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NEW CLASSES OF GRACEFUL AND ODD GRACEFUL GRAPHS

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ABSTRACT: In this Paper we proved that the Graphs $G = P_m(QS_n)$ for every $m \geq 2$, $n \geq 1$, $G = S_t \cup K_{m,n}$, $t > 3$ for every m, n, $G = S_t \cup K_{m,n} \cup P_u$, $t \geq 4$, $u \geq 3$ for every m, n are Odd Graceful graphs and $G = S_m(QS_n)$, for every$m \geq 4$, $n \geq 1$ is Graceful graph.

***AMS Subject Classification*:** 05C78.

***Keywords*:** Odd graceful labeling, Graceful labeling, Path related graphs, Complete bipartite graph, Star graph, Quadrilateral snake graph.

1. INTRODUCTION

A function f is called an Graceful labeling of a graph G with q edges. If f is an injection from the vertices of G to the set $\{0, 1, 2, ..., q\}$ such that when each edge uv is assigned the label $|f(u) - f(v)|$, the resulting edge labels are distinct.

A graph which admits a Graceful labeling is called a Graceful Graph.

A Graph G with 'q' edges to be Odd Graceful if there is an injection f from $V(G)$ to $\{0, 1, 2, 3, ..., 2q - 1\}$ such that when each edge xy is assigned the label $|f(x) - f(y)|$, the resulting edge labels are $\{1, 3, 5, ..., 2q - 1\}$.

A Graph which admits an Odd Graceful labeling is called an Odd graceful graph.

Many mathematicians have constructed a larger graceful graph from certain standard graphs by using various graph operations. Join and product operations are used extensively among the graphs such as paths, cycles, stars, complete graphs, completebipartite graphs, complement of complete graphs and graceful trees etc., to get larger graceful or harmonious graph etc., (refer Acharya and Gill (1981), Balakrishnan and Kumar (1994), Bu (1994), Frucht and Gallian (1988), Grace (1983), Jungreis and Reid (1992)). On the other hand many copies of certain standard graphs, such as complete graphs, complete bipartite graphs, cycles etc., are adjoined at one common vertex have been proved to be graceful or harmonious or felicitous (refer Bermond *et al.*, (1978), Bodendiek *et al.*, (1975), Huang and Skiena (1994), Kathiresan (1992), Koh *et al.*, (1979)). Similarly, many copies of certain graphs, like complete graph K_4, edge deleted subgraphs of complete graph K_4, cycle C_n with $n - 3$ consecutive chords etc., (refer Delorme (1980), Sethuraman and Dhavamani (2000), Sethuraman and Kishore (1999)) are adjoined at one common edge and the resultant graphs are proved to be graceful. For an exhaustive survey of these topics one may refer to the excellent survey paper of Gallian (2000). We introduce a new method of construction of graph.

We consider the notation QS_n as Quadrilateral Snake with 'n' number of C_4 attached in series connection as defined in the following diagram.

Figure 1.1 Quadrilateral Snake Graph QS_4

Theorem 1: The Graph $G = P_m(QS_n)$ for every $m \geq 2$, $n \geq 1$ is Odd Graceful graph.

Proof: Let $G = P_m(QS_n)$ = has $m(3n + 1)$ vertices and $m(4n + 1) - 1$ edges.

The graph G is Quadrilateral snake attached with each vertex of path P_m, n is the number of C_4 attached in series connection. The graph G as shown in the following diagram

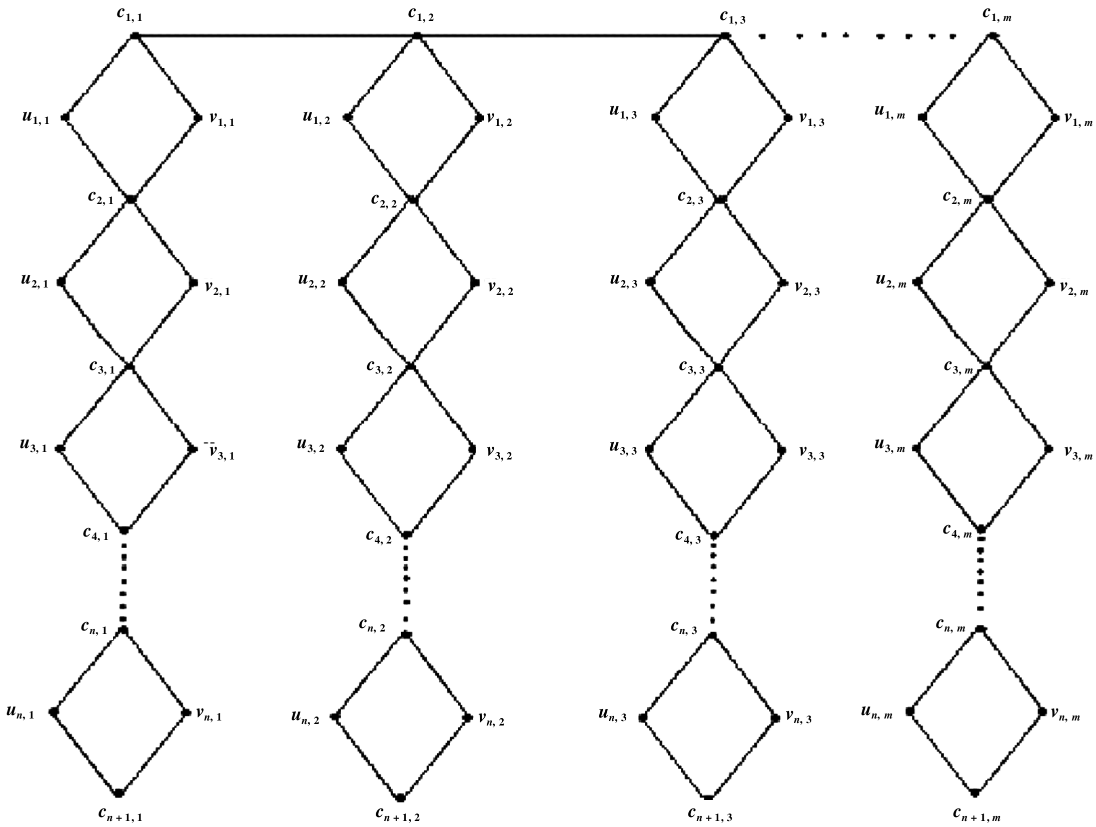

Figure 1.2: The Graph $G = P_m(QS_n)$

The Odd Graceful labeling for vertices of G is defined by

$$c_{i,j} = \begin{cases} (j-1)(4n+1)+2(i-1), & i=1,2,3,...,n+1, \quad j=1,2,3,...,m, \quad j \text{ is odd} \\ (4n+1)(2m-j+2)-8n+2i-5, & i=1,2,3,...,n+1, \quad j=1,2,3,...,m, \quad j \text{ is even} \end{cases}$$

$$u_{i,j} = \begin{cases} 2m(4n+1)-3-4n(j-1)-2(i-1)-(j-1), & i=1,2,3,...,n, \quad j=1,2,3,...,m, \quad j \text{ is odd} \\ 4n+(4n+1)(j-2)-2(i-1), & i=1,2,3,...,n, \quad j=1,2,3,...,m, \quad j \text{ is even} \end{cases}$$

$$v_{i,j} = \begin{cases} 2m(4n+1)-3-4n-(j-1)\ (4n+1)-2(i-1), & i=1,2,3,...,n, \quad j=1,2,3,...,m, \quad j \text{ is odd} \\ 8n+(4n+1)(j-2)-2(i-1), & i=1,2,3,...,n, \quad j=1,2,3,...,m, \quad j \text{ is even} \end{cases}$$

from the above assignment the labeling of vertices and edges are distinct.

Hence the Graph $G = P_m(QS_n)$ is Odd Graceful graph.

Example: $G = P_5(QS_4)$

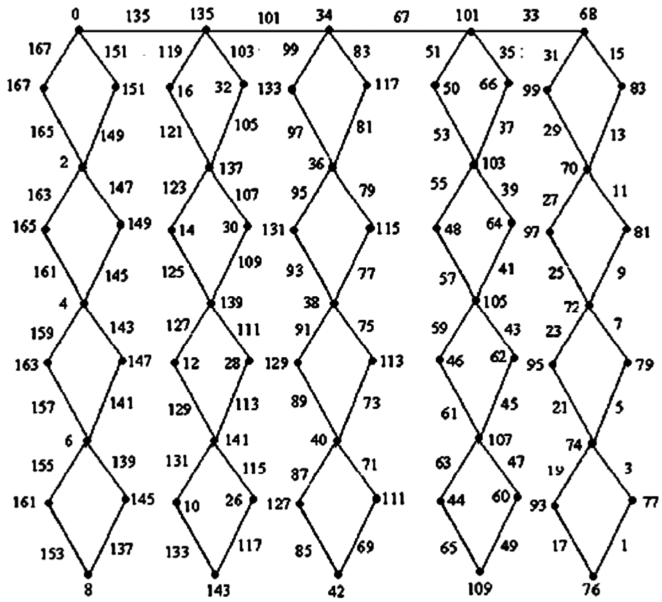

Figure 1.3: Odd Graceful Labeling of Graph $G = P_5(QS_4)$

Theorem 2: The Graph $G = S_m(QS_n)$ for every $m \geq 4$, $n \geq 1$ is Graceful graph.

Proof: Let $G = S_m(QS_n)$ has $t(3n + 1) + 1$ vertices and $t(4n + 1)$ edges. The graph G is Quadrilateral snake attached with each pendent vertex of Star S_m.

't' be the number of pendent vertex of Star S_m. The graph G as shown in the following diagram.

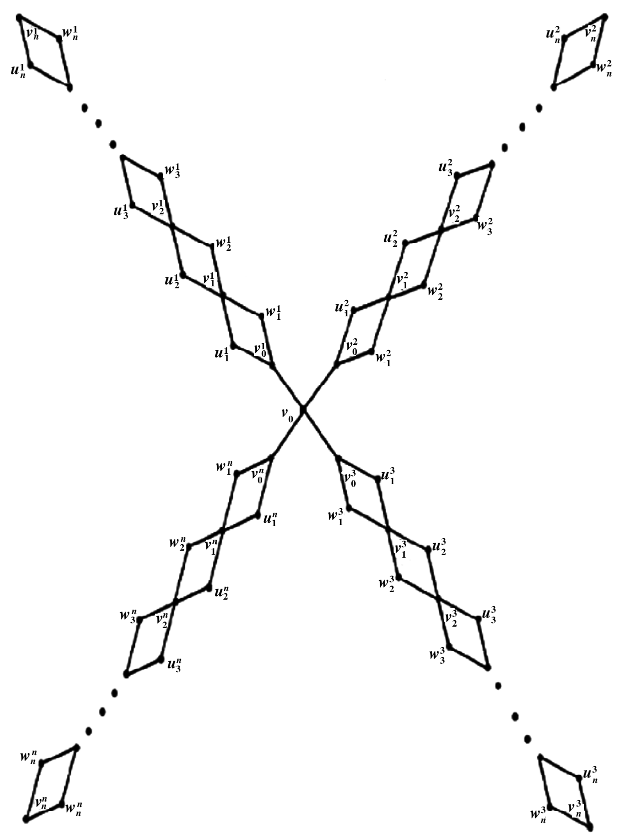

Figure 2.1: The Graph $G = S_m(QS_n)$

Graceful labeling for vertices of G is defined by

$$v_0 = 0$$

$$v_i^j = \begin{cases} (4n+1)(t-j+1)-2i, & 0 \le i < n, \quad 1 \le j < t \\ (4n+1)(t-j+1)-2i+3, & i = n, \quad j = t \end{cases}$$

$$u_i^j = \begin{cases} (4n+1)(j-1)+2i-1, & 1 \le i < n, \quad 1 \le j < t \\ 2+nt-2n+t-1, & i = n, \quad j = t \end{cases}$$

$$w_i^j = \begin{cases} (4n+1)(j-1)+2i, & 1 \le i < n, \quad 1 \le j < t \\ (4n+1)(j-1)+2i+2, & i = n, \quad j = t \end{cases}$$

From the above assignment the labeling of vertices and edges are distinct.

Hence the Graph $G = S_m(QS_n)$ is Graceful graph.

Example: $G = S_9(QS_4)$

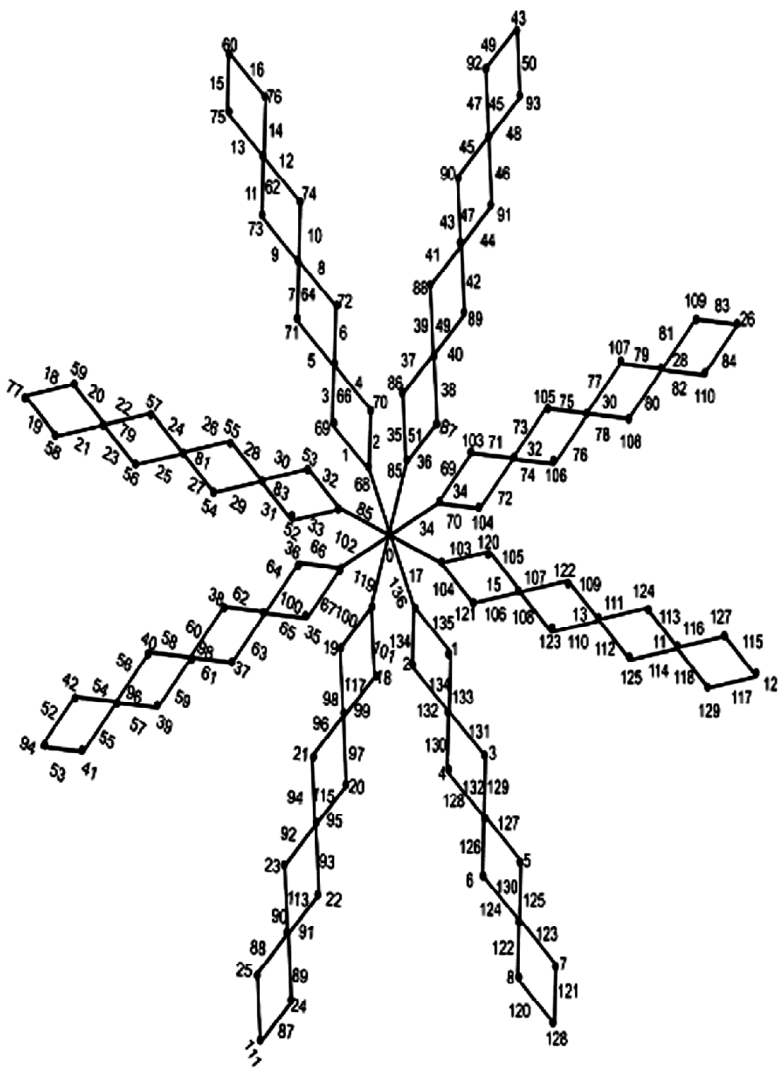

Figure 2.2: Odd Graceful Labeling of Graph $G = S_9(QS_4)$

Theorem 3:

The Graph $G = S_t \cup K_{m,n}$, $t > 3$, $\forall m, n$ is Odd Graceful graph

Proof: Let $G = S_t \cup K_{m,n}$ has $m + n + t$ vertices and $mn + t - 1$ edges.

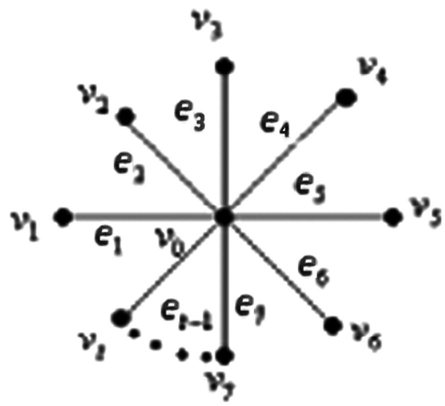

Figure 3.1: The Graph S_t

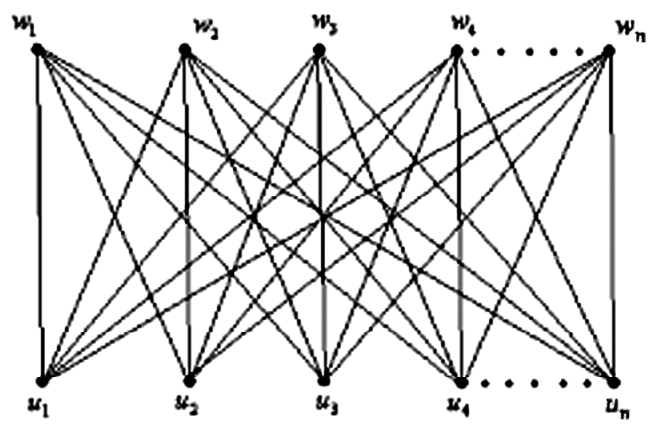

Figure 3.2: The Graph $K_{m,n}$

Odd Graceful labeling for vertices of $G = S_t \cup K_{m,n}$ is defined by

$$v_0 = 0$$

$$v_i = 2mn + 2t - 2i - 1, \qquad i = 1, 2, 3, ..., t - 1$$

$$u_j = 2mn - 2j + 4, \qquad j = 1, 2, 3, ..., m$$

$$w_k = 3 + 2mk - 2m, \qquad k = 1, 2, 3, ..., n$$

Edge labelings are defined by

$$e_i = 2mn + 2t - 2i - 1, \qquad i = 1, 2, 3, ..., t - 1$$

$$e_{i,j} = 2mn - 2i + 1 - 2mj + 2m, \quad i = 1, 2, 3, ..., m \;\&\; j = 1, 2, 3, ..., n$$

From the above assignment, the labeling of vertices and edges are distinct.

Hence the graph $G = S_t \cup K_{m,n}$ is Odd Graceful graph.

Example: $G = S_5 \cup K_{3,6}$

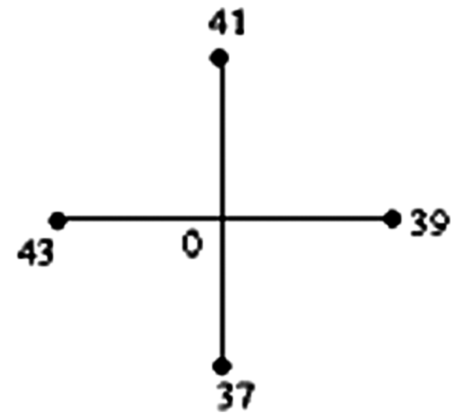

Figure 3.3: Odd Graceful Labeling of Graph S_5

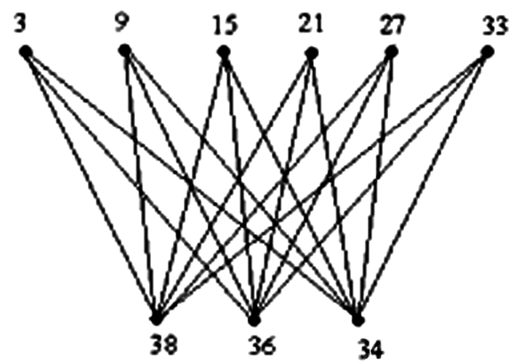

Figure 3.4: Odd Graceful Labeling of The Graph $K_{3,6}$

Odd graceful labeling of The Graph $G = S_5 \cup K_{3,6}$

Theorem 4: The Graph $G = S_t \cup K_{m,n} \cup P_u$, $u \geq 4$, $t \geq 3$, for every m, n is Odd Graceful graph.

Proof: Let $G = S_t \cup K_{m,n} \cup P_u$ has $t + m + n + u$ vertices and $t + mn + u - 2$ edges

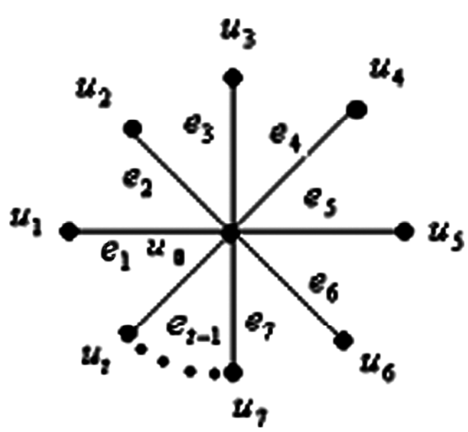

Figure 4.1: The Graph S_t

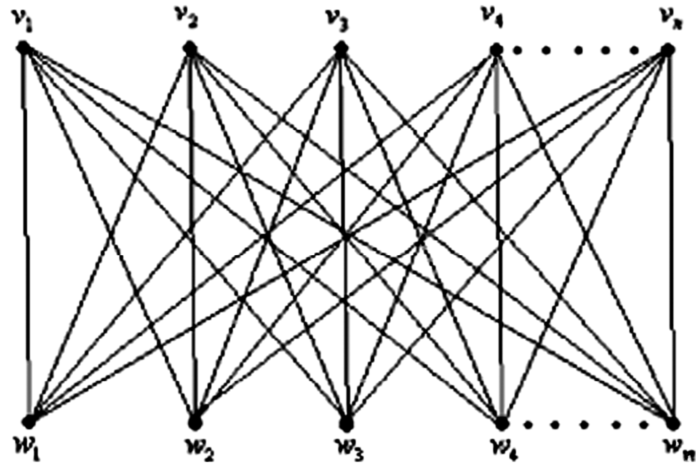

Figure 4.2: The Graph $K_{m,n}$

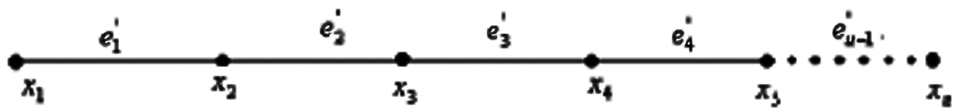

Figure 4.3: The Graph P_u

The Graph $G = S_t \cup K_{m,n} \cup P_u$

Odd Graceful labeling for vertices of $G = S_t \cup K_{m,n} \cup P_u$ is defined by

$$u_0 = 0$$

$$u_i = 2t + 2mn + 2u - 2i - 3, \qquad i = 1, 2, 3, \ldots, t-1$$

$$v_j = 3 + 2n(j-1), \qquad j = 1, 2, 3, \ldots, n$$

$$w_k = 2mn + 2u - 2k + 2, \qquad k = 1, 2, 3, \ldots, m$$

$$x_{2p-1} = \begin{cases} & u \text{ is odd}, \quad p = 1, 2, 3, \ldots, \frac{u+1}{2} \\ 2mn - 2m + 2p & \\ & u \text{ is even}, \quad p = 1, 2, 3, \ldots, \frac{u}{2} \end{cases}$$

$$x_{2p} = \begin{cases} & u \text{ is odd}, \quad p = 1, 2, 3, \ldots, \frac{u-1}{2} \\ 2mn + 2u - 2m - 2p + 1 & \\ & u \text{ is even}, \quad p = 1, 2, 3, \ldots, \frac{u}{2} \end{cases}$$

Edge labelings are defined by

$$e_i = 2t + 2mn + 2u - 2i - 3, \qquad i = 1, 2, 3, \ldots, t-1$$

$$e_i' = 2u - 2i - 1, \qquad i = 1, 2, 3, \ldots, u-1$$

$$e_{i,j} = 2mn + 2u - 2i - 2mj + 2m - 1, \qquad i = 1, 2, 3, \ldots, n, \; j = 1, 2, 3, \ldots, n$$

From the above assignment the labeling of vertices and edges are distinct.

Hence the Graph $G = S_t \cup K_{m,n} \cup P_u$ is Odd Graceful graph.

Example: $G = S_9 \cup K_{3,5} \cup P_9$

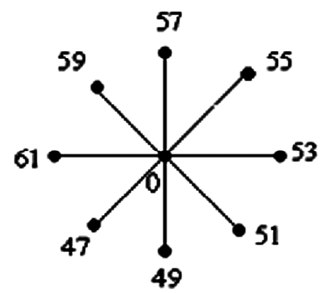

Figure 4.4: Odd Graceful Labeling of the Graph S_9

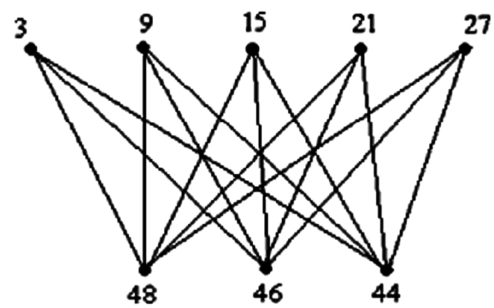

Figure 4.5: Odd Graceful Labeling of Graph $K_{3,5}$

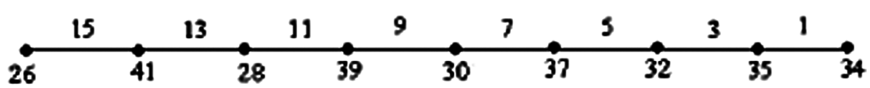

Figure 4.6: Odd Graceful Labeling of Graph P_9

Odd Graceful labeling for vertices of $G = S_9 \cup K_{3,5} \cup P_9$

2. CONCLUSION

In this Paper we have given Odd Graceful labeling for the graph $G = P_m(QS_n)$ for every $m \geq 2$, $n \geq 1$, $G = S_t \cup K_{m,n}$, $t > 3$, $\forall m, n$, $G = S_t \cup K_{m,n} \cup P_u$, $t \geq 4$, $u \geq 3$, for every m, n are Odd Graceful graphs and $G = S_m(QS_n)$, for every $m \geq 4$, $n \geq 1$ is Graceful graph. We pose that " Any other graph construction that satisfies the Odd Graceful graphs.

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