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GENERALIZED DISPERSION OF ATMOSPHERIC AEROSOLS ON UNSTEADY CONVECTIVE DIFFUSION IN COUPLE STRESS FLUID BOUNDED BY ELECTRODES

P. Meena Priya & Nirmala P. Ratchagar

ABSTRACT: A generalized dispersion model is used to obtain exact solution for unsteady convective diffusion in couple stress fluid bounded by electrodes. The effect of the couple stress parameter 'a' on the most dominant dispersion coefficient is clearly depicted. The mean concentration distribution is obtained as a function of axial distance, time and couple stress '*a*'. It is shown that the effect of couple stress is predominate only for small values.

Keywords: Couple stress fluid, Electrodes, Generalized dispersion.

1. INTRODUCTION

Knowledge based on air pollution and its impact on human health is growing at a tremendous pace worldwide because of their threat to future of mankind, overtaking infectious diseases and nutritional disorders. Air pollution is defined as the presence in the outdoor atmosphere of one or more contaminants (pollutants) in quantities and duration that can injure human, plant or animal life or property (materials) or which unreasonably interferes with the enjoyment of life.

The growth of population, indiscriminate establishment of industries, automobiles and so on, are the cause for the atmospheric pollution besides power shortage. There is a growing concern that this pollution together with accelerated growth and development of nations may alter the climate due to the release of large amount of soot, carbon emission and other pollutants into the atmosphere in the form of ultrafine dust particles which are suspended in the atmosphere called aerosols [1]. The dispersion of aerosols have two important principle ways that (i) extremely small aerosol particles can be removed from an aerosol (ii) The particles can collide with other particles and grow into larger particles enough to be removed by gravity or electrical or aerodynamic forces or they can migrate to surfaces, stick to those surfaces and thus be removed [3]. The process by which these particles migrate, either to a surface or to one another is called diffusion and their motion is described as dispersion.

The dispersion of air pollutants in the atmosphere is governed by the processes of molecular diffusion and convection and is mainly affected by various meterological conditions such as wind, temperature inversion, foggy atmosphere, topography of the terrain etc and types of number of sources [2]. In the nature, transition between ionosphere and atmosphere of the earth is a region having poorly conducting fluid in which the electrical forces dominate in driving the fluid (Malkus and Veronis [9]). Further the electrical forces in thunderstorm also appear to be as important as the fluid forces at some stages of critical forces [4]. In this region the aerosols are of nano size. The electrical conductivity of atmospheric fluid is a function of temperature [8]. In the study of pollutant dispersion, freely suspended atmospheric pollutants which execute microrotation is called micropolar fluid. When micro rotation balances natural vorticity of the micropolar fluid is called couple stress fluid [11]. In the dispersion of atmospheric aerosols, we are generally interested to find the concentration of the pollutant, after being emitted from its source at a specified location and time. The concentration depends upon number of sources, their locations, emission rates and other meterological and atmospheric conditions [10]. The main objective of this paper is to study the dispersion of aerosols valid for all time by using generalized dispersion model of Gill and Sankarasubramanian [5].

2. MATHEMATICAL FORMULATION

The physical configuration shown in the Fig. 1 consists of an infinite horizontal couple stress fluid layer bounded on both sides by electro-conducting impermeable rigid plates embeded with electrodes located at $y = 0$ and $y = h$ and electric potentials $\phi = \frac{V}{h} x$ at $y = 0$ and $\phi = \frac{V}{h}(x - x_0)$ at $y = h$ are maintained on these boundaries where V is potential.

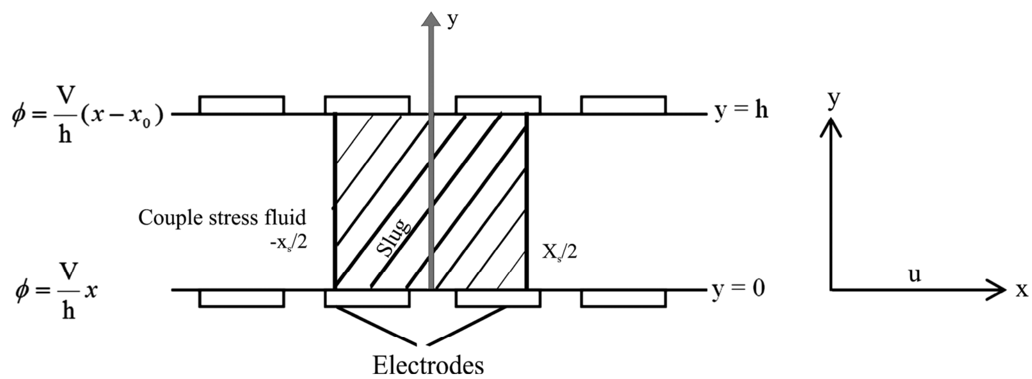

Figure 1: Physical Configuration

In this paper, we make the assumption that the electrical conductivity (σ) is negligibly small and hence the magnetic field is negligible. This assumption makes the electric field $\vec{E}$, to be conservative.

$$\text{ie.} \quad \vec{E} = -\nabla\phi \tag{1}$$

The basic equations are,

Conservation of mass for an incompressible flow

$$\nabla \cdot \vec{q} = 0 \tag{2}$$

Conservation of momentum

$$\rho\left(\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\,\vec{q}\right) = -\nabla p + \mu\nabla^2\vec{q} - \lambda\nabla^4\vec{q} + \rho_e\vec{E} \tag{3}$$

where λ is a couple stress parameter.

Conservation of species

$$\frac{\partial C}{\partial t} + (\vec{q} \cdot \nabla)\,C = D\nabla^2 C \tag{4}$$

Conservation of charges

$$\frac{\partial \rho_e}{\partial t} + (\vec{q} \cdot \nabla)\,\rho_e + \nabla \cdot \vec{J} = 0 \tag{5}$$

Maxwell's equation

$$\nabla \cdot E = \frac{\rho_e}{\epsilon_0} \qquad \text{(Gauss law)} \tag{6}$$

$$\nabla \times E = 0 \qquad \text{(Faraday's law)} \tag{7}$$

The equation (7) is zero because in a poorly conducting fluid, induced magnetic field is negligible and there is no applied magnetic field.

$$J = \sigma E \quad \text{(Ohm's law)} \tag{8}$$

The above equations are solved using the following boundary conditions on velocity and potential are,

No slip condition,
$$\left.\begin{array}{l} u = 0 \quad \text{at} \quad y = 0 \\ u = 0 \quad \text{at} \quad y = h \end{array}\right\} \tag{9}$$

couple stress condition, $\frac{d^2u}{dy^2} = 0$ at $y = 0$ and $y = h$ (10)

$$\left.\begin{array}{ll} \phi = \frac{V}{h}x & \text{at} \quad y = 0 \\ \phi = \frac{V}{h}(x - x_0) & \text{at} \quad y = h \end{array}\right\} \tag{11}$$

In Cartesian form, using the above approximation equation (3) becomes

$$0 = -\frac{\partial p}{\partial x} + \mu\nabla^2 u - \lambda\nabla^2 u + \rho_e E_x, \qquad \nabla^2 = \frac{\partial^2}{\partial y^2}$$

In a poorly conducting fluid, the electrical conductivity is assumed to vary linearly with temperature and increases with temperature in the form

$$\sigma = \sigma_0[1 + \alpha_h(T_b - T_0)] \tag{12}$$

where α_h is the volumetric coefficient of expansion.

We assume the flow is fully developed and unidirectional in the *x*-direction. This means the velocity is independent of time and all physical quantities except pressure and concentrtion are independent of *x*, so that the velocity and temperature will be functions of *y* only. Using the following dimensionless quantities, $\eta = \frac{y}{h}$; $u^* = \frac{u}{\frac{\nu}{h}}$; $E_x^* = \frac{E_x}{\frac{V}{h}}$; $\rho_e^* = \frac{\rho_e}{\frac{\epsilon_0 V}{h^2}}$; $P^* = \frac{P}{\rho\left(\frac{\nu}{h}\right)^2}$; $x^* = \frac{x}{h}$; where *V* is electric potential, we get electric potential through electrodes.

Equations (3) to (11) becomes,

$$\frac{d^4u}{d\eta^4} - \frac{\mu}{\lambda}\cdot h^2 \cdot \frac{d^2u}{d\eta^2} - \frac{\rho_e E_x \epsilon_0 V^2 h^2}{\lambda\nu} = -\frac{\rho\nu h^2}{\lambda}\cdot\frac{\partial P}{\partial x}.$$

We assume that the fluid with pollutants is isotropic and homogenous so that molecular diffusivity *D*, viscosity μ are all constants [7].

$$\frac{d^4u}{d\eta^4} - a^2\frac{d^2u}{d\eta^2} - a^2 W_e \rho_e E_x = a^2 P \tag{13}$$

where $W_e = \frac{\epsilon_0 V^2}{\mu}, P = \frac{-\partial p}{\partial x}, l = \sqrt{\frac{\lambda}{\mu}}, a = \frac{h}{l}$ is the couple stress parameter.

Equation (5) becomes, $\nabla \cdot J = 0$

Using equation (1) we get,

$$\sigma(\nabla^2\phi) + \nabla\phi\cdot\nabla\sigma = 0 \tag{14}$$

The boundary conditions on velocity, couple stress and electric potential after dimensionless are

$$\left.\begin{array}{l} u=0 \quad \text{at} \quad \eta=0 \\ u=0 \quad \text{at} \quad \eta=1 \end{array}\right\} \tag{15}$$

$$\frac{d^2u}{d\eta^2}=0 \quad \text{at} \quad \eta=0,1 \tag{16}$$

$$\left.\begin{array}{ll} \phi=x & \text{at} \quad \eta=0 \\ \phi=x-x_0 & \text{at} \quad \eta=1 \end{array}\right\} \tag{17}$$

The solution for ϕ, according to (14) depends on σ which in turn depends on the temperature T_b as in (12). In a poorly conducting fluid, $\sigma \ll 1$ and hence any perturbation on it is negligible and hence it depends on the conduction temperature T_b namely,

$$\frac{d^2T_b}{d\eta^2}=0 \tag{18}$$

with the boundary conditions

$$\left.\begin{array}{ll} T_b=T_0 & \text{at} \quad \eta=0 \\ T_b=T_1 & \text{at} \quad \eta=1 \end{array}\right\} \tag{19}$$

$$\text{is} \quad T_b-T_0=\Delta T\eta \tag{20}$$

Therefore equation (12) becomes

$$\begin{array}{l} \sigma=\sigma_0[1+\alpha_h\Delta T\eta]=\sigma_0(1+\alpha\eta)=\sigma_0 e^{\alpha\eta} \\ \sigma \approx e^{\alpha\eta} \ (\text{since } \alpha \ll 1) \end{array} \tag{21}$$

Where $\alpha=\alpha_h\Delta T$.

Then (14) using (21) we get

$$\frac{d^2\phi}{d\eta^2}+\alpha\frac{d\phi}{d\eta}=0 \tag{22}$$

Its solution satisfying the boundary condition (11) is

$$\phi=x-\frac{x_0}{1-e^{-\alpha}}[1-e^{-\alpha\eta}] \tag{23}$$

Using the dimensionless quantities and equation (23), equations (6), (7) and (8) reduce to

$$\rho_e=\nabla\cdot\vec{E}=-\nabla^2\phi=-\frac{x_0\alpha^2e^{-\alpha\eta}}{1-e^{-\alpha}}; \quad E_x=-1$$

Therefore

$$\rho_e E_x=\frac{x_0\alpha^2e^{-\alpha\eta}}{1-e^{-\alpha}} \tag{24}$$

The solution of equation (13) satisfying the condition (9) is

$$u = C_1\eta + C_2 + C_3 e^{\alpha\eta} + C_4 e^{-\alpha\eta} + \frac{a_0 e^{-\alpha\eta}}{\alpha^2(\alpha^2 - a^2)} - \frac{\eta^2}{2}\cdot P \tag{25}$$

where $a_0 = \frac{W_e x_0 \alpha^2}{1 - e^{-\alpha}}$.

The average velocity is given by,

$$\bar{u} = \frac{1}{2}\int_0^1 u\,d\eta$$

$$\bar{u} = \frac{k_1 k_2}{4} - \frac{k_3 k_2}{4} + \frac{k_4}{4} - \frac{k_4}{6} + \frac{k_1}{2}\left(1 + \frac{1}{\alpha} + \frac{e^{-\alpha}}{\alpha}\right)$$

$$+\frac{k_3}{2}\left(1 - \frac{1}{a} - \frac{e^{\alpha}}{a}\right) + \frac{k_5}{2}\left(1 - \frac{e^{-\alpha}}{a} - \frac{1}{a}\right) - \frac{k_6}{a}(\cosh a + 1) \tag{26}$$

Hence C, the concentration of aerosol in the atmosphere which diffuse in a fully developed flow, can be written as

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right). \tag{27}$$

The initial and boundary conditions for solving equation (27)

$$C(0, x, y) = C_0 \quad \text{for} \quad |x| \le \frac{1}{2}x_s, \tag{28a}$$

$$C(0, x, y) = 0 \quad \text{for} \quad |x| > \frac{1}{2}x_s, \tag{28b}$$

$$\frac{\partial C}{\partial y}(t, x, 0) = 0, \tag{28c}$$

$$\frac{\partial C}{\partial y}(t, x, h) = 0, \tag{28d}$$

$$\mathrm{C}(t,\infty, y) = \frac{\partial C}{\partial y}(t,\infty, y) = 0. \tag{28e}$$

Where C_0 is the concentration of the initial slug input of length x_s and equation (28a, 28b) represent the initial concentrations equation (28e) specifies that the concentration does not reach points far away downstream and equations (28c, 28d) specifies that there is no transfer of mass flux at the walls. We make equations (27) and (28a-e) dimensionless using

$$\theta = \frac{C}{C_0};\quad u^* = \frac{u}{\bar{u}};\quad \eta = \frac{y}{h};\quad X = \frac{Dx}{h^2\bar{x}};\quad X_s = \frac{Dx_s}{h^2\bar{u}};\quad P_e = \frac{\bar{u}h}{D};\quad \tau = \frac{tD}{h^2} \tag{29}$$

and obtain

$$\frac{\partial\theta}{\partial\tau} + u^*\frac{\partial\theta}{\partial\tau} = \frac{1}{P_e^2}\cdot\frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial\eta^2}. \tag{30}$$

Where $u^* = \frac{u}{\bar{u}}$, nondimensional velocity of atmospheric fluid. We define the axial coordinate moving with the average velocity of flow as $x_1 = x - \bar{u}t$ which in dimensionless from is $X_1 = X - \tau$, where $X_1 = \frac{x_1 D}{h^2 \bar{u}}$. Then equation (30) becomes

$$\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X_1} = \frac{1}{P_e^2} \cdot \frac{\partial^2 \theta}{\partial X_1^2} + \frac{\partial^2 \theta}{\partial \eta^2} \tag{31}$$

with $U = \frac{u - \bar{u}}{\bar{u}}$.

The non-dimensional initial and boundary conditions from equations (28a-28e) are given by

$$\theta(0, X_1,) = C_0 \quad \text{for} \quad |x| \le \frac{1}{2} x_s, \tag{32a}$$

$$\theta(0, X_1,) = C_0 \quad \text{for} \quad |x| \le \frac{1}{2} x_s, \tag{32b}$$

$$\frac{\partial \theta}{\partial \eta}(\tau, X_1, 0) = 0, \tag{32c}$$

$$\frac{\partial \theta}{\partial \eta}(\tau, X_1, 1) = 0, \tag{32d}$$

$$\theta(\tau, \infty, \eta) = \frac{\partial \theta}{\partial \eta}(\tau, \infty, \eta) = 0. \tag{32e}$$

The solution of equation (31) subject to the conditions (32a-e) are written, following Gill and Sankarasubramanian [6], as a series expansion in the form

$$\theta(\tau, X_1, \eta) = \theta_m(\tau, X_1) + f_1(\tau, \eta) \frac{\partial \theta_m}{\partial X_1}(\tau, X_1) + f_2(\tau, \eta) \frac{\partial^2 \theta_m}{\partial X_1^2}(\tau, X_1) + \ldots \tag{33}$$

Where θ_m is the dimensionless cross sectional average concentration and is given by

$$\theta_m(\tau, X_1) = \int_0^1 \theta(\tau, X_1, \eta)\, d\eta \tag{34}$$

Here equation (33) signifies that the difference between θ and its mean θ_m can be accounted by the convective and diffusive contributions. This is based on the observation of Taylor [12].

Integrating equation (31) with respect to η in [0, 1] and using the definition of θ_m given in equation (34) we get,

$$\frac{\partial \theta_m}{\partial \tau} = \frac{1}{P_e^2} \cdot \frac{\partial^2 \theta_m}{\partial X_1^2} + \int_0^1 \frac{\partial^2 \theta}{\partial \eta^2} d\eta - \frac{\partial}{\partial X_1} \int_0^1 U \theta d\eta \tag{35}$$

Substituting equation (33) in equation (35) we get

$$\frac{\partial \theta_m}{\partial \tau} = \frac{1}{P_e^2} \cdot \frac{\partial^2 \theta_m}{\partial X_1^2} - \frac{\partial}{\partial X_1} \int_0^1 U \left(\theta_m(\tau, X_1) + f_1(\tau, \eta) \frac{\partial \theta_m}{\partial X_1}(\tau, X_1) + \ldots \right) d\eta \tag{36}$$

One can introduce the generalized dispersion model with time-dependent dispersion coefficient as

$$\frac{\partial \theta_m}{\partial \tau} = K_1 \frac{\partial \theta_m}{\partial X_1} + K_2 \frac{\partial^2 \theta_m}{\partial X_1^2} + K_3 \frac{\partial^3 \theta_m}{\partial X_1^3} + \ldots \tag{37}$$

Comparing equation (36) and (37) we get

$$K_i(\tau) = \frac{\delta_{i2}}{P_i^2} - \int_0^1 U f_{i-1}(\tau, \eta)\, d\eta, \qquad (i = 1, 2, 3, \ldots) \tag{38}$$

Here $f_{-1} = 0$ and δ_{i2} is the Kroneckar delta.

Equation (37) is solved subject to the condition

$$\theta_m(0, X_1) = 1 \qquad |X| \le \frac{1}{2} X_s,$$

$$\theta_m(0, X_1) = 0 \qquad |X| > \frac{1}{2} X_s,$$

$$\theta_m(\tau, \infty) = 0 \tag{39}$$

Substituting equation (33) in equation (37) and following Gill and Sankarasubramanian (1970) using

$$\frac{\partial^{K+1}\theta_m}{\partial\tau\partial X_1^K} = \sum_{K=1}^{\infty} K_i(\tau) \frac{\partial^{K+i}\theta_m}{\partial X_1^{K+i}}.$$

We obtain

$$\left[\frac{\partial f_1}{\partial\tau} - \frac{\partial^2 f_1}{\partial\eta^2} + U + K_1(\tau)\right]\frac{\partial\theta_m}{\partial X_1} + \left[\frac{\partial f_2}{\partial\tau} - \frac{\partial^2 f_2}{\partial\eta^2} + U f_1 + K_1(\tau) f_1 + K_2(\tau) - \frac{1}{P_e^2}\right]\frac{\partial^2\theta_m}{\partial X_1^2}$$
$$+ \sum_{K=1}^{\infty}\left[\frac{\partial f_{K+2}}{\partial\tau} - \frac{\partial^2 f_{K+2}}{\partial\eta^2} + U f_{K+1} + K_1(\tau) f_{K+1} + \left(K_2(\tau) - \frac{1}{P_e^2}\right) f_K + \sum_{i=3}^{K+2} K_i f_{K+2-i}\right] = 0 \tag{40}$$

Equating the coefficients of $\frac{\partial^K\theta_m}{\partial X_1^K}$ ($K = 1, 2, 3, \ldots$) in equations (40) to zero, we obtain the following set of partial differential equations:

$$\frac{\partial f_1}{\partial\tau} = \frac{\partial^2 f_1}{\partial\eta^2} - U - K_1(\tau) \tag{41}$$

$$\frac{\partial f_2}{\partial\tau} = \frac{\partial^2 f_2}{\partial\eta^2} - U f_1 - K_1(\tau) f_1 - K_2(\tau) + \frac{1}{P_e^2} \tag{42}$$

$$\frac{\partial f_{K+2}}{\partial\tau} = \frac{\partial^2 f_{K+2}}{\partial\eta^2} - U f_{K+1} - K_1(\tau) f_{K+1} - \left(K_2(\tau) - \frac{1}{P_e^2}\right) f_K + \sum_{i=3}^{K+2} K_i f_{K+2-i} \tag{43}$$

We note that to evaluate K_i's we need to know the f_K's. For this, solve equation (43) for f_K's subject to the initial and boundary conditions.

$$f_K(o, \eta) = \delta_{K_0}, \qquad K = 1, 2, 3, \ldots \tag{44a}$$

$$\frac{\partial f_K}{\partial\eta}(\tau, 0) = 0 \tag{44b}$$

$$\frac{\partial f_K}{\partial\eta}(\tau, 1) = 0 \qquad K = 1, 2, 3, \ldots \tag{44c}$$

Further, $$\int_0^1 f_K(\tau, \eta)\, d\eta = 0 \qquad K = 1, 2, 3, \ldots \tag{44d}$$

From equation (38) for $i = 1$, using $f_0 = 1$, we get K_1 as

$$K_1(\tau) = 0. \tag{45}$$

From equation (38) and (45) we get

$$K_2(\tau) = \frac{1}{P_e^2} - \int_0^1 U f_1 \, d\eta \tag{46}$$

First, we have to solve equation (41) for satisfying the conditions (44a) because equation (42) and (46) require f_1 to find $K_2(\tau)$. For this, as before we write

$$f_1 = f_{10}(\eta) + f_{11}(\tau, \eta). \tag{47}$$

Where $f_{10}(\eta)$ corresponds to an infintely wide slug which is independent of τ satisfying

$$\frac{df_{10}}{d\eta} = 0 \quad \text{at} \quad \eta = 0 \tag{48a}$$

$$f_{10} = 0 \quad \text{at} \quad \eta = 1 \quad \text{and} \tag{48b}$$

$$\int_0^1 f_{10} \, d\eta = 0 \tag{48c}$$

$f_{11}(\eta, \tau)$ is τ dependent. Then equation (41) using (47) becomes

$$\frac{d^2 f_{10}}{d\eta^2} = \frac{1}{F_1}[(k_1k_2 - k_3k_2 + k_4)\eta - k_4\eta^2 + k_1e^{\alpha\eta} + (k_5 - k_3 - k_6)\, e^{a\eta} + k_6e^{a\eta} + k_6e^{-a\eta} + F_0] \tag{49}$$

$$\frac{\partial f_{11}}{\partial \tau} = \frac{\partial^2 f_{11}}{\partial \eta^2} \tag{50}$$

Solution of equation (49) satisfying the conditions (48a, 48b)

$$f_{10} = \frac{1}{F_1}\left[(k_1k_2 - k_3k_2 + k_4)\frac{\eta^3}{6} - k_4\frac{\eta^4}{12} + \frac{k_1e^{-\alpha\eta}}{\alpha^2} - (k_5 - k_3 + k_6)\frac{e^{a\eta}}{a^2} + \frac{k_6e^{-a\eta}}{a^2} + \frac{F_0\eta^2}{2}\right] + \frac{1}{F_1}\left[\frac{k_1}{\alpha} - \frac{(k_5 - k_3 - k_6)}{a} + \frac{k_6}{a}\right]\eta$$
$$- \frac{1}{F_1}\left[(k_1k_2 - k_3k_2 + k_4)\frac{1}{6} - \frac{k_4}{12} + \frac{k_1e^{-\alpha}}{\alpha^2} - (k_5 - k_3 + k_6)\frac{e^{a}}{a^2} + k_6\frac{e^{-a}}{a} + \frac{F_0}{2} + \frac{k_1}{\alpha} - \frac{(k_5 - k_3 - k_6)}{a} + \frac{k_6}{a}\right] \tag{51}$$

Equation (50) is the well-known heat conduction equation and is solved by using separation of variables.

The solution of equation (50) satisfying the condition $f_{11}(\tau, \eta) = -f_{10}(\eta)$ is given by

$$f_{11} = \sum_{n=1}^{\infty} A_n e^{-\lambda_n^2 \tau} \cos \lambda_n \eta \tag{52}$$

where

$$A_n = -\int_0^1 f_{10}(\eta) \cos(\lambda_n \eta) \, d\eta \tag{53}$$

and $\lambda_n = n\pi$. Hence

$$f_1 = \frac{1}{F_1}\left[(k_1k_2 - k_3k_2 + k_4)\frac{\eta^3}{6} - k_4\frac{\eta^4}{12} + \frac{k_1e^{-\alpha\eta}}{\alpha^2} - (k_5 - k_3 + k_6)\frac{e^{a\eta}}{a^2}\right.$$
$$\left. + \frac{k_6e^{-a\eta}}{a^2} + \frac{F_0\eta^2}{2}\right] + F_2\eta + F_3 + \sum A_n e^{-\lambda_n^2\tau}\cos(\lambda_2\eta) \tag{54}$$

and
$$K_2(\tau) = \frac{1}{P_e^2} - \int_0^1 f_1 \cdot U d\eta \tag{55}$$

Substituting for $f_1(\eta)$ from equation (54) in equation (55) and completing integration, we get the dispersion coefficient. The lengthy expression of the same is calculated using Mathematica 7.0 software and numerical results are obtained given in the appendix.

Similarly, $K_3(\tau)$, $K_4(\tau)$ and so on are obtained and we found that $K_i(\tau)$, $i > 2$ are negligibly small compared to $K_2(\tau)$. Hence the dispersion model (37) now leads to

$$\frac{\partial\theta_m}{\partial\tau} = K_2\frac{\partial^2\theta_m}{\partial X_1^2}.$$

Whose solution is $\theta m(X_1, \tau) = \frac{1}{2}\left[erf\left(\frac{\frac{X_S}{2} + X_1}{2\sqrt{T}}\right) + erf\left(\frac{\frac{X_S}{2} - X_1}{2\sqrt{T}}\right)\right].$

Where $T = \int_0^\tau K_2(y)\,dy$ and $erf(x)\frac{1}{\pi}\int_0^x e^{-z^2}dz.$

3. RESULTS AND DISCUSSIONS

One of the by products of modern development is the atmospheric pollution. This pollution has great impact on future of mankind, overtaking infections diseases and nutritional disorders. Many scientists and engineers are working hard to find causes for atmospheric pollution and methods to control them. Therefore, it is essential to monitor the atmospheric pollution due to pollutants.

The axial dispersion in a couple stress fluid bounded by electrodes is studied using generalized dispersion model. The dominent dispersion coefficient given by (55) is computed for different values of couple stress 'a' and the dimensionless time 'τ'. The results are graphically represented in Fig. 2. It show that for non zero values of electric number 'W_e' that is in the presence of electric field, the dispersion coefficient $K_2(\tau) - P_e^{-2}$ increases with the increase in 'a'. Further, $K_2(\tau) - P_e^{-2}$, values are spaced out for small values of time $\tau \ll 0.45$ and becomes steady and are closer when $\tau \ll 0.75$. Figs 3 and 4 depict the mean concentration θ_m with axial distance x for different values of 'a' for fixed τ. These figures reveal that there is marked variation of concentration with the axial distance and time. It is apparent from these figures that the effect of increasing 'a' is to decrease the peak value of the mean concentration. This implies that the concentration is more distributed in the x-direction for larger and larger values of Peclet number 'P_e'. Figure 5 is the plot of mean concentration versus τ for different values of 'a'. We observe from these figures that the concentration at the points outside the input slug ($x > X_s$) are obviously less than those at the points inside the slug. We also see that it decreases with the increase in τ for points inside and outside the slug.

The parameters that influence K_2 and θ_m are the couplestress parameter 'a' and electric number 'W_e'. Here the behaviour of K_2 is to increase with the increase in 'a'. We see that as τ increases θ_m tends asymptotically to zero. The proposed model and analysis presented here also suggests that the pollutants can be removed from the atmosphere and this rate of removal would depend upon the rate of introduction of the pollutant and other parameters.

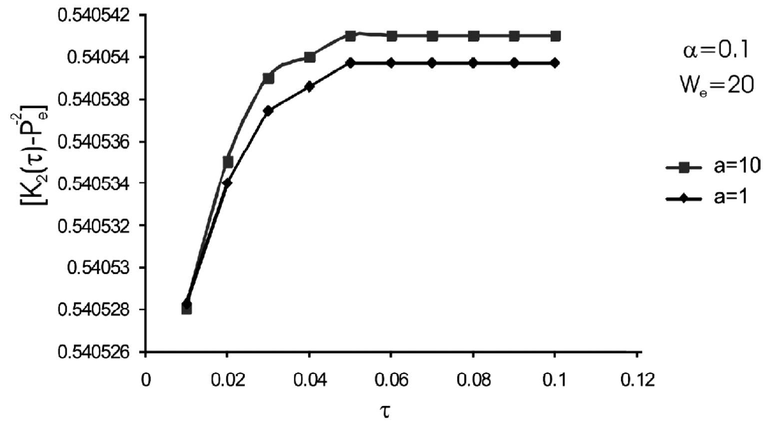

Figure 2: Effect of Couple Stress on Unsteady Dispersion Coefficient ($K_2(\tau)$)

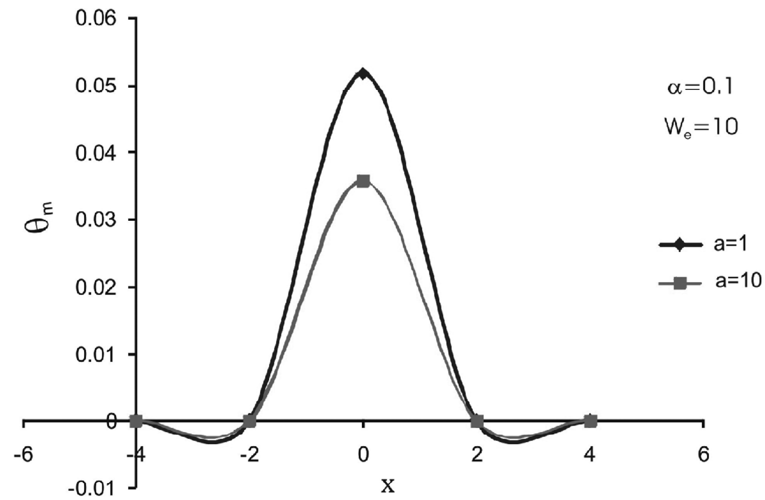

Figure 3: Mean Concentration (θ_m) Varying Along Axial Distance for Different Values of 'a' and for Fixed $X_s = 0.019$, $\tau = 0.3$ and $P_e = 10$

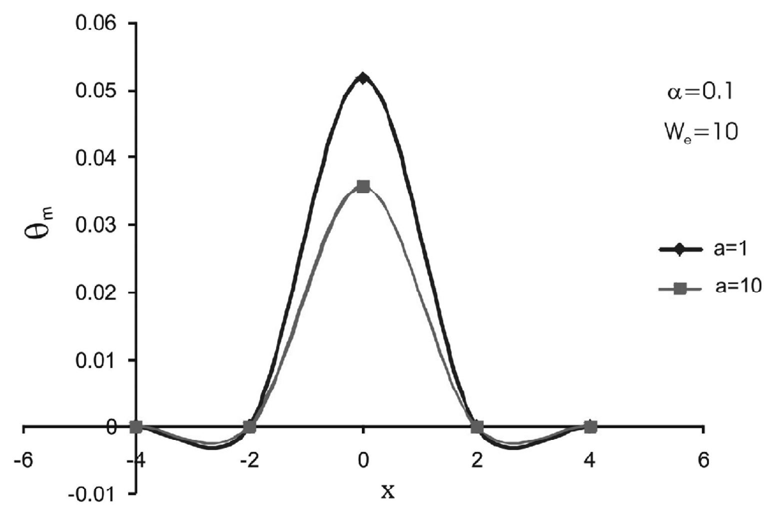

Figure 4: Mean Concentration m Varying Along Axial Distance for Different Values of 'a' and for Fixed $X_s = 0.019$, $\tau = 0.03$, $P_e = 10$

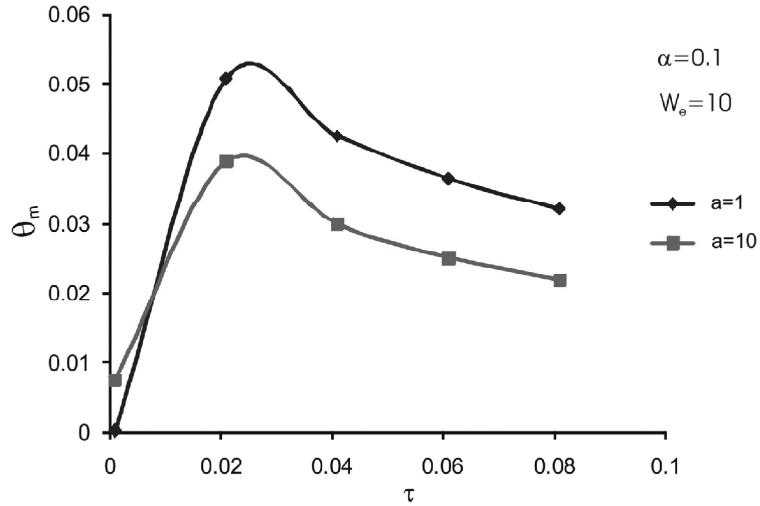

Figure 5: Mean Concentration θ_m Varying Along Dimensionlesstime τ at X_s = 0.019, X = 0.1 and P_e = 10

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P. Meena Priya[*] & Nirmala P. Ratchagar[**]
Mathematics Section, Faculty of Engineering and Technology,
Annamalai University, Annamalainagar-608 002, India.
E-mails: [*]meenapriya au@yahoo.com
[**]nirmalapasala@yahoo.co.in

APPENDIX

$$k_1 = \frac{a_0}{\alpha^2(\alpha^2 - a^2)}$$

$$k_2 = 1 - e^{-\alpha}$$

$$k_3 = \frac{a_0}{a^2(\alpha^2 - a^2)}$$

$$k_4 = \frac{P}{2}$$

$$k_5 = \frac{P}{a^2}$$

$$k_6 = \frac{1}{2\,\mathrm{Sin}h[a]}\left(\frac{a_0(e^{-\alpha} - e^a}{a^2(\alpha^2 - a^2)} + \frac{a^2P(e^a - 1)}{a^4}\right)$$

$$k_7 = \frac{\alpha + 1 + e^{-\alpha}}{2\alpha}$$

$$k_8 = \frac{a - 1 - e^{-a}}{2a}$$

$$k_9 = \frac{1 + \mathrm{Cos}h[a]}{a}$$

$$k_{10} = \frac{1}{2} - \frac{e^{-\alpha}}{\alpha^2} - \frac{1}{\alpha}$$

$$k_{11} = \frac{1}{2} - \frac{e^a}{a^2} + \frac{1}{a}$$

$$k_{12} = \frac{2\,\mathrm{Sin}h[a]}{a^2} - \frac{2}{a}$$

s_2 = Subscript [λ, y] = $y\pi$

s_3 = Sum [s_2, {y, 1, 1000, 1}]

$$A = \left(\frac{1}{F_1}\left((k_1k_2 - k_3k_2 + k_4)\frac{1}{2} - \frac{k_4}{3} - k_1\frac{e\hat{} - \alpha}{\alpha} + (k_5 - k_3 - k_6)\frac{e^a}{a} - k_6\frac{e^{-a}}{a} + F_0\right)\left(-\frac{\mathrm{Cos}[s_3]}{s_3\hat{}2}\right.\right.$$

$$\left.\left. - F_2\left(\frac{\mathrm{Cos}[s_3]}{s_3\hat{}2}\right) + \frac{1}{(s_3\hat{}2)F_1}\left(\frac{-k_1}{\alpha} + \frac{(k_5 - k_3 - k_4)}{a} - \frac{k_6}{a}\right) + \frac{F_2}{s_3\hat{}2_1}\right)$$

$$F_0 = \left(k_3 + k_1k_7 + k_5k_8 + k_6k_9 - k_1 - k_3k_8 - k_5 - \frac{3}{12}(k_1k_2 - k_3k_2) + \frac{k_4}{12}\right)$$

$$F_1 = \left(\frac{3}{12}(k_1k_2 - k_3k_2) + \frac{k_4}{12} - k_1k_7 + k_3k_8 - k_5k_8 - k_6k_9\right)$$

$$F_2 = \frac{1}{F_1}\left(\frac{k_1}{\alpha} - \frac{(k_5 - k_3 - k_6)}{a} + \frac{k_6}{a}\right)$$

$$F_3 = -\frac{1}{F_1}\left((k_1 k_2 - k_3 k_2 + k_4)\frac{1}{6} - \frac{k_4}{12} + \frac{k_1 e^{-\alpha}}{\alpha^2} - (k_5 - k_3 + k_6)\frac{e^a}{a^2} + k_6 \frac{e^{-a}}{a^2} + \frac{F_0}{2} + \frac{k_1}{\alpha} - \frac{(k_5 - k_3 - k_6)}{a} + \frac{k_6}{a}\right)$$

$$F_4 = (k_1 k_2 - k_3 k_2 + k_4)$$

$$F_5 = (k_5 - k_3 - k_6)$$

$$F_6 = \left(\frac{F_3 F_5}{F_1} - \frac{F_0 F_5}{F_1^2 a^2}\right)$$

$$F_7 = \left(\frac{F_3 k_6}{F_1} + \frac{F_0 k_6}{F_1^2 a^2}\right)$$

$$F_8 = \left(\frac{F_3 k_1}{F_1} + \frac{F_0 k_1}{F_1^2 \alpha^2}\right)$$

$$F_9 = \left(-\frac{F_5 k_1}{F_1^2 a^2} + \frac{F_5 k_1}{F_1^2 \alpha^2}\right)$$

$$F_{10} = \left(\frac{k_1 k_6}{F_1^2 a^2} + \frac{k_1 k_6}{F_1^2 \alpha^2}\right)$$

$$F_{11} = \left(\frac{F_0 F_5}{2F_1^2} + \frac{F_5 k_4}{F_1^2 a^2}\right)$$

$$F_{12} = \left(\frac{F_0 k_6}{2F_1^2} - \frac{k_4 k_6}{F_1^2 a^2}\right)$$

$$F_{13} = \left(\frac{F_0 k_1}{2F_1^2} - \frac{k_1 k_4}{F_1^2 \alpha^2}\right)$$

$$F_{14} = \left(\frac{F_4^2}{6F_1^2} - \frac{7F_0 k_4}{12F_1^2}\right)$$

$$F_{15} = \left(\frac{F_0 F_2}{F_1} + \frac{F_3 F_4}{F_1} + \frac{1}{F_1}(\mathrm{Cos}[e\text{^} - s_3\text{^}s_1])\, F_0(s_3\ \mathrm{Sum}[A,\{n,1,1000,1\}])\right)$$

$$F_{16} = \left(\frac{F_0^2}{2F_1^2} + \frac{F_2 F_4}{F_1} - \frac{F_3 k_4}{F_1} + \frac{1}{F_1}(\mathrm{Cos}[e\text{^} - s_3\text{^}s_1])\, F_3(s_3)\ \mathrm{Sum}[A,\{n,1,1000,1\}]\right)$$

$$F_{17} = \left(\frac{F_2 F_5}{F_1} - \frac{F_4 F_5}{F_1^2 a^2} + \frac{1}{F_1}(\mathrm{Cos}[e\text{^} - s_3\text{^}s_1])\, F_5(s_3\ \mathrm{Sum}[A,\{n,1,1000,1\}])\right)$$

$$F_{18} = \left(\frac{F_2 k_1}{F_1} + \frac{F_4 k_1}{F_1^2 \alpha^2} + \frac{1}{F_1} (\mathrm{Cos}\,[e^\wedge - s_3])\, k_1 (s_3 \ \mathrm{Sum}\,[A, \{n, 1, 1000, 1\}]) \right)$$

$$F_{19} = \left(\frac{2F_0 F_4}{3F_1^2} - \frac{F_2 k_4}{F_1} - \frac{1}{F_1} (\mathrm{Cos}\,[e^\wedge - s_3{}^\wedge s_1])\, k_1 s_3 \ \mathrm{Sum}\,[A, \{n, 1, 1000, 1\}] \right)$$

$$F_{20} = \left(\frac{F_2 k_6}{F_1} + \frac{F_4 k_6}{F_1^2 a^2} + \frac{1}{F_1} (\mathrm{Cos}\,[e^\wedge - s_3{}^\wedge s_1])\, k_6 s_3 \ \mathrm{Sum}\,[A, \{n, 1, 1000, 1\}] \right)$$

$$K_2(\tau) = \frac{1}{pe^\wedge 2} + \left(\left(\left(- \left(\frac{1}{12 F_1^2 a^2 \alpha^2} \left(\frac{6k_1^2 a^2}{\alpha} - \frac{6e^{-2\alpha} k_1^2 a^2}{\alpha} + \frac{6F_5^2 \alpha^2}{a} - \frac{6e^{2a} F_5^2 \alpha^2}{a} + \frac{6k_6^2 \alpha^2}{a} - \frac{6e^{-2a} k_6^2 \alpha^2}{a} \right.\right.\right.\right.\right.$$

$$+ \frac{1}{70} (14 F_1^2 (12 F_{14} + 5(6F_{15} + 4F_{16} + 3F_{19})) + 840 F_0 F_1 F_3 + 5k_4(-7F_4 + 2k_4))\, a^2 \alpha^2$$

$$\frac{12(F_5(2k_4 + F_4 a) + F_1^2 a^2 (-2F_{11} + F_{17} a - F_{16} a^2))\alpha^2}{a^3} + \frac{12(-2k_4 k_6 + F_4 k_6 a + F_1^2 a^2 (2F_{12} + F_{20} a + F_7 a^2))\alpha^2}{a^3}$$

$$+ \frac{1}{a^3} e^a \,(12 F_1^2 a^2 (-F_{17} a + F_{17} a^2 + F_6 a^2 + F_{11}(2 - 2a + a^2)) - F_5(-2F_4 a(-6 + 6a - 3a^4 + a^3)$$

$$+ k_4 (24 - 24a + 12a^2 - 4a^3 + a^2)))\alpha^2 + \frac{1}{a^3} e^{-a} \,(24 + 24a + 12a^2 + 4a^3 + a^4) - 2(F_4 k_6 a (6 + 6a + 3a^2 + a^3)$$

$$+ 6F_1^2 a^2 (F_7 a^2 + F_{20}(a + a^2) + F_{12}(2 + 2a + a^2))))\,\alpha^2 - \frac{12 F_1^2 F_9 a^2 \alpha^2}{a - \alpha} + \frac{12 e^{a^2 - \alpha} F_1^2 F_9 a^2 \alpha^2}{a - \alpha}$$

$$+ \frac{12 F_1^2 F_{10} a^2 \alpha^2}{a + \alpha} - \frac{12 e^{-a - \alpha} F_1^2 F_{10} a^2 \alpha^2}{a + \alpha} + \frac{12 a^2 (k_1(-2k_4 + F_4 \alpha) + F_1^2 \alpha^2 (2F_{13} + \alpha(F_{18} + F_8 \alpha)))}{\alpha^3}$$

$$+ \frac{1}{\alpha^3} e^{\alpha} a^2 \,(-12 F_1^2 \alpha^2 (\alpha (F_{18} + F_{18}\alpha + F_8 \alpha) + F_{13}(2 + 2\alpha + \alpha^2)) + k_1(-2F_4 \alpha (6 + 6\alpha + 3\alpha^2 + \alpha^3)$$

$$+ k_4 (24 + 24\alpha + 12\alpha^2 + 4\alpha^3 + \alpha^4)))))))$$