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EFFECT OF MAGNETIC FIELD ON THE PERISTALTIC PUMPING OF A JEFFREY FLUID IN A CHANNEL WITH VARIABLE VISCOSITY

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ABSTRACT: In the present paper we studied the peristaltic flow of a Jeffrey fluid in a two-dimensional channel in the presence of transverse magnetic field. The flow is examined in a wave frame of reference moving with the velocity of the wave. The problem is formulated using perturbation expansion in terms of viscosity parameter α. The governing equations are developed upto first order in the viscosity parameter α. The expressions for the velocity and pressure gradient have been obtained. The effects of Hartmann number M, viscosity parameter α, material parameter λ_1 and amplitude ratio ϕ on the pumping characteristics and friction force studied through graphs in detail.

AMS 2000 Mathematics Subject Classification: 76Z05, 76D05.

Keywords: Peristaltic transport, Jeffrey fluid, Hartmann number, Variable viscosity.

1. INTRODUCTION

Peristalsis is a series of coordinated, rhythmic muscle contractions. It is an automatic and vital process that moves food through the digestive tract, urine from the kidneys through the ureters into the bladder, and bile from the gallbladder into the duodenum. The transport phenomenon created by peristalsis is an interesting problem because of its application in understanding many physiological transport processes through vessels under peristaltic motion. Roller and finger pumps using viscous fluids also operate on this principle. Here the tube is passive but is compressed by rotating rollers, by a series of mechanical fingers or by a nutating plate.

A number of analytical (Burns and Parkes [1], Fung and Yih [2], Jaffrin [3], Ramachandra Rao and Usha [4], Shapiro *et al*., [5], Siddiqui and Schwarz [6], Zien and Ostrach [7]) as well as numerical and experimental (Latham [8], Weinberg *et al*., [9], Takabatake and Ayukawa [10], Takabatake *et al*., [11], Tang and Rankin [12]) studies of peristaltic flows of different fluids have been reported. Several review articles have been written (Jaffrin and Shapiro [13], Rath [14]).

The complex rheology of biological and physiological flows has also motivated a number of studies involving non-Newtonian fluid. The power-law model was used by Raju and Devanathan [15], Shukla and Gupta [16], Becker [17] and Subba Reddy *et al*., [18] to investigate Shear-thinning and Shear-thickening effects. Raju and Devanathan [19] and Bohme and Friedrich [20] investigated the effects of viscoelasticity. Siddiqui *et al*., [21] used the second-order fluid model to study the effects of normal stresses non-Newtonian flows. Abd El Hakeem *et al*., [22] have investigated the peristaltic flow of a fluid with variable viscosity under the effect of magnetic field.

The effect of moving magnetic field on blood flow was studied by Sud *et al*., [23] and they have observed that the effect of suitable moving magnetic filed accelerates the speed of blood. Prasad and Ramacharyulu [24] have observed that by considering the blood as an electrically conducting fluid constitutes a suspension of red cell in plasma. Also, Agrawal and Anwaruddin [25] studied the effect of magnetic field on the peristaltic flow of blood using long wavelength approximation method and observed for the flow of blood in arteries with arterial stenosis or arteriosclerosis, that the influence of magnetic field may be utilized as blood pump in carrying out cardiac operations. Li *et al*., [26] have studied an impulsive magnetic filed in the combined therapy of patients with stone fragments in the upper urinary tract. The peristaltic transport of blood under effect of a magnetic field in non uniform channels was studied by Mekheimer [27]. Hayat *et al*., [28] have analyzed the influence of an endoscope on the peristaltic flow of a Jeffrey under the effect of magnetic field in a tube. Peristaltic motion of a Jeffery fluid under the effect of a magnetic field in a tube was discussed by Hayat and Ali [29].

In this paper, we analyze the peristaltic flow of a Jeffrey fluid (non-Newtonian fluid) with variable viscosity under the effect of a magnetic field in a two-dimensional channel. The flow is examined in a wave frame of reference moving with the velocity of the wave. The problem is formulated using perturbation expansion in terms of viscosity parameter α. The governing equations are developed upto first order in the viscosity parameter α. The expressions for the velocity and pressure gradient have been obtained. The effects of Hartmann number M, viscosity parameter α, material parameter λ_1 and amplitude ratio ϕ on the pumping characteristics and friction force studied in detail.

2. MATHEMATICAL FORMULATION

We consider an incompressible and electrically conducting Jeffrey fluid with variable viscosity in a two-dimensional channel of width $2a$. The walls of the cannel are flexible and non-conducting. The sinusoidal wave trains propagate on the channel walls with constant speed c and propped the fluid along the walls. In rectangular coordinate system (X, Y), the geometry of the wall surface is described by

$$H(X,t) = a + b\cos\left[\frac{2\pi}{\lambda}(X - ct)\right] \tag{2.1}$$

where b is the wave amplitude, λ is the wave length, c is the velocity of propagation and x is the direction of wave propagation. Figure 1 depicts the physical model of the problem.

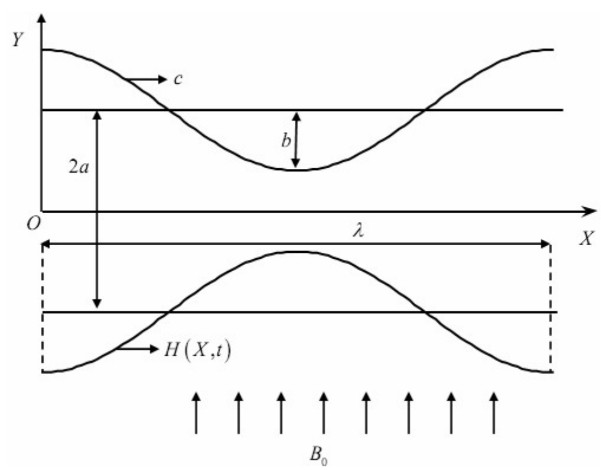

Figure 1: The Physical Model

A uniform magnetic field of strength B_0 is applied in the transverse direction to the flow. The induced magnetic field is neglected by assuming small magnetic Reynolds number. The electric field is taken zero. Under the assumptions that the channel length is an integral multiple of the wavelength λ and the pressure difference across the ends of the channel is a constant, the flow becomes steady in the wave frame (x, y) moving with velocity c away from the fixed (laboratory) frame (X, Y). The transformation between these two frames is given by

$$x = X - ct, \quad y = Y, \quad u = U - c, \quad p(x) = P(X, t). \tag{2.2}$$

where U and V are velocity components in the laboratory frame and u and v are velocity components in the wave frame.

The equations governing the flow in a wave frame are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.3}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}-\sigma B_0^2(u+c) \tag{2.4}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\frac{\partial S_{xy}}{\partial x}+\frac{\partial S_{yy}}{\partial y} \tag{2.5}$$

where ρ is the density, p is the pressure, and σ is the electrical conductivity.

The constitutive equation for the extra stress tensor S is

$$S=\frac{\mu(y)}{1+\lambda_1}[\dot{\gamma}+\lambda_2\ddot{\gamma}] \tag{2.6}$$

where λ_1 is the ratio of relaxation to retardation times, λ_2 is the retardation time, $\mu(y)$ is the viscosity function, γ is the shear rate and dots over the quantities indicate differentiation with respect to time.

Using the following non dimensional quantities

$$\bar{x}=\frac{x}{\lambda},\quad \bar{y}=\frac{y}{a},\quad \bar{u}=\frac{u}{c},\quad \bar{v}=\frac{v}{c\delta},\quad \delta=\frac{a}{\lambda},\quad \bar{p}=\frac{pa^2}{\mu_0 c\lambda},\quad \bar{t}=\frac{ct}{\lambda},\quad h=\frac{H}{a},\quad \bar{S}=\frac{aS}{\mu_0 c},$$

where μ_0 is the viscosity, in the Equations (2.3)-(2.5), we get (dropping bars)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.7}$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\delta\frac{\partial S_{xx}}{\partial x}+\frac{\partial S_{xy}}{\partial y}-M^2(u+1) \tag{2.8}$$

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\delta^2\frac{\partial S_{xy}}{\partial x}+\frac{\partial S_{yy}}{\partial y} \tag{2.9}$$

where

$$S_{xx}=\frac{2\delta\mu(y)}{1+\lambda_1}\left[1+\frac{\lambda_2 c\delta}{a}\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\right]\frac{\partial u}{\partial x},$$

$$S_{xy}=\frac{\mu(y)}{1+\lambda_1}\left[1+\frac{\lambda_2 c\delta}{a}\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\right]\left(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x}\right),$$

$$S_{yy}=\frac{2\delta\mu(y)}{1+\lambda_1}\left[1+\frac{c\delta\lambda_2}{a}\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\right]\frac{\partial v}{\partial y},$$

$\mathrm{Re}=\frac{\rho ac}{\mu_0}$ is the Reynolds number and $M=aB_0\sqrt{\frac{\sigma}{\mu_0}}$ is the Hartmann number.

The corresponding non-dimensional boundary conditions are

$$\frac{\partial u}{\partial y} = 0 \qquad \text{at} \qquad y = 0, \tag{2.10}$$

$$u = -1 \qquad \text{at} \qquad y = h. \tag{2.11}$$

Using the long wavelength ($\delta \ll 1$) and low Reynolds number ($\text{Re} \to 0$) assumptions, the Equations (2.8) and (2.9) becomes

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left[\frac{\mu(y)}{1+\lambda_1}\frac{\partial u}{\partial y}\right] - M^2(u+1), \tag{2.12}$$

$$\frac{\partial p}{\partial y} = 0, \tag{2.13}$$

From Eq. (2.13), $p \neq p(y)$, that is p is a function of x only. So that Eq. (2.12) can be rewritten as

$$(1+\lambda_1)\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left[\mu(y)\frac{\partial u}{\partial y}\right] - N^2(u+1), \tag{2.14}$$

where $N = M\sqrt{(1+\lambda_1)}$.

The effect of viscosity variation on peristaltic flow can be investigated for any given function $\mu(y)$. For the present investigation, we assume the viscosity variation in the dimensionless from as

$$\mu(y) = e^{-\alpha y} \quad \text{or} \quad \mu(y) = 1 - \alpha y \quad \text{for} \quad \alpha \ll 1 \tag{2.15}$$

The non dimensional volume flow rate q in a wave frame of reference is given by

$$q = \int_0^h u\, dy. \tag{2.16}$$

The instantaneous flux $Q(X, t)$ in the laboratory frame is

$$Q(X,t) = \int_0^h U dy = \int_0^h (u+1)\, dy = \int_0^h u dy + \int_0^h dy = q + h. \tag{2.17}$$

The time averaged flux over one period $T(= \lambda/c)$ of the peristaltic wave is

$$\bar{Q} = \frac{1}{T}\int_0^T Q dt = \int_0^1 (q+h)\, dx = q + 1. \tag{2.18}$$

3. SOLUTION

The Equation (2.14) is non-linear and its closed form solution is not possible. Hence, we linearize this equation in terms of α, since α is small for the type of flow under consideration. So we expand u, $\frac{dp}{dx}$ and q as

$$u = u_0 + \alpha u_1 + O(\alpha^2), \tag{3.1}$$

$$\frac{dp}{dx} = \frac{dp_0}{dx} + \alpha\frac{dp}{dx} + O(\alpha^2), \tag{3.2}$$

$$q = q_0 + \alpha q_1 + O(\alpha^2). \tag{3.3}$$

Substituting from Equations (3.3) and (3.2) in the Equations (2.14), (2.10) and (2.11), we get

3.1 System of Order Xero

$$(1+\lambda_1)\frac{\partial p_0}{\partial x}=\frac{\partial^2 u_0}{\partial y^2}-N^2(u_0+1) \tag{3.4}$$

with the boundary conditions

$$\frac{\partial u_0}{\partial y}=0 \qquad \text{at} \qquad y=0 \tag{3.5}$$

$$u_0=-1 \qquad \text{at} \qquad y=h. \tag{3.6}$$

3.2 System of Order One

$$\frac{\partial^2 u_1}{\partial y^2}-N^2 u_1=(1+\lambda_1)\frac{\partial p_1}{\partial x}+y\frac{\partial^2 u_0}{\partial y^2}+\frac{\partial u_0}{\partial y^2} \tag{3.7}$$

with the boundary conditions

$$\frac{\partial u_1}{\partial y}=0 \qquad \text{at} \qquad y=0 \tag{3.8}$$

$$u_1=0 \qquad \text{at} \qquad y=h. \tag{3.9}$$

3.3 Solution of Order Zero

Solving Eq. (3.4) using the boundary conditions (3.5) and (3.6), we get

$$u_0=\frac{(1+\lambda_1)}{N^2}\frac{dp_0}{dx}\left[\frac{\cosh Ny}{\cosh Nh}-1\right]-1. \tag{3.10}$$

The volume flow rate q_0 in a wave frame is given by

$$q_0=\int_0^h u_0 dy=\frac{(1+\lambda_1)}{N^3}\frac{dp_0}{dx}\left[\frac{\sinh Nh-Nh\cosh Nh}{\cosh Nh}\right]-h. \tag{3.11}$$

3.4 Solution of Order One

Substituting Eq. (3.10) in Eq. (3.7) and solving it using the boundary conditions (3.8) and (3.9), we get

$$u_1=\frac{(1+\lambda_1)}{N^2}\frac{dp_1}{dx}\left[\frac{\cosh Ny}{\cosh Nh}-1\right]+\frac{1}{4N\cosh Nh}\frac{dp_0}{dx}[y^2\sinh Ny-h^2\tanh Nh\cosh Ny] \tag{3.12}$$

The volume flow rate q_1 in a wave frame is given by

$$q_1=\int_0^h u_1 dy=\frac{(1+\lambda_1)}{N^3}\frac{dp_1}{dx}\frac{[\sinh Nh-Nh\cosh Nh]}{\cosh Nh}$$

$$+\frac{(1+\lambda_1)}{4N^4\cosh^2 Nh}\frac{dp_0}{dx}\{N^2h^2-hN\sinh 2Nh+2\cosh Nh(\cosh Nh-1)\} \tag{3.13}$$

Substituting from Equations (3.11) and (3.13) into Eq. (3.3), we get

$$q=\frac{(1+\lambda_1)}{N^3}\left(\frac{dp}{dx}\right)\frac{(\sinh Nh-Nh\cosh Nh)}{\cosh Nh}$$

$$+\alpha\frac{(1+\lambda_1)}{\cosh^2 Nh}\frac{dp_0}{dx}\frac{1}{4N^4}\{N^2h^2-Nh\sinh 2Nh+2\cosh Nh(\cosh Nh-1)\} \tag{3.14}$$

Solving Eq. (3.14) for $\frac{dp}{dx}$ using $\frac{dp_0}{dx} = \frac{dp}{dx} - \alpha \frac{dp_1}{dx}$ and neglecting $O(\alpha^2)$ terms, we obtain

$$\frac{dp}{dx} = \frac{(q+h)\,N^3 \cosh Nh}{(1+\lambda_1)\,[\sinh Nh - Nh \cosh Nh]} \left[1 - \frac{\alpha}{4N \cosh Nh}\,\Xi\right], \tag{3.15}$$

where

$$\Xi = \frac{N^2 h^2 - Nh \sinh 2Nh + 2\cosh Nh\,(\cosh Nh - 1)}{\sinh Nh - Nh \cosh Nh}.$$

The pressure rise Δp per one wave length is given by

$$\Delta p = \int_0^1 \frac{dp}{dx}\,dx. \tag{3.16}$$

The friction force on the channel wall is given by

$$F = \int_0^1 h\left(-\frac{dp}{dx}\right) dx. \tag{3.17}$$

4. RESULTS AND DISCUSSIONS

A regular perturbation series in terms of the viscosity parameter (α) is used to obtain solution to the field equations for peristaltic flow of a Jeffery fluid in an axisymmetric tube. Since the integrals in equations (3.16) and (3.17) are not integrable in closed form, we have evaluated it numerically using in **MATLAB 7.0** package. The values of various parameters for the transport of mucus in the small intestine, as reported in Shukla *et al*., [30] and Srivastava *et al*., [31] are $c = 2$ cm/min, $\alpha = 1.25$ cm, $\lambda = 8.01$ cm. The values of viscosity parameter α as reported in Srivastava *et al*., [31] are $\alpha = 0$ and $\alpha = 0.1$. It may be noted that the theory of long wave length and zero Reynolds number of the present investigation remains applicable here, since the radius of the small intestine is very small compared with the wave length.

The variation of pressure rise Δp with time averaged flux $\bar{Q}$ for different vales of viscosity parameter α with $M = 1$, $\lambda_1 = 0.3$ and $\phi = 0.6$ is depicted in Fig. 2. It is observed that, in the pumping region ($\Delta p > 0$) the time averaged flux $\bar{Q}$ is decreases with an increase in viscosity parameter α, whereas in the free pumping region ($\Delta p = 0$) as well as in the co-pumping region ($\Delta p < 0$), $\bar{Q}$. Increases with increasing α.

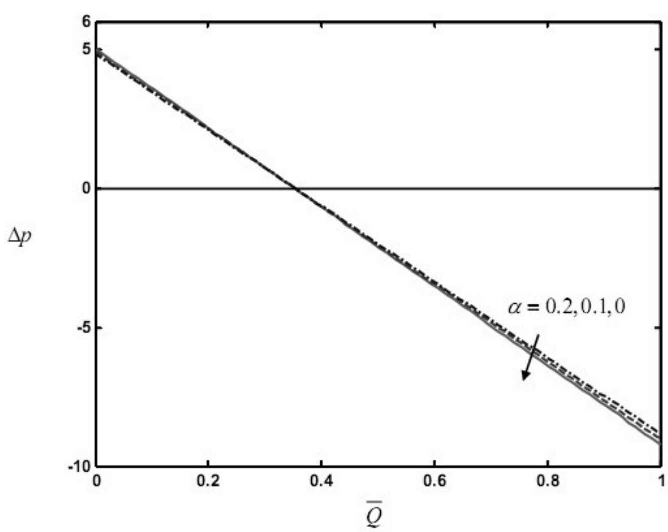

Figure 2: The Variation of Pressure Rise Δp with Time Averaged Flux $\bar{Q}$ for Different Vales of Viscosity Parameter α with $M = 1$, $\lambda_1 = 0.3$ and $\phi = 0.6$

Figure 3 illustrates the variation of pressure rise Δp with time averaged flux $\bar{Q}$ for different vales of λ_1 with $M = 1$, $\alpha = 0.1$ and $\phi = 0.6$. It is found that, the $\bar{Q}$ decreases with increasing λ_1 in both the pumping region and free pumping region. In the co-pumping region, $\bar{Q}$ increases as λ_1 increases for appropriately chosen $\Delta p\,(< 0)$. Further, it is observed that, the pumping is more for Newtonian fluid ($\lambda_1 \to 0$) than that of Jeffrey fluid ($\lambda_1 > 0$).

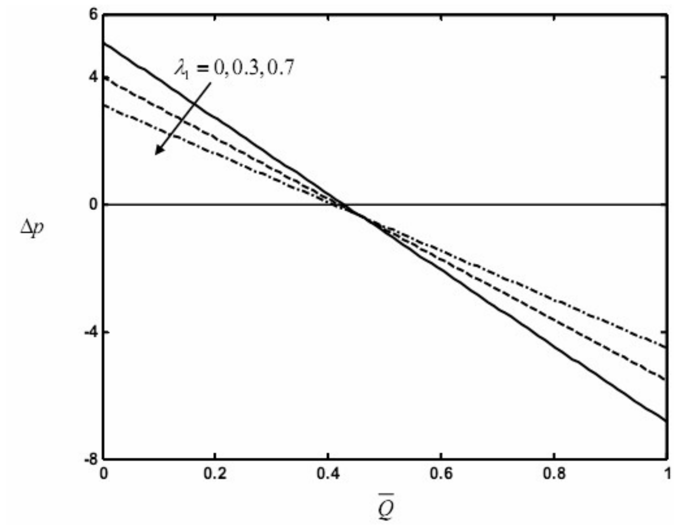

Figure 3: The Variation of Pressure Rise Δp with Time Averaged Flux $\bar{Q}$ for Different Vales of λ_1 with $M = 1$, $\alpha = 0.1$ and $\phi = 0.6$

The variation of pressure rise Δp with time averaged flux $\bar{Q}$ for different vales of Hartmann number M with $\alpha = 0.1$, $\lambda_1 = 0.3$ and $\phi = 0.6$ is shown in Fig. 4. It is noted that, any two pumping curves intersect in the first quadrant. To the left of this point, the $\bar{Q}$ increases and to the right of this point it decreases with increasing Hartmann number M.

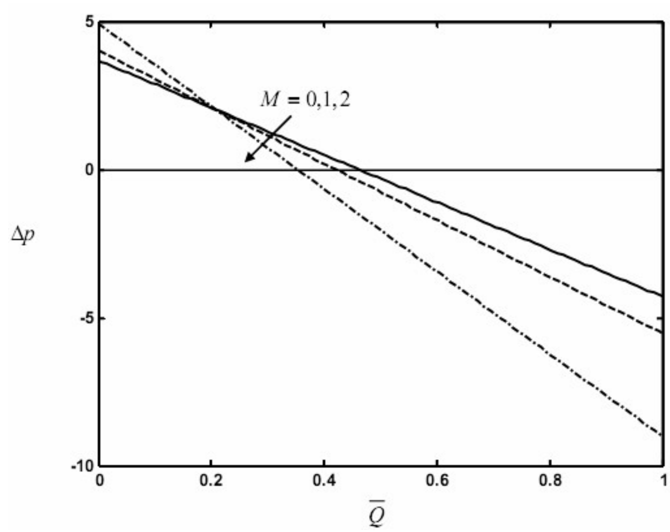

Figure 4: The Variation of Pressure Rise Δp with Time Averaged Flux $\bar{Q}$ for Different Vales of Hartmann Number M with $\alpha = 0.1$, $\lambda_1 = 0.3$ and $\phi = 0.6$

Figure 5 shows the variation of pressure rise Δp versus time averaged flux $\bar{Q}$ for different vales of amplitude ratio ϕ with $M = 1$, $\lambda_1 = 0.3$ and $\alpha = 0.1$. It is observed that, the $\bar{Q}$ increases with increasing ϕ in both the pumping region and free pumping region. An interesting observation is that in the co-pumping region, as ϕ increases $\bar{Q}$ decreases for appropriately chosen $\Delta p\,(< 0)$.

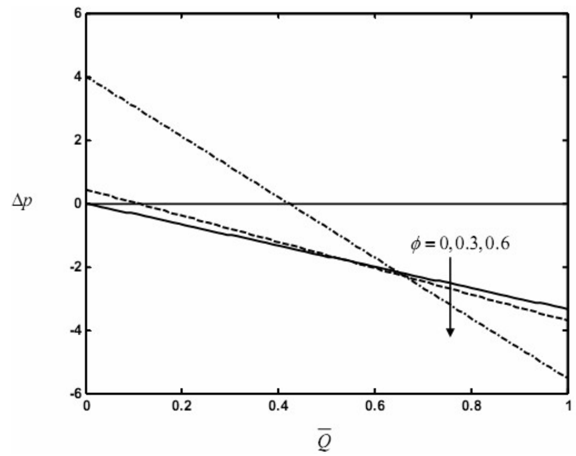

Figure 5: The Variation of Pressure Rise Δp Versus Time Averaged Flux $\overline{Q}$ for Different Vales of Amplitude Ratio ϕ with $M = 1$, $\lambda_1 = 0.3$ and $\alpha = 0.1$

The friction force F versus time averaged flux $\overline{Q}$ for different vales of viscosity parameter α with $M = 1$, $\lambda_1 = 0.3$ and $\phi = 0.6$ is depicted in Fig. 6. It is found that, the friction force F initially increases and then decreases with an increase in the viscosity parameter α.

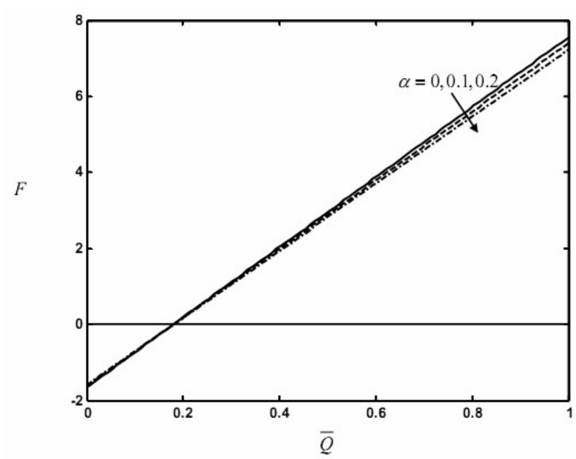

Figure 6(i): The Friction Force F Versus Time Averaged Flux $\overline{Q}$ for Different Values of Viscosity Parameter α with $M = 1$, $\lambda_1 = 0.3$ and $\phi = 0.6$

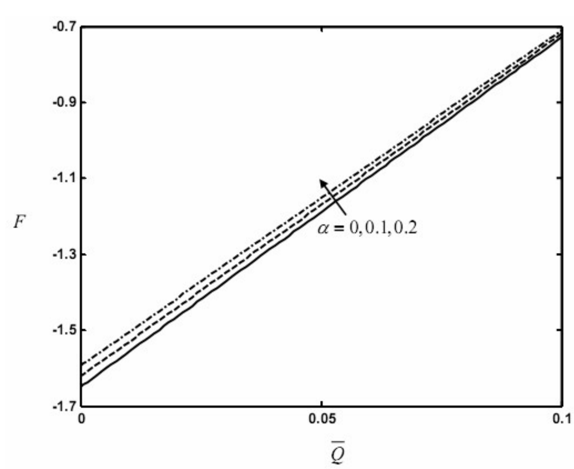

Figure 6(ii): Enlargement of (i)

Figure 7 shows the variation of friction force F with time averaged flux $\bar{Q}$ for different vales of λ_1 with $M = 1$, $\alpha = 0.1$ and $\phi = 0.6$. It is observed that, the friction force F initially increases and then increases with increasing λ_1.

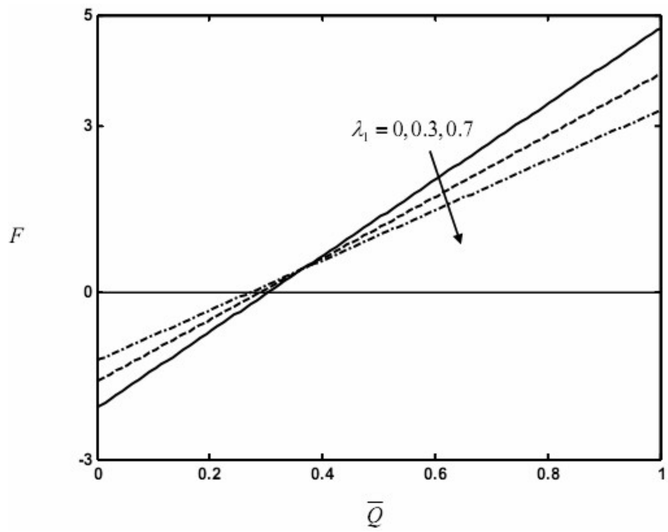

Figure 7: The Variation of Friction Force *F* with Time Averaged Flux $\bar{Q}$ for Different Vales of λ_1 with $M = 1$, $\alpha = 0.1$ and $\phi = 0.6$

The variation of friction force F with time averaged flux $\bar{Q}$ for different vales of Hartmann number M with $\alpha = 0.1$, $\lambda_1 = 0.3$ and $\phi = 0.6$ is presented in Fig. 8. It is found that, as the Hartmann number M increases the magnitude of the friction force F increases.

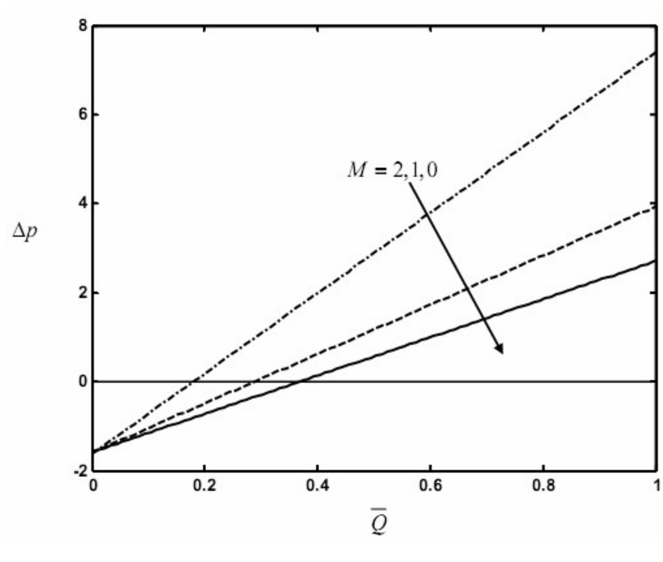

Figure 8: The Variation of Friction Force *F* with Time Averaged Flux $\bar{Q}$ for Different Vales of Hartmann Number *M* with $\alpha = 0.1$, $\lambda_1 = 0.3$ and $\phi = 06$

The variation of friction force F with time averaged flux $\bar{Q}$ for different vales of amplitude ratio ϕ with $M = 1$, $\lambda_1 = 0.3$ and $\alpha = 0.1$ is shown in Fig. 9. It is observed that, the friction force F first decreases and then increases with increasing ϕ.

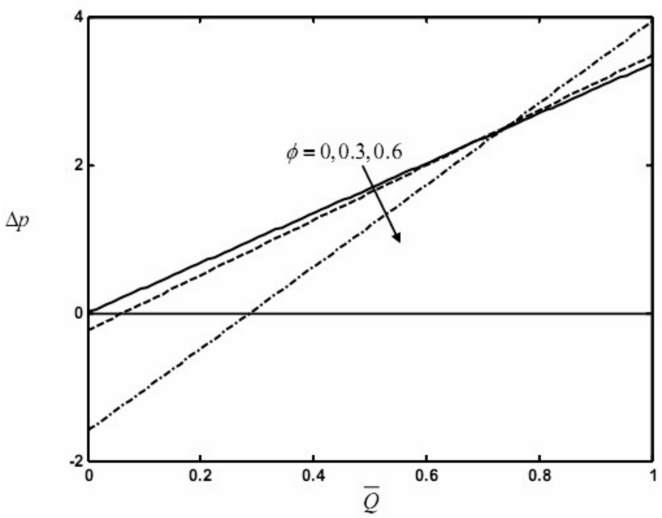

Figure 9: The Variation of Friction Force F with Time Averaged Flux $\overline{Q}$ for Different Vales of Amplitude Ratio ϕ with $M = 1$, $\lambda_1 = 0.3$ and $\alpha = 0.1$

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