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INTUITIONISTIC FUZZY LATTICES

ABSTRACT: *We provide a general development of the theory of intuitionistic fuzzy sublattices. We prove structural theorems for intuitionistic fuzzy sublattices and intuitionistic fuzzy ideals (filter) generated by an intuitionistic fuzzy set. Various intuitionistic fuzzy analogs of the results of classical theory are established. Also we deal with the homomorphic images and pre-images of intuitionistic fuzzy sublattices.*

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1. INTRODUCTION

The theory of fuzzy sets proposed by Zadeh[24] in 1965 has achieved a great success in various fields. After that time, some autors[20, 21] applied this concept to group theory. In particular, Yuan and Wu[21] investigated fuzzy ideals and fuzzy congruences on a distributive lattice. Moreover, Ajmal and Thomas[1] applied the notion of fuzzy sets to lattice theory.

With the research of fuzzy sets, in 1986, Atanassov[2] presented intuitionistic fuzzy sets which are very effective to deal with vaguenes. The concept of the intuitionistic fuzzy sets is a generalization of one of the fuzzy sets. Recently, Çoker and his colleagues [5,6,8], Lee and Lee[17], and Hur and his colleagues[13] introduced the concept of intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets and investigated some of it's properties. In 1989, Biswas[4] introduced the concept of intuitionistic fuzzy subgroup and studied some of it's properties. In 2003, Banerjee and Basnet [3] investigated intuitionistic fuzzy subgroups and intuitionistic fuzzy ideals using intuitionistic fuzzy sets. Also, Hur and his colleagues[9-12] studied variuos propoerties of intuitionistic fuzzy subgroupoids, intuitionistic fuzzy subgroups, intuitionistic fuzzy subrings and intuitionistic fuzzy topological groups. In particular, Hur and his colleagues[14] introduced the concept of intuitionistic fuzzy ideals and intuitionistic fuzzy bi-ideal of a semigroup and studied some of their properties. And Yon and Kim[20] introduced the notion of intuitionistic fuzzy sublattice, filters and ideals, and investigated some of their properties. Moreover, Hur and his colleagues[16] applied the concept of intuitionistic fuzzy sets to congruences on a lattice.

In this paper, We discuss the important concept of an intuitionistic fuzzy sublattice or an intuitionistic fuzzy ideal generated by an intuitionistic fuzzy set. In fact, an intuitionistic fuzzy sublattice generated by an intuitionistic fuzzy set is also characterized by the level subsets of the given intuitionistic fuzzy set. Moreover, the notion of an intuitionistic fuzzy convex sublattice is also introduced and discussed. A pleasing feature of our investigation in this direction is that the unique representation theorem for convex sublattice is successfully extended to intuitionistic fuzzy setting. In the last section, we deal with the homomorphic images and pre-images of intuitionistic fuzzy sublattices. Also, we prove that the homomorphic image of an intuitionistic fuzzy prime ideal is an intuitionistic fuzzy prime ideal, provided that it has the sup-property or is IF-invariant.

2. PRELIMINARIES

We will list some concept needed in the later sections.

For sets X, Y and Z, $f = (f_1, f_2) : X \to Y \times Z$ is called a *complex mapping* if $f_1 : X \to Y$ and $f_2 : X \to Z$ are mappings.

Throughout this paper, we will denote the unit interval $[0; 1]$ as I. And $L = (L, +, .)$ denotes a lattice(See[10]), where " + " and " . " denote the sup and the inf, respectively.

Definition 1.1 [2,5]. Let X be a nonempty set. A complex mapping $A = (\mu_A, \nu_A)$: $X \to I \times I$ is called an *intuitionistic fuzzy set*(in short, IFS) in X if $\mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$, where the mapping $\mu_A : X \to I$ and $\nu_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each $x \in X$ to A, respectively. In particular, 0~ and 1~ denote the *intuitionistic fuzzy empty set* and the *intuitionistic fuzzy whole set* in a set X defined by $0\text{~}(x) = (0, 1)$ and $1\text{~}(x) = (1, 0)$ for each $x \in X$, respectively.

We will denote the set of all IFSs in X as IFS(X).

Definition 1.2 [2]. Let X be a nonempty sets and let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be an IFSs in X. Then

(1) $A \subset B$ if and only if $\mu_A \leq \mu_B$ and $\nu_A \geq \nu_B$.

(2) $A = B$ if and only if $A \subset B$ and $B \subset A$.

(3) $A^c = (\nu_A, \mu_A)$.

(4) $A \cap B = (\mu_A \wedge \mu_B, \nu_A \vee \nu_B)$.

(5) $A \cup B = (\mu_A \vee \mu_B, \nu_A \wedge \nu_B)$.

Definition 1.3 [5]. Let $\{A_i\}_{i \in J}$ be an arbitrary family of IFSs in X, where $A_i = (\mu_{Ai}, \nu_{Ai})$ for each $i \in J$. Then

(1) $\cap A_i = (\wedge \mu_{A_i}, \vee \nu_{A_i})$.

(2) $\cup A_i = (\vee \mu_{A_i}, \wedge \nu_{A_i})$.

2. INTUITIONISTIC FUZZY SUBLATTICES AND THEIR CHARACTERIZATIONS

In this section, we discuss some basic results. In the process, some well-known basic concepts of lattice theory are extended to the intuitionistic fuzzy setting. After systematically introducing the notions of intuitionistic fuzzy ideal, intuitionistic fuzzy filter, intuitionistic fuzzy prime ideal and intuitionistic fuzzy dual prime ideal, we provide their characterizations. Also we establish the fact that the concept of level subsets is going to play an important role in the theory of intuitionistic fuzzy lattices, as in the case in the theory of intuitionistic fuzzy groups and intuitionistic fuzzy rings.

Definition 2.1[20]. Let $A \in$ IFI(L). Then A is called an *intuitionistic fuzzy sublattice* (in short, IFSL) of L if it satisfies the following conditions:

(i) $\mu_A(x + y) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(x + y) \leq \nu_A(x) \vee \nu_A(y)$ for any $x, y \in L$.

(ii) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$ for any $x, y \in L$.

We will denote the set of all IFSLs of L as IFSL(L).

Result 2.A [15, Proposition 3.2]. Let $A \in \text{IFS}(L)$ and let $x, y \in L$. Then the following are equivalent:

(1) If $x \le y$, then $\mu_A(x) \ge \mu_A(y)$ and $\nu_A(x) \le \nu_A(y)$.

(2) $\mu_A(xy) \ge \mu_A(x) \vee \mu_A(y)$ and $\nu_A(xy) \le \nu_A(x) \wedge \nu_A(y)$.

(3) $\mu_A(x + y) \ge \mu_A(x) \vee \mu_A(y)$ and $\nu_A(x + y) \le \nu_A(x) \vee \nu_A(y)$.

Result 2.B [The dual of Result 2.A, 15, Proposition 3.3]. Let $A \in \text{IFS}(L)$ and let $x, y \in L$. Then the following are equivalent:

(1) If $x \le y$, then $\mu_A(x) \le \mu_A(y)$ and $\nu_A(x) \ge \nu_A(y)$.

(2) $\mu_A(xy) \le \mu_A(x) \vee \mu_A(y)$ and $\nu_A(xy) \ge \nu_A(x) \wedge \nu_A(y)$.

(3) $\mu_A(x + y) \le \mu_A(x) \vee \mu_A(y)$ and $\nu_A(x + y) \ge \nu_A(x) \wedge \nu_A(y)$.

Definition 2.2 [20]. Let $A \in$IFSL(L). Then A is called an

(i) *intuitionistic fuzzy ideal*(in short, *IFI*) of L if for any $x, y \in L$ $x \le y$ in L implies $\mu_A(x) \ge \mu_A(y)$ and $\nu_A(x) \le \nu_A(y)$.

(ii) *intuitionistic fuzzy filter*(in short, IFF) of L if for any $x, y \in L$ $x \le y$ in L implies $\mu_A(x) \le \mu_A(y)$ and $\nu_A(x) \ge \nu_A(y)$.

We will denote the set of all IFSLs[resp. IFFs] of S as IFI(L)[resp. IFF(L)].

Result 2.C[15, Theorem 3.4]. Let $A \in$ IFSL(L). Then $A \in$ IFI(L) [resp. IFF(L)] if and only if it satisfies any are of the conditions in Result 2.A [resp. Result 2.B].

Definition 2.3 [20]. Let $A \in$ IFI(L)[resp. IFF(L)]. Then A is called an *intuitionistic fuzzy prime ideal*(in short, *IFPI*) [resp. prime filter(in short, *IFPF*)] of L if for any $x, y \in L$, $\mu_A(xy) \le \mu_A(x) \vee \mu_A(y)$ and $\nu_A(xy) \ge \nu_A(x) \wedge \nu_A(y)$ [resp. $\mu_A(x + y) \le \mu_A(x) \vee \mu_A(y)$ and $\nu_A(x + y) \ge \nu_A(x) \wedge \nu_A(y)$].

We will denote the set of all IFPIs [resp. IFPFs] of L as IFPI(L)[IFPF(L)].

The following are the immediate results of Results 2.A and 2.B, and Definition 2.3.

Proposition 2.4. Let $A \in$ IFI(L). Then the following are equivalent:

(1) $A \in$ IFPI(L).

(2) $A(xy) = (\mu_A(x) \vee \mu_A(y), \nu_A(x) \wedge \nu_A(y))$ for any $x, y \in L$.

(3) $A(xy) = A(x)$ or $A(y)$ for any $x, y \in L$.

Proposition 2.4 [The dual of Proposition 2.4]. Let $A \in \mathrm{IFF}(L)$. Then the following are equivalent:

(1) $A \in \mathrm{IFPF}(L)$.

(2) $A(x + y) = (\mu_A(x) \vee \mu_A(y), \nu_A(x) \wedge \nu_A(y))$ for any $x, y \in L$.

(3) $A(x + y) = A(x)$ or $A(y)$ for any $x, y \in L$.

The following results are easily seen.

Corollary 2.4-1. If $A \in \mathrm{IFI}(L)$ such that $A^c \in \mathrm{IFF}(L)$, then $A, A^c \in \mathrm{IFPI}(L)$.

Corollary 2.4-2. Let $A \in \mathrm{IFS}(L)$. That $A \in \mathrm{IFPI}(L)$ if and only if $A^c \in \mathrm{IFPF}(L)$.

Definition 2.5[9]. Let X be a set, let $A \in \mathrm{IFS}(X)$ and let $(\lambda, \mu) \in I \times I$ with $\lambda + \mu \leq 1$. Then the set $A^{(\lambda,\mu)} = \{x \in X : \mu_A(x) \geq \lambda \text{ and } \nu_A(x) \leq \mu\}$ is called a (λ, μ)-*level subset* of A.

In the following results, we characterize intuitionistic fuzzy sublattices, intuitionistic fuzzy ideals[filters], intuitionistic fuzzy prime ideals and [resp. prime filters] in terms of level subsets.

Theorem 2.6. Let $A \in \mathrm{IFS}(L)$. Then $A \in \mathrm{IFSL}(L)$ if and only if for each $(\lambda, \mu) \in \mathrm{Im}\, A$, $A^{(\lambda,\mu)}$ is a sublattice of L. Equivalently, $A \in \mathrm{IFSL}(L)$ if and only if each nonempty level subset $A^{(\lambda,\mu)}$ is a sublattice. In this case, $A^{(\lambda,\mu)}$ is called a *level sublattice* of L.

Proof. We prove here the second assertion of the theorem. The proof of the first is the same, except for trivial modifications.

($\Rightarrow$): Suppose $A \in \mathrm{IFSL}(L)$. Let $A^{(\lambda,\mu)}$ be any nonempty level subset of A and let $x, y \in A^{(\lambda,\mu)}$. Then $\mu_A(x) \geq \lambda$, $\nu_A(x) \leq \mu$ and $\mu_A(y) \geq \lambda$, $\nu_A(y) \leq \mu$. Thus

$$\mu_A(x + y) \geq \mu_A(x) \wedge \mu_A(y) \geq \lambda,\ \nu_A(x + y) \leq \nu_A(x) \vee \nu_A(y) \leq \mu$$

and

$$\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y) \geq \lambda,\ \nu_A(xy) \leq \nu_A(x) \vee \nu_A(y) \leq \mu$$

So $x + y \in A^{(\lambda,\mu)}$ and $xy \in A^{(\lambda,\mu)}$. Hence $A^{(\lambda,\mu)}$ is a sublattice of L.

($\Leftarrow$): Suppose the necessary condition holds. For any $x, y \in L$, let $A(x) = (\lambda, \mu)$ and $A(y) = (s, t)$. Without loss of generality, we can assume that $s \leq \lambda$ and $t \geq \mu$. Then $A^{(\lambda,\mu)} \subset A^{(s,t)}$. Since $x \in A^{(\lambda,\mu)}$ and $y \in A^{(s,t)}$, $x, y \in A^{(s,t)}$. Since $A^{(s,t)}$ is a sublattice of L, $x + y \in A^{(s,t)}$ and $xy \in A^{(s,t)}$. Thus

$\mu_A(x + y) \geq s$, $\nu_A(x + y) \leq t$ and $\mu_A(xy) \geq s$, $\nu_A(xy) \leq t$.

So

$$\mu_A(x + y) \geq s = \mu_A(x) \wedge \mu_A(y),\ \nu_A(x + y) \leq t = \nu_A(x) \vee \nu_A(y)$$

and

$$\mu_A(xy) \geq s = \mu A(x) \wedge \mu_A(y),\ \nu_A(xy) \leq t = \nu_A(x) \vee \nu_A(y).$$

Hence $A \in$ IFSL(L). This completes the proof.

It is easy to verify that the set of all level sublattices of an IFSL of a lattice forms a chain. However, in contrast of the chain level subgroups of an intuitionistic fuzzy subgroup, the chain of level sublattices of an IFSL may no contain a least element.

In intuitionistic fuzzy group theory, it is well-known that an intuitionistic fuzzy subgroup of a group attains its supremum at the identity element of the given group. However, since an intuitionistic fuzzy sublattice may neither attain its supremum nor its infimum at any element of the lattice, the situation is different in the case of an intuitionistic fuzzy sublattice of a lattice.

Example 2.7. Let $L = \mathbb{N}$. We define a complex mapping $A : L \to I \times I$ as follows: for each $x \in L$,

$$A(x) = \left(1 - \frac{1}{n+1}, \alpha_n\right) \text{ for all } x \in \, < 2^n > \, \sim \, < 2^{n+1} >$$

for each fixed nonnegative integer n, where $< 2^n >$ denote the set of all positive integers which are multiple of 2^n and $\alpha_n \in I$ with $\alpha_n \leq \frac{1}{n+1}$. Then clearly $A \in$ IFS(L). Moreover, we can see that $A \in$ IFSL(L) with the following chain of level sublattices:

$$\ldots \subset \, < 2^3 > \subset < 2^2 > \subset < 2 > \subset L.$$

On the other hand, "1" is the least element of L and $A(1) = (0, 1)$. But $(\vee_{x \in L} \mu_A(x), \wedge_{x \in L} \nu_A(x)) = (1, 0)$, which is not attained anywhere by A.

Example 2.8. Let $L = \mathbb{N} \times \mathbb{N}$. We define subsets L_i of L as follows:

$$L_1 = \{(2, 2), (2, 3), (3, 2), (3, 3)\}$$

$$L_n = \{x \in L : x \leq (n + 1, n + 1)\}$$

for each positive integer $n \geq 2$. Now we define a complex mapping $A : L \to I \times I$ as follows: for each $x \in L$,

$A(x) = (1, 0)$ for all $x \in L_1$,

$A(x) = \left(\frac{1}{n}, \alpha_n\right)$ for all $x \in L_n \sim L_{n-1}$, where $n \in \mathbb{N}$ with $n \geq 2$ and $\alpha_n \in I$ with $\alpha_n + \frac{1}{n} \leq 1$.

Then clearly $A \in \text{IFS}(L)$. Moreover, we can see that $A \in \text{IFSL}(L)$. But $(\vee_{x\in L}\, \mu_A(x), \wedge_{x\in L}\, \nu_A(x)) = (1, 0)$, which is not attained at the least element (1, 1) of L by A and infimum is not attained anywhere.

Example 2.9. Let $L = \{\theta, \mathbb{N}\} \cup \{\{n\} : n \in \mathbb{N}\}$. Then clearly L is a lattice under the ordering of set inclusion with θ as its least element and $\mathbb{N}$ the greatest element (See Example 3.13 in [1]). Now consider all the finite sublattices of L of the form

$L_1 = \{\theta, \mathbb{N}\}$,

$L_n = \{\theta, \mathbb{N}\} \cup \{\ \{i\} : i \leq n - 1\}$, for each $n \in \mathbb{N}$ and $n \geq 2$.

We define a complex mapping $A : L \to I \times I$ as follows: for each $x \in L$,

$A(x) = (1, 0)$ for $x \in L_1$,

$A(x) = \left(\frac{1}{n}, \alpha_n\right)$ for $x \in L_n \sim L_{n-1}$, where $\alpha_n \in I$ with $\alpha_n + \frac{1}{n} \leq 1$ (See Example 3.1 in [15]).

Also we define a complex mapping $B : L \to I \times I$ as follows: for each $x \in L$,

$B(x) = (0, 1)$ for $x \in L_1$,

$B(x) = \left(1 - \frac{1}{n}, \beta_n\right)$ for $x \in L_n \sim L_{n-1}$, where $\beta_n \in I$ with $\beta_n + \frac{1}{n}$.

Then clearly $A, B \in \text{IFS}(L)$. Moreover we can see that $A, B \in \text{IFSL}(L)$. But A does not attain its infimum, where as B does not attain it supremum.

Result 2.D[15, Theorem 3.5 and 3.6]. Let $A \in$ IFSL(L). Then $A \in$ IFI(L)[resp. $A \in$ IFF(L)] if and only if for each (λ, μ) 2 Im A, $A^{(\lambda,\mu)}$ is an ideal[resp. filter] of L.

Equivalently, for each $A \in$ IFS(L), $A \in$ IFI(L)[resp. $A \in$ IFF(L)] if and only if each nonempty level subset $A^{(\lambda,\mu)}$ is an ideal[resp. filter] of L. In this case, $A^{(\lambda,\mu)}$ is called a *level ideal*[resp. *filter*] of L.

Theorem 2.10. Let $A \in$ IFI(L)[resp. IFF(L)]. Then $A \in$ IFPI(L)[resp. IFPF(L)] if and only if for each $(\lambda, \mu) \in$ Im A, $A^{(\lambda,\mu)}$ is a prime ideal[resp. dual prime ideal] of L.

Equivalently, for each $A \in$ IFS(L), $A \in$ IFPI(L)[resp. IFPF(L)] if and only if each nonempty level subset $A^{(\lambda,\mu)}$ is a prime ideal[resp. dual prime ideal] of L.

Proof.($\Rightarrow$): Suppose $A \in$ IFPI(L). Let $(\lambda, \mu) \in$ Im A and a, $b \in A^{(\lambda,\mu)}$. Then $\mu_A(ab) \geq \lambda$ and $\nu_A(ab) \leq \mu$. Since $A \in$ IFPI(L), by Proposition 2.4, $A(ab) = A(a)$ or $A(b)$. Thus

$$\mu_A(a) \geq \lambda,\ \nu_A(a) \leq \mu \text{ or } \mu_A(b) \geq \lambda,\ \nu_A(b) \leq \mu.$$

So $a \in A^{(\lambda,\mu)}$ or $b \in A^{(\lambda,\mu)}$. Moreover, by Result 2. D, it is clear that $A^{(\lambda,\mu)}$ is an ideal of S. Hence $A^{(\lambda,\mu)}$ is prime.

($\Leftarrow$): Suppose each level ideal $A^{(\lambda,\mu)}$ is prime. Assume that $A \notin$ IFPI(L). Then, by Proposition 2.4, there exist a, $b \in L$ such that $A(ab) \neq A(a)$ and $A(ab) = A(b)$. Since $a \in$ IFI(L),

$$\mu_A(ab) > \mu_A(a),\ \nu_A(ab) < \mu_A(a)$$

and

$$\mu_A(ab) > \mu_A(b),\ \nu_A(ab) < \mu_A(b).$$

Let $A(ab) = (\lambda, \mu)$. Then $ab \in A^{(\lambda,\mu)}$, but $a \notin A^{(\lambda,\mu)}$ and $b \notin A^{(\lambda,\mu)}$. This contradicts the fact that $A^{(\lambda,\mu)}$ is prime. Hence $A \in$ IFPI(L).

The following can be easily verified.

Proposition 2.11. Let $\{A_\alpha\}_{\alpha\in\Gamma} \subset$ IFSL(L)[resp. IFI(L) and IFF(L)]. Then $\cap_{\alpha\in\Gamma} A_\alpha \in$ IFSL(L)[resp. IFI(L) and IFF(L)].

Definition 2.12. Let $A \in$ IFS(L). Then the least intuitionistic fuzzy sublattice[resp. ideal and filter] of L containing A is called the intuitionistic fuzzy sublattice sublattice[resp. ideal and filter] *generated by* A.

Let A be a nonzero intuitionistic fuzzy set in a lattice. Then the existence of a least intuitionistic fuzzy sublattice containing A is ensured by Proposition 2.11. We denote the intuitionistic fuzzy sublattice generated by a nonzero intuitionistic fuzzy set A as $[A]$ and the same notation $[A^{(\lambda,\mu)}]$ is used for a sublattice generated by a level subset $A^{(\lambda,\mu)}$. Similarly, we denote the intuitionistic fuzzy ideal[resp. filter] generated by A as (A)[resp. $[A]$].

In the following result, we construct the intuitionistic fuzzy sublattice generated by an intuitionistic fuzzy set in a specified way. We recall from lattice theory that the sublattice generated by any subset H of a lattice L consists of the lattice polynomial functions of the elements of H. That is, if $[H]$ is the sublattice generated by H, then $[H] = \{a : a = P(h_0, h_1, \ldots, h_{n-1}), n \geq 1, h_i \in H\}$, for some n-ary polynomial P(see[17]).

Theorem 2.13. Let $A \in \text{IFS}(L)$ and let $(\lambda, \mu), (s, t) \in I \times I$ such that $s > \lambda$ and $t < \mu$. We define complex mapping $A^* : S \to I \times I$ as follows: for each $x \in L$,

$$A^*(x) = (\lambda, \mu) \text{ if } x \in [A^{(\lambda,\mu)}] \text{ and } x \notin [A^{(s,t)}].$$

then $A^* = [A]$.

Proof. It is clear that $A^* \in \text{IFS}(L)$. Let $(\lambda, \mu) \in \text{Im } A^*$ and let $x \in A^{*(\lambda,\mu)}$. Then clearly either $A^*(x) = (\lambda, \mu)$ or $\mu_{A*}(x) > \lambda$ and $\nu_{A*}(x) < \mu$.

Case(i): Suppose $A^*(x) = (\lambda, \mu)$. Then, by the definition of A^*, $x \in [A^{(\lambda,\mu)}]$. Thus $A^{*(\lambda,\mu)} \subset [A^{(\lambda,\mu)}]$.

Case(ii): Suppose $\mu_A^*(x) > \lambda$ and $\nu_A^*(x) < \mu$. Let $A^*(x) = (s, t)$. Then, by the definition of A^*, $x \in [A^{(s,t)}]$. Since $[A^{(s,t)}] \subset [A^{(\lambda,\mu)}]$, $x \in [A^{(\lambda,\mu)}]$. Thus $[A^{*(\lambda,\mu)}] \subset [A^{(\lambda,\mu)}]$. So, in all, $[A^{*(\lambda,\mu)}] \subset [A^{(\lambda,\mu)}]$. Now let $x \in [A^{(\lambda,\mu)}]$ and assume that $x \notin [A^{*(\lambda,\mu)}]$. Then $\mu_{A*}(x) < \lambda$ and $\nu_{A*}(x) > \mu$. Let $A^*(x) = (s, t)$. Then

$x \in [A^{(s,t)}]$ and $x \notin [A^{(\lambda_0,\mu_0)}]$ for $\lambda_0 > s$ and $\mu_0 < t$.

Since $\lambda > s$ and $\mu < t$, $x \in [A^{(s,t)}]$ and $x \notin [A^{(\lambda,\mu)}]$. This contradicts the fact that $x \in [A^{(\lambda,\mu)}]$. Thus $x \in [A^{*(\lambda,\mu)}]$. So $[A^{(\lambda,\mu)}] \subset [A^{*(\lambda,\mu)}]$. Hence $[A^{*(\lambda,\mu)}] = [A^{(\lambda,\mu)}]$.

It is clear that $A^* \in IFSL(L)$ by Theorem 2.6.

Assume that $A \not\subset A^*$. Then there exists $x \in L$ such that $A(x) = (S, T)$, $\mu_A(x) > \mu_{A*}(x)$ and $\nu_A(x) < \nu_{A*}(x)$. Let $A^*(x) = (\lambda, \mu)$. Then $x \in [A^{(\lambda,\mu)}]$ and $x \notin [A^{(\lambda0,\mu0)}]$ for each

$\lambda_0 > \lambda$ and $\mu_0 < \mu$. Since $s > \lambda$ and $t < \mu$, $x \notin [A^{(s,t)}]$. This contradicts the fact that $x \in A^{(s,t)}$. So $A \subset A^*$.

Now let $B \in$ IFSL(L) such that $A \subset B$ and let $A^*(x) = (s, t)$. Then $x \in [A^{(s,t)}]$ and $x \notin [A^{(\lambda,\mu)}]$ for $\lambda > s$ and $\mu < t$. Thus x is a lattice polynomial function. So x can be written as

$$x = P(h_1, h_2, \ldots, h_k), \text{ where } h_i \in A^{(s,t)},\ i = 1, 2, \ldots, k.$$

Then it can be proved by induction on the rank of x and by using the definition of intuitionistic fuzzy sublattice that

$$\begin{aligned}\mu_B(x) &\geq \mu_B(h_1) \wedge \mu_B(h_2) \wedge \ldots \wedge \mu_B(h_k)\\ &\geq \mu_A(h_1) \wedge \mu_A(h_2) \wedge \ldots \wedge \mu_A(h_k) \geq s\end{aligned}$$

and

$$\begin{aligned}\nu_B(x) &\leq \nu_B(h_1) \vee \nu_B(h_2) \vee \ldots \vee \nu_B(h_k)\\ &\leq \nu_A(h_1) \vee \nu_A(h_2) \vee \ldots \vee \nu_A(h_k) \leq t.\end{aligned}$$

So $\mu_B(x) \geq s = \mu^*_A(x)$ and $\nu_B(x) \leq t = \nu^*_A(x)$. Hence $A^* \subset B$. Therefore $A^* = [A]$. This completes the proof.

Example 2.14. Let $L = \mathbb{N} \times \mathbb{N}$ be the lattice of Example 2.8. We define a complex mapping $A : L \to I \times I$ as follows: for each $(m, n) \in L$,

$$A((m, n)) = \left(\frac{1}{m+n}, \frac{1}{2} - \frac{1}{m+n+2}\right).$$

For each $n \in \mathbb{N}$, consider the sublattice L_n of L defined as follows:

$L_n = \{x \in L : x \leq (n, n)\}$.

Then the level subsets $A^{(\lambda,\mu)}$ of A are given by

$$A^{\left(\frac{1}{2},\frac{1}{4}\right)} = \{(1,1)\} = L_1$$

and

$$A^{\left(\frac{1}{r},\frac{r}{2(r+2)}\right)} = \{(m,n) \in L : m+n \leq r\} \quad \text{for each } r \in \mathbb{N}.$$

Thus it follows that

$$[A^{\left(\frac{1}{2},\frac{1}{4}\right)} = A^{\left(\frac{1}{2},\frac{1}{4}\right)} = L_1$$

and

$$A^{\left(\frac{1}{r},\frac{r}{2(r+2)}\right)} = L_{r-1} \quad \text{for each } r \in \mathbb{N}.$$

Moreover, we can easily verify that $A^* = [A]$ is given by

$$A^*(x) = \left(\frac{1}{2},\frac{1}{4}\right) \text{ for } x \in L_1$$

and

$$A^*(x) = \left(\frac{1}{n+1},\frac{n+1}{2(r+3)}\right) \text{ for } x \in L_n \sim L_{n-1} \text{ and } n \geq 2.$$

On the other hand, $A((3, 2)) = \left(\frac{1}{5},\frac{5}{14}\right)$ but $A^*((3, 2)) = \left(\frac{1}{4},\frac{1}{3}\right)\cdot$ Thus $A \neq A^*$ and it is clear that $A \subset A^*$.

In a similar way as in Theorem 2.13, we can obtain the intuitionistic fuzzy ideal [resp. dual ideal] generated by the intuitionistic fuzzy set. We make use of the fact that an ideal generated by a subset H of L will be of the form $(H] = \{a : a \leq h_1 + h_2 + \ldots + h_n, n \geq 1 \text{ and } h_i \in H\}$ (see[7]).

Theorem 2.15. Let $A \in \mathrm{IFS}(L)$. We define a complex mapping $A^* : L \to I \times I$ as follows: for each $x \in L$,

$A^*(x) = (\lambda, \nu)$ if $x \in (A^{(\lambda,\mu)}]$ and $x \notin (A^{(s,t)}]$ for $s > \lambda$ and $t < \mu$.

Then $A^* = (A]$.

Theorem 2.15′[The dual of Theorem 2.15]. Let $A \in \mathrm{IFS}(L)$. We define a complex mapping $A^* : L \to I \times I$ as follows : for each $x \in L$,

$A^*(x) = (\lambda, \nu)$ if $x \in [A^{(\lambda,\mu)})$ and $x \notin [A^{(s,t)})$ for $s > \lambda$ and $t < \mu$.

Then $A^* = [A)$.

3. INTUITIONISTIC FUZZY CONVEXITY AND ITS CHARACTERIZATIONS

Convex sublattice occupy an important place in the theory of lattices. In this section, we extend this notion to the intuitionistic fuzzy setting and provide its characterizations. It is proved that an intersection of an intuitionistic fuzzy ideal and an intuitionistic fuzzy filter is an intuitionistic fuzzy convex sublattice and every intuitionistic fuzzy convex sublattice has a unique representation of this type. This is an intuitionistic fuzzy analog of a famous result of lattice theory.

Definition 3.1[15]. Let $A \in$ IFSL(L). Then A is said to be *intuitionistic fuzzy convex* (in short, *IFCL*) if for each interval $[a, b] \subset L$ and each $x \in [a, b]$,

$$\mu_A(x) \geq \mu_A(a) \wedge \mu_A(b) \text{ and } \nu_A(x) \leq \nu_A(a) \vee \nu_A(b).$$

We will denote the set of all IFCLs of L as IFCL(L).

Result 3.A [15, Proposition 3.8]. In a lattice, every IFI(IFF) is an IFCL.

Result 3.B[15, Theorem 3.9]. Let $A \in$ IFSL(L). Then $A \in$ IFCL(L) if and only if for each $(\lambda, \mu) \in$ Im A, $A^{(\lambda,\mu)}$ is a convex sublattice of L.

Result 3.C[15, Theorem 3.10]. If $\{A\alpha\}_{\alpha\in\Gamma} \subset$ IFCL(S), then $\cap_{\alpha\in\Gamma} A_\alpha \in$ IFCL(L).

Result 3.D [7, Lemma 1]. Let J and H be nonempty subsets of L.

(1) J is an ideal of L if and only if $a, b \in J$ implies that $a + b \in L$ and $a \in J$, $x \in L$, $x \leq a$ imply that $x \in J$.

(2) J = [H] if and only if for each $x \in J$ there exists an integer $n \geq 1$ and there exist $h_0, h_1, \ldots, h_{n-1} \in H$ such that $x \leq h_0 \vee h_1 \vee \ldots h_{n-1}$.

Result 3.E[7, Lemma 6]. Let J be an ideal and let D be a filter. If $J \cap D \neq \theta$, then $J \cap D$ is a convex sublattice, and every convex sublattice can be expressed in this form in one and only one way.

Now, we will establish an anolog of Result 3.E in terms of intuitionistic fuzzy sets.

Proposition 3.2. Let $A \in$ IFI(L) and let $B \in$ IFF(L). If $A \cap B \neq 0\sim$, then $A \cap B \in$ IFCL(L). And every intuitionistic fuzzy convex sublattice can be expressed in this form in one and only one way.

Proof. The first statement is obvious from Results 3.A and 3.C. Let $A \in \mathrm{IFCL}(L)$. Then clearly $A \subset (A] \cap [A)$. For each $x \in L$, let $(A](x) = (\lambda_0, \mu_0)$ and let $[A)(x) = (s_0, t_0)$. Without loss of generality, we can assume that $\lambda_0 \leq s_0$ and $\mu_0 \geq t_0$. Then, by Theorem 2.15,

$$x \in (A^{(\lambda_0,\mu_0)}] \text{ and } x \notin (A^{(\lambda,\mu)}] \text{ for } \lambda > \lambda_0 \text{ and } \mu < \mu 0.$$

Thus, by Result 3.D(2), there exists $y_1 \in A^{(\lambda_0,\mu_0)}$ such that $x \leq y$. Similarly, by Theorem 2.15′,

$$x \in [A^{(\lambda_0,\mu_0)}) \text{ and } x \notin [A^{(\lambda,\mu)}) \text{ for } \lambda > s_0 \text{ and } \mu < t_0.$$

Thus there exists $y_2 \in A^{(s_0,t_0)}$ such that $y_2 \leq x$. Since $A^{(s_0,t_0)} \subset A^{(\lambda_0,\mu_0)}$ $y_1, y_2 \in A^{(\lambda_0,\mu_0)}$. Since $A \in \mathrm{IFCL}(L)$, by Result 3. B, $A^{(\lambda_0,\mu_0)}$ is a sublattice of L. Then $x \in A^{(\lambda_0,\mu_0)}$. Thus $\mu_A(x) \geq \lambda_0$ and $\nu_A(x) \leq \mu_0$. So

$$\mu_A(x) \geq \lambda_0 = \mu_{(A]}(x) \wedge \mu_{[A)}(x) = \mu_{(A]\cap[A)}(x)$$

and

$$\nu_A(x) \leq \mu_0 = \nu_{(A]}(x) \vee \nu_{[A)}(x) = \nu_{(A]\cup[A)}(x).$$

Hence $(A] \cap [A) \subset A$. Therefore $A = (A] \cap [A)$.

Now, we show that this representation is unique. Let $A = B \cap C$, where $B \in \mathrm{IFI}(L)$ and $C \in \mathrm{IFF}(L)$. Since $A \subset B$, $(A] \subset B$. For each $a \in L$, let $B(a) = (s_0, t_0)$. Then $a \in B^{(s_0,t_0)}$. since $A \subset B$, $A^{(s_0,t_0)} \subset B^{(s_0,t_0)}$. Let $b \in A^{(s_0,t_0)}$. Since $B \in \mathrm{IFI}(L)$, by Result 2.D, $B^{(s_0,t_0)}$ is an ideal of L. Then, by Result 3.D(1),

$$a + c \in B^{(s_0,t_0)} \text{ and } b \in [A^{(s_0,t_0)}).$$

Since $b \leq a + b$ and $[A^{(s_0,t_0)})$ is a filter of L, by the dual of Result 3.D(1), $a + b \in [A^{(s_0,t_0)})$. Also, we can see that $[A^{(s_0,t_0)}) = [A)^{(s_0,t_0)}$. Thus $\mu_{[A)}(a + b) \geq s_0$ and $\nu_{[A)}(a + b) \leq t_0$. Since $[A) \subset C$, $\mu_C(a + b) \geq s_0$ and $\nu_C(a + b) \leq t_0$. Moreover, $\mu_B(a + b) \geq s_0$ and $\nu_B(a + b) \leq t_0$. So

$$\mu_A(a + b) = \mu_{B\cap C}(a + b) = \mu_B(a + b) \wedge \mu_C(a + b) \geq s_0$$

and

$\nu_A(a+b) = \nu_{B\cap C}(a+b) = \nu_B(a+b) \vee \nu_C(a+b) \le t_0$.

Then $a+b \in A^{(s_0,t_0)}$. Thus $a+b \in (A^{(s_0,t_0)}]$. Since $a \le a+b$ and $(A^{(s_0,t_0)}]$ is the ideal generated by $A^{(s_0,t_0)}$, $a \in (A^{(s_0,t_0)}]$. Since $(A^{(s_0,t_0)}] = (A]^{(s_0,t_0)}$, $a \in (A]^{(s_0,t_0)}$. Thus $\mu_{(A]}(a) \ge s_0 = \mu_B(a)$ and $\nu_{(A]}(a) \le s_0 = \nu_B(a)$. So $B \subset (A]$. Hence $(A] = B$. The dual argument establishes that $[A) = C$. Hence the representation is unique. This completes the proof.

4. HOMOMORPHISM AND INTUITIONISTIC FUZZY SUBLATTICES

In this section, we discuss the algebraic nature of homomorphic image and preimages of different types of intuitionistic fuzzy sublattice. To begin with, we will list some of the results of [20]. Now, without any restriction on the homomorphism, we prove that the homomorphic image of an intuitionistic fuzzy sublattice is indeed an intuitionistic fuzzy sublattice and we also improve several other results.

Definition 4.1[9]. Let X be a set and let $A \in \mathrm{IFS}(X)$. Then A is said to be *sup-property* if for each $Y \subset P(X)$, there exists $y_0 \in Y$ such that $A(y_0) = (\vee_{y\in Y} \mu_A(y), \wedge_{y\in Y} \nu_A(y))$.

Definition 4.2[5]. Let X and Y be nonempty sets and let $f : X \to Y$ be a mapping. Let $A = (\mu_A, \nu_A)$ be an IFS in X and $B = (\mu_B, \nu_B)$ be an IFS in Y. Then

(1) the *preimage* of B under f, denoted by $f^{-1}(B)$, is the IFS in X defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\nu B)),$$

where $f^{-1}(\mu_B) = \mu_B \circ f$.

(2) the *image* of A under f, denoted by $f(A)$, is the IFS in Y defined by

$$f(A) = (f(\mu_A), f(\nu A)),$$

where for each $y \in Y$

$$\mu_{f(A)}(y) = f(\mu_A)(y) = \begin{cases} \vee_{x\in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \ne \theta, \\ 0 & \text{if } f^{-1}(y) = \theta, \end{cases}$$

and

$$\nu_{f(A)}(y) = f(\nu_A)(y) = \begin{cases} \wedge_{x \in f^{-1}(y)} \nu_A(x) & \text{if } f^{-1}(y) \neq \theta, \\ 0 & \text{if } f^{-1}(y) = \theta. \end{cases}$$

In particular, if $f : L \to L'$ is a lattice homomorphism, $A \in$ IFS(L) and $B \in$ IFS(L'), then $f(A)$ is called the *homomorphic image* of A under f and $f^{-1}(B)$ is called the *homomorphic preimage* of B, where L and L' denote lattices, respectively.

Result 4.A[20, Theorems 2.18, 2.19, 2.20 and 2.21]. Let $f : L \to L'$ be a lattice homomorphism.

(1) If f is surjective and $A \in$ IFSL(L)[resp. IFI(L) and IFF(L)], then $f(A) \in$ IFSL(L')[resp. IFI(L') and IFF(L')].

(2) If $B \in$ IFSL(L')[resp. IFI(L') and IFF(L')], then $f^{-1}(B) \in$ IFSL(L)[resp. IFI(L) and IFF(L)].

Lemma 4.3. Let L and L' be lattices, let $f : L \to L'$ be a surjection and let A be an IFSL with sup property in L. Then

$$(f(A))^{(\lambda,\mu)} = f(A^{(\lambda,\mu)}) \text{ for each } (\lambda, \mu) \in I \times I \text{ with } \lambda + \mu \leq 1.$$

Proof. Let $(\lambda, \mu) \in I \times I$ with $\lambda + \mu \leq 1$ and let $y \in (f(A))^{(\lambda,\mu)}$. Then

$$\mu_{f(A)}(y) = \vee_{x \in f^{-1}(y)} \mu_A(x) \geq \lambda \text{ and } \nu_{f(A)}(y) = \wedge_{x \in f^{-1}(y)} \nu_A(x) \leq \mu.$$

Since A has sup property, there exists $x_0 \in f^{-1}(y)$ such that

$$\mu_A(x_0) = \vee_{x \in f^{-1}(y)} \mu_A(x) \textit{ and } \nu_A(y) = \wedge_{x \in f^{-1}(y)} \nu_A(x).$$

Thus $\mu_A(x_0) \geq \lambda$ and $\nu_A(x_0) \leq \mu$. So $x_0 \in A^{(\lambda,\mu)}$. Since $x_0 \in f^{-1}(y)$, $y \in f(A^{(\lambda,\mu)})$. Then $(f(A))^{(\lambda,\mu)} \subset f(A^{(\lambda,\mu)})$. Now let $y \in f(A^{(\lambda,\mu)})$. Then there exists $x_0 \in A(\lambda,\mu)$ such that $y = f(x_0)$. Since $x_0 \in A^{(\lambda,\mu)}$, $\mu_A(x_0) \geq \lambda$ and $\nu_A(x_0) \leq \mu$. Thus $\mu_{f(A)}(y) = \vee_{x \in f^{-1}}(y)\ \mu_A(x) \geq \lambda$ and $\nu_{f(A)}(y) = \wedge_{x \in f^{-1}}(y)\ \nu_A(x) \leq \mu$.

So $y \in (f(A))^{(\lambda,\mu)}$, i.e., $f(A^{(\lambda,\mu)}) \subset (f(A))^{(\lambda,\mu)}$. Hence $(f(A))^{(\lambda,\mu)} = f(A^{(\lambda,\mu)})$. This completes the proof.

Proposition 4.4. Let $f : L \to L'$ be a lattice epimorphism. If A is an intuitionistic fuzzy prime ideal[resp. prime filter] with the sup property in L, then $f(A) \in$ IFPI(L')[resp. IFPF(L')].

Proof. Let A be an intuitionistic fuzzy prime ideal with sup property in L. Then, Result 4.A(1), $f(A) \in \mathrm{IFI}(L')$. Let $(\lambda \mu) \in I \times 1$ with $\lambda + \mu \leq 1$ such that $(f(A))^{(\lambda,\mu)} \neq \theta$. Since A has the sup property, by Lemma 4.2, $(f(A))^{(\lambda,\mu)} = f(A^{(\lambda,\mu)})$. Thus $A^{(\lambda,\mu)} \neq \theta$. Since $A \in \mathrm{IFPI}(L)$, by Theorem 2.10, $A^{(\lambda,\mu)}$ is a a prime ideal of L. So $(f(A))^{(\lambda,\mu)}$ is prime ideal of L'. Hence, by Theorem 2.10, $f(A) \in \mathrm{IFPI}(L')$. Similarly, we can see that $f(A) \in \mathrm{IFPF}(L')$.

Definition 4.5[4]. Let $f : L \to L'$ be a mapping and let $A \in \mathrm{IFS}(L)$, where L and L' are lattices. Then A is said to be *IF-invariant* if $f(x) = f(y)$ implies $A(x) = A(y)$ for any $x, y \in L$.

Recall 4.B[3, Proposition 6.6]. Let $f : L \to L'$ be a mapping and let A be an IF-invariant intuitionistic fuzzy set in L. Then $f^{-1}(f(A)) = A$.

Proposition 4.6. Let $f : L \to L'$ be a lattice epimorphism and let A be an IF-invariant intuitionistic fuzzy prime ideal[resp. prime filter]. Then $f(A) \in \mathrm{IFPI}(L')$[resp. $\mathrm{IFPF}(L')$].

Proof. Assume that $f(A) \notin \mathrm{IFPI}(L')$. Then there exist $f(a), f(b) \in L'$ such that

$$f(A)(f(a) \,.\, f(b)) \neq f(A)(f(a)) \text{ and } f(A)(f(a)) \,.\, f(b)) \neq f(A)(f(b)).$$

Since $f(A) \in \mathrm{IFI}(L')$,

$$\mu_{f(A)}(f(a) \,.\, f(b)) > \mu_{f(A)}(f(a)) v_{f(A)}(f(a) \,.\, f(b)) < v_{f(A)}(f(a))$$

and

$$\mu_{f(A)}(f(a) \,.\, f(b)) > \mu_{f(A)}(f(b)) v_{f(A)}(f(a)) \,.\, f(b)) < v_{f(A)}(f(b)).$$

Since f is a lattice homomorphism,

$$\mu_{f^{-1}(f(A))}(a.b) > \mu_{f^{-1}(f(A))}(a)\, v_{f^{-1}(f(A))}(a.b) < v_{f^{-1}(f(A))}(a)$$

and

$$\mu_{f^{-1}(f(A))}(a.b) > \mu_{f^{-1}(f(A))}(b)\, v_{f^{-1}(f(A))}(a.b) < v_{f^{-1}(f(A))}(b).$$

Since A is IF-invariant, by Result 4.B,

$$\mu_A(a \,.\, b) > \mu_A(a)\,,\; v_A(a \,.\, b) < v_A(a)$$

and

$\mu_A(a.b) > \mu_A(b)$, $v_A(a.b) < v_A(b)$. This contradicts the fact that $A \in$ IFPI(L), Hence $f(A) \in$ IFPI(L'). Similarly, we can see that $f(A) \in$ IFPF(L').

From Proposition 4.5 and Result 4.A(2), we obtain the following result.

Theorem 4.6. Let $f : L \to L'$ be a lattice epimorphism and let $\mathcal{A}$[resp. $\mathcal{A}'$] be the set of all IF-invariant intuitionistic fuzzy prime ideals[resp. filters] of L. Then there is a one-to-one correspondence between $\mathcal{A}$[resp. $\mathcal{A}'$] and $IFPI(L')$[resp. $IFPF(L')$].

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