat transfer is more affected by the viscous dissipation as compared to skin friction. Due to viscous dissipation, the fluid near the wall heats up and its temperature becomes more than the wall, although originally the wall was at higher temperature [$T_\infty = 28.7^oC$, $T_w = 38.7^0C$, $\Delta T_w = 10.0$]. Thus, the cooler free stream is unable to cool the hot wall due to the heat cushion provided by frictional heating. This results in the reduction of the heat transfer (from the wall to the fluid). Further, it is observed that, when $Ec \neq 0$, $Nu\,(Re_L)^{-1/2}$ becomes negative indicating the reversal of the direction of heat transfer from the initial, wall to fluid, to fluid to wall. Similar trend has been observed in the case of constant fluid properties parallel flow past a flat plate at zero incidence [7]. However, in the absence of viscous dissipation ($Ec = 0$) heat transfer takes place in the usual way (from wall to the fluid).

Figure 6 shows the effect of mass transfer parameter (A) on $C_f(Re_L)^{1/2}$ and $Nu\,(Re_L)^{-1/2}$, when time $t^* = 2.0$. Whatever may be the value of A, both skin friction and heat transfer are found to increases. Further, it is also

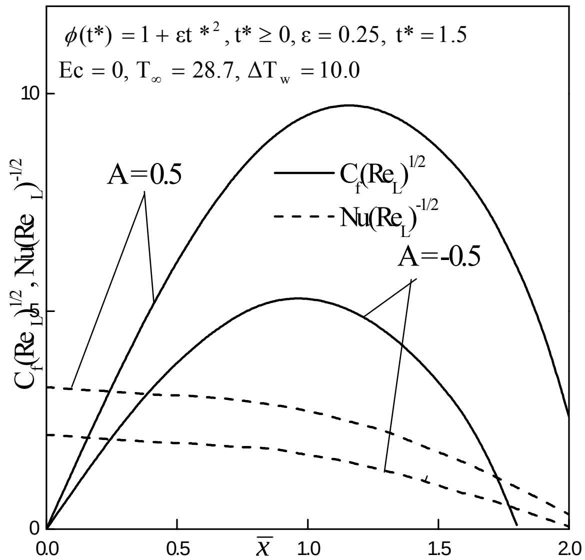

Figure 6: Effect of Mass Transfer on Skin Friction and Heat Transfer Coefficients

found that suction ($A > 0$) increases both $C_f(\mathrm{Re}_L)^{1/2}$ and $Nu(\mathrm{Re}_L)^{-1/2}$ while injection ($A < 0$) does the reverse. From the Fig. 6 it is observed that $C_f(\mathrm{Re}_L)^{1/2}$ increases from zero to a maximum up to a value of $\bar{x} = 1.0$ in the case of injection ($A < 0$) and up to $\bar{x} = 1.1$ in the case of suction ($A > 0$), and then decreases as further increases. On the other hand, $Nu(\mathrm{Re}_L)^{-1/2}$ continues to decrease with the increase of streamwise location ($\bar{x}$).

4. CONCLUSIONS

Under the assumption of temperature-dependent viscosity and Prandtl number, the unsteady axisymmetric nonsimilar (ethanol) boundary layer flow has been studied when the external flow velocity varies arbitrarily with time. The viscosity and Prandtl number are assumed to vary as an inverse liner function of temperature which is more realistic for moderate temperature difference between the surface and the ambient liquid.

Results indicate that the skin friction coefficient increases in the case of variable viscosity as compared to constant viscosity, where as the effect of variable viscosity on the heat transfer coefficient is just opposite. The unsteadiness and suction cause the point of zero skin friction to move downstream. In general injection reduces both skin friction and heat transfer while suction does the reverse. The heat transfer is found to depend strongly on viscous dissipation, but the skin friction is little affected by it. Both momentum and thermal boundary layer thicknesses decrease boundary-layer in the stream wise location.

From our study, it can be concluded that the effect of variation of viscosity and Prandtl number with temperature has to be taken into consideration to avoid significant errors in the prediction of skin friction coefficient and heat transfer rate at the wall.

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