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ON INSTUITIONISTIC FUZZY D-ALGEBRA

ABSTRACT: *In this paper, we introduce the notion of intuitionistic fuzzy d-subalgebra, level intuitionistic fuzzy sets and intuitionistic fuzzy d-ideals in d-algebras, and investigated some of their results.*

Keywords: *Intuitionistic fuzzy d-subalgebra, level intuitionistic fuzzy sets, intuitionistic fuzzy d-ideals in d-algebras,homomorphism.*

1. INTRODUCTION

The two classes of abstract algebras: *BCK-algebra* and *BCI-algebras* were introduced by Imai and Iseki [4,5]. It is known that the notion of *BCI-algebras* is a generalization of *BCK-algebras*. Negger, Jun and Kim [7, 8] introduced the class of d-algebras which is another generalization of *BCK-algebras*, and investigated relations between d-algebras and *BCK-algebras*. After the introduction of the concept of fuzzy sets by Zadeh [10] several researches were conducted on the generalizations of the notation of fuzzy sets. Xi[9] introduced the notion of fuzzy *BCK-algebras*. Jun and Meng[6] studied fuzzy *BCK-algebra*. Ahn and Lee [1] studied fuzzy subalgebra of *BG-algebras*. Akram and Dar [2] studied on fuzzy d-algebras. The idea of '*intuitionistic fuzzy set*' was first published by Atanassov [3] as a generalization of the notion of fuzzy set. Zaraandi and Bourumand Saeid [11] studied intuitionistic fuzzy ideals of *BG-Algebras*. In this paper, we introduce the notion of intuitionistic fuzzy d-subalgebra, level intuitionistic fuzzy sets and intuitionistic fuzzy d-ideals in d-algebras, and investigated some of their results.

2. PRELIMINARIES

Definition 2.1: An algebra $(X; *, 0)$ of type $(2, 0)$ is called a *BCK-algebra* if it satisfies the following conditions:

1. $((x * y) * (x * z)) * (z * y) = 0$,

2. $((x * (x * y)) * y = 0$,

3. $x * x = 0$,

4. $0 * x = 0$,

5. $x * y = 0$ and $y * x = 0 \Rightarrow x = y$, for all $x, y \in X$.

Remark: A partial ordering "$\leq$" on X can be defined by $x \leq y$ if and only if $x * y = 0$.

Definition 2.2: A nonempty set X with a constant 0 and a binary operation $*$ is called a *d-algebra*, if it satisfies the following axioms:

d1. $x * x = 0$,

d2. $0 * x = 0$,

d3. $x * y = 0$ and $y * x = 0 \Rightarrow x = y$ for all $x, y \in X$.

Example 2.3: Let $X = \{0, 1, 2\}$ be a set with the following table:

Table 1

*	**0**	**1**	**2**
0	0	0	0
1	2	0	2
2	2	0	1

by usual calculation, it is clear that $(X; *, 0)$ is a d-algebra.

Hereafter in this paper throughout X denotes d-algebra, unless otherwise specified.

Definition 2.4: Let X be a d-algebra and I be subset of X , then I is called *d-ideal* of X if it satisfies following conditions.

I1. $0 \in I$

I2. $x * y \in I$ and $y \in I \Rightarrow x \in I$

I3. $x \in x, y \in I \Rightarrow x * y \in$ (*i.e.*) $I \times X \subseteq I$

Definition 2.5: Let S be a non-empty subset of a d-algebra X ,then S is called *d-subalgebra* of X if $x * y \in S$, for all $x, y \in S$.

Definition: A fuzzy subset of X is a function $\mu : X \to [0, 1]$.

Definition 2.6 : An Intuitionistic fuzzy set (IFS) A non-empty set X is an object having the form

$$A = \{(x, \alpha_A (x), \beta_A (x))/x \in X\},$$

Where the function $\alpha_A : x \to [0, 1]$ and $\beta_A : X \to [0, 1]$ denoted the degree of membership and the degree of non-membership respectively and $0 \leq \alpha_A(x) + \beta_A (x) \leq 1$ for all $x \in X$.

An IFS $A = \{x, \alpha_A (x), \beta_A (x))/x \in X\}$, the X can be identified to an ordered pair (α_A, β_A) in $I^X \times I^X$. For the sake of simplicity, we shall use the symbol $A = (\alpha_A, \beta_A)$ for the IFS $A = \{(x, \alpha_A (x), \beta_A(x))/x \in X\}$.

Definition 2.7: A fuzzy set μ in d-algebra X is called a *fuzzy d-subalgebra* of X if it satisfies $\mu (x * y) \geq \min \{\mu (x), \mu (y)\}$, for all $x, y \in X$.

3. INTUITIONISTIC FUZZY D-SUBALGEBRA

Definition 3.1: An IFS $A = (\alpha_A, \beta_A)$ in X is called an *intuitionistic fuzzy d-subalgebra* of X if it satisfies

1. $\alpha_A,(x * y) \geq \min \{\alpha_A(x), \alpha_A (y)\}$
2. $\beta_A,(x * y) \leq \max\{(\beta_A(x), \beta_A (y)\}$

Example 3.2: Let $X = \{0, 1, 2\}$ be a d-algebra with the following table:

Table 2

*	**0**	**1**	**2**
0	0	0	0
1	2	0	2
2	2	0	1

Define a IFS $A = (\alpha_A, \beta_A)$ in X as follows:

$$\alpha_A (0) = \alpha_A(1) = 0.7 > 0.3 = \alpha_A (2)$$
$$\beta_A (0) = \beta_A(1) = 0.2 < 0.5 = \beta_A (1)$$

by usual calculation we know that $A = (\alpha_A, \beta_A)$ is an IF d-subalgebra of X.

Definition 3.3: An IFS A is called level IFS if

$$A_t = \{x \in X / \alpha_A(x) \geq t \text{ and } \beta_A(x) \leq t\}, \text{ for } 0 \leq t \leq 1.$$

Remarks:

1. The upper and lower level set of α_A is defined by $\alpha^t_A = \{x \in X \mid \alpha_A(x) \geq t\}$ and $\alpha_{A,t} = \{x \in X \mid \alpha_A(x) \leq t\}$ respectively.
2. Two level IFSets A_s and A_t are IF equal ($A_s \doteq A_t$) if $\alpha^s_A = \alpha^t_A$ and $\beta_{A,s} = \beta_{a,t}$.
3. Two level IFSets A_s and A_t are IF subsets ($A_s \dot{\subseteq} A_t$) if and $\alpha^s_A \subseteq \alpha^t_A$ and $\beta_{A,s} \supseteq \beta_{a,t}$. (($\mathrm{A_s} \dot{\supseteq} \mathrm{A_t}$) if $\alpha^s_A \supseteq \alpha^t_A$ and $\beta_{A,s} \subseteq \beta_{A,t}$).

Proposition 3.4: For every intuitionistic fuzzy d-subalgebra of X (i) $\alpha_{\mathrm{A}}(0) \geq \alpha_A(x)$ (ii) $\beta_A(0) \leq \beta_A(x)$, for all $x \in X$.

Proof: It is easy and straightforward.

Proposition 3.5: An IFS A of a d-algebra X is a intutionistic fuzzy d-algebra if and only if for every $0 \leq t \leq 1$ the level IFS A_t is either empty or a subalgebra of X.

Proof: Suppose that A is a IF d-subalgebra of X and $A_t \neq 0$. For every $x, y \in A_t$

$$\alpha_A(x * y) \geq \min\{\alpha_A(x), \alpha_A(y)\} = t \text{ and}$$
$$\beta_A(x * y) \leq \max\{\beta_A(x), \beta_A(y)\} = t$$

Hence $x * y \in A_t$

Conversely, take $t = \min\{\alpha_A(x), \alpha_A(y)\}$ and $t = \max\{\beta_A(x), \beta_A(y)\}$ for every $x, y \in X$. If $x, y \in A_t$ then $\alpha_A(x * y) \geq t = \min\{\alpha_A(x), \alpha_A(y)\}$ and $\beta_A(x * y) \leq t = \max\{\beta_A(x), \beta_A(y)\}$. Hence A is IF d-subalgebra.

Proposition 3.6: Any IF d-subalgebra of a d-algebra X can be realized as a level sub d-algebra of some IF d-subalgebra of X.

Proof: Let A be a IF d-subalgebra of X.

Define

$$\alpha_A(x) = \beta_A(x) = \begin{cases} t & \text{if } x \in A \\ 0 \text{ or } 1 & \text{if } x \notin A \end{cases}$$

If $x, y \in A$, then $\alpha_A(x * y) \geq \min\{\alpha_A(x), \alpha_A(y)\} = t$ and

$$\beta_A(x * y) \leq \max(\beta_A(x), \beta_A(y) = t$$

Hence $x * y \in A_t$

If $x, y \notin A$, then $\alpha_A(x * y) \geq \min\{\alpha_A(x), \alpha_A(y)\} = 0$ and

$\beta_A(x * y) \leq \max(\beta_A(x), \beta_A(y) = 0$

Hence $x * y \in A_t$.

If atmost one of $x, y \in A$, then $\alpha_A(x * y) \geq \min\{\alpha_A(x), \alpha_A(y)\} = t$

$\beta_A(x * y) \leq \max(\beta_A(x), \beta_A(y) = \mathrm{t}$

Hence $(x * y) \in A_t$

Proposition 3.7: Let A_s and A_t $(s < t)$ be are two IF d-subalgebra of a d-algebra X. Then $A_s \doteq A_t$ if and only if there is no $x \in X$ such that $s \leq \alpha_A(x) < t$ and $s > \beta_A(x) \geq t$.

Proof: Suppose that $A_s \doteq A_t$, for some $s < t$. If there exist $x \in X$ such that $s \leq \alpha_A(x) < t$ and $s > \beta_A(x) \geq t$, then $A_s \dot{\subseteq} A_t$ which is a contradiction.

Conversely, suppose that there is no $x \in X$ such that $s \leq \alpha_A(x) < t$ and $s > \beta_A(x) \geq t$. If $x \in A_s$, then $\alpha_A(x) \geq s$ and $s > \beta_A(x) < s$, also $\alpha_A(x) < t$ and $\beta_A(x) \geq t$. Thus $\alpha_A^s \subseteq \alpha_A^t$ and $\beta_{A,s} \supseteq \beta_{A,t}$. The converse inclusion is obvious since $s < t$. Hence $A_s \doteq A_t$.

4. INTUITIONISTIC FUZZY D-IDEALS

Definition 4.1: An IFS $A = (\alpha_A, \beta_A)$ in X is called an *IF d-ideal* of X if it is satisfies

(i) $\alpha_A(0) \geq \alpha_A(x)$ $\beta_A(0) \leq \beta_A(x)$,

(ii) $\alpha_A(x) \geq \min\{\alpha_A(x * y), \alpha_A(x)\}$, $\beta_A(x) \leq \max\{\beta_A(x * y), \beta_A(x)\}$,

(iii) $\alpha_A(x * y) \geq \min\{\alpha_A(x), \alpha_A(y)\}$, $\beta_A(x * y) \leq \max\{\beta_A(x), \beta_A(y)\}$.

Clearly, every IF d-ideal of a d-algebra is an IF d-subalgebra of X.

Example 4.2: Let $X = \{0, 1, 2\}$ be a d-algebra with the following table:

Table 3

*	**0**	**1**	**2**
0	0	0	0
1	2	0	2
2	2	0	1

Define a IFS $A = \{\alpha_A, \beta_A)$ in X as follows:

$$\alpha_A(0) = \alpha_A(2) = 1 \qquad \alpha_A(1) = t$$

$$\beta_A(0) = \beta_A(2) = 1 \qquad \beta_A(1) = s$$

where $0 \leq t \leq 1$, $0 \leq s \leq 1$, and $t + s = 1$. By usual calculation we know that $A = (\alpha_A, \beta_A)$ is an IF d-ideal of X.

Theorem 4.3: Let $A = (\alpha_A, \beta_A)$ in X be an IF d-ideal of X. If $x * y \leq z$ then

(i) $\alpha_A(x) \geq \min\{\alpha_A(y), \alpha_A(z)\}$, $\beta_A(x) \leq \max\{\beta_A(y), \beta_A(z)\}$ and

(ii) $\alpha_A(0) \geq \alpha_A(z)$, $\beta_A(0) \leq \beta_A(z)$, for all $x, y, z \in X$.

Proof: Let $x, y, z \in X$ such that $x * y \leq z$. (If $x * y \leq z$ then $(x * y) * z = 0$.) Then

(i) It is easy to prove

that $\alpha_A(x) \geq \min\{\alpha_A(y), \alpha_A(z)\}$, $\beta_A(x) \leq \max\{\beta_A(y), \beta_A(z)\}$.

(ii) $\alpha_A((x * y) * z) \geq \min\{\alpha_A(x * y), \alpha_A(z)\}$

$\alpha_A(0) \geq \min\{\min\{\alpha_A((x * y) * z), \alpha_A(z)\}, \alpha_A(z)\}$

$\alpha_A(0) \geq \min\{\min\{\alpha_A(0), \alpha_A(z)\}, \alpha_A(z)\}$

$\alpha_A(0) \geq \min\{\alpha_A(z), \alpha_A(z)\}$

$\alpha_A(0) \geq \alpha_A(z)$

$\beta_A((x * y) * z) \leq \max\{\beta_A(x * y), \beta_A(z)\}$

$\beta_A(0) \leq \max\{\max\{\beta_A((x * y) * z), \beta_A(z)\}, \beta_A(z)\}$

$\beta_A(0) \leq \max\{\max\{\beta_A(0), \beta_A(z)\}, \beta_A(z)\}$

$\beta_A(0) \leq \max\{\beta_A(z), \beta_A(z)\}$

$\beta_A(0) \geq \beta_A(z)$

Theorem 4.4: Let $A = (\alpha_A, \beta_A)$ in X be an IF d-ideal of X. If $x \leq y$ then

(i) $\alpha_A(0) \geq \alpha_A(x) \geq \alpha_A(y)$ and,

(ii) $\beta_A(0) \leq \beta_A(x), \leq \beta_A(y)$, for all $x, y \in X$.

Proof:

For $x, y \in X$ and $x \leq y$ then $x * y = 0$

$$\alpha_A(x) \geq \min \{\alpha_A (x * y), \alpha_A (y)\}$$

$$\alpha_A(x) = \min \{\alpha_A (0), \alpha_A (y)\}$$

$$\alpha_A(x) = \alpha_A (y)$$

and

$$\alpha_A(x * y) \geq \min \{\alpha_A (x), \alpha_A (y)\}$$

$$\alpha_A(0) = \min \{\alpha_A (x), \alpha_A (y)\}$$

$$\alpha_A(0) = \alpha_A (x)$$

Hence $\alpha_A(0) \geq \alpha_A (x) \geq \alpha_A (y)$

Similarly we can prove (ii).

Definition 4.5: Let $A = (\alpha_A, \beta_A)$ and $B = (\alpha_B, \beta_B)$ be two IF sets of a d-algebra X. Then the cartesian product of $A \times B : X \times X \to [0, 1]$ is defined by,

$(\alpha_A \times \alpha_B) (x, y) = \min \{\alpha_A (x), \alpha_B(y)$ and $(\beta_A \times \beta_B) (x, y) = \max \{\beta_A (x), \beta_B(y)\}$, for all $x, y \in X$.

Theorem 4.6: If $A = (\alpha_A, \beta_A)$ and $B = (\alpha_B, \beta_B)$ are IF d-ideal of a d-algebra X. Then $A \times B$ is a IF d-Ideal of X.

Proof: For any $(x, y) \in X \times X$, we have

(i) $(\alpha_A \times \alpha_B) (0, 0) = \min \{\alpha_A (0), \alpha_B(0)\}$

$$\geq \min \{\alpha_A (x), \alpha_B(y)\}$$

$$= (\alpha_A \times \alpha_B) (x, y)$$

and

$(\beta_A \times \beta_B) (0, 0) = \max \{\beta_A (0), \beta_B(0)\}$

$$\geq \max \{\beta_A (x), \beta_B(y)\}$$

$$= (\alpha_A \times \alpha_B) (x, y)$$

(ii) Let (x_1, x_2) and $(y_1, y_2) \in X \times X$, then

$$\begin{aligned}
(\alpha_A \times \alpha_B)(x_1, x_2) &= \min\{\alpha_A(x_1), \alpha_B(x_2)\} \\
&\geq \min\{\min\{\alpha_A(x_1 * y_1), \alpha_A(y_1)\} \min\{\alpha_B(x_2 * y_2), \alpha_B(y_2)\}\} \\
&= \min\{\min\{\alpha_A(x_1 * y_1), \alpha_B(x_2 * y_2)\}, \min\{\alpha_A(y_1), \alpha_B(y_2)\}\} \\
&= \min\{(\alpha_A \times \alpha_B)(x_1 * y_1, x_2 * y_2), (\alpha_A \times \alpha_B)(y_1, y_2)\} \\
\text{and} \quad &= \min\{(\alpha_A \times \alpha_B)((x_1, x_2) * (y_1 * y_2)), (\alpha_A \times \alpha_B)(y_1, y_2)\}
\end{aligned}$$

$$\begin{aligned}
(\beta_A \times \beta_B)(x_1, x_2) &= \max\{\beta_A(x_1), \beta_B(x_2)\} \\
&\leq \max\{\beta_A(x_1 * y_1), \beta_B(y_1)\} \max\{\beta_A(x_2 * y_2), \beta_B(y_2)\} \\
&= \max\{\max\{\beta_A(x_1 * y_1), \beta_B(x_2 * y_2)\}, \max\{\beta_A(y_1), \beta_B(y_2)\}\} \\
&= \max\{(\beta_A \times \beta_B)(x_1 * y_1, x_2 * y_2), (\beta_A \times \beta_B)(y_1, y_2)\} \\
&= \max\{(\beta_A \times \beta_B)((x_1, x_2) * (y_1 * y_2)), (\beta_A \times \beta_B)(y_1, y_2)\}
\end{aligned}$$

(iii) Let (x_1, x_2) and $(y_1, y_2) \in X \times X$, then

$$\begin{aligned}
(\alpha_A \times \alpha_B)(x_1, x_2) * (y_1, y_2) &= (\alpha_A \times \alpha_B)((x_1 * y_1, x_2 * y_2)) \\
&= \min\{\alpha_A(x_1 * y_1), \alpha_B(x_2 * y_2)\} \\
&\geq \min\{\min\{\alpha_A(x_1), \alpha_A(y_1)\} \min\{\alpha_B(x_2), \alpha_B(y_2)\}\} \\
&= \min\{\min\{\alpha_A(x_1), \alpha_B(x_2)\}, \min\{\alpha_A(y_1), \alpha_B(y_2)\}\} \\
&= \min\{(\alpha_A \times \alpha_B)(x_1, x_2), (\alpha_A \times \alpha_B)(y_1, y_2)\}
\end{aligned}$$

and

$$\begin{aligned}
(\beta_A \times \beta_B)(x_1, x_2) * (y_1, y_2) &= (\beta_A \times \beta_B((x_1 * y_1, x_2 * y_2,)\} \\
&= \max\{\beta_A(x_1 * y_1), \beta_B(x_2 * y_2)\} \\
&\leq \max\{\max\{\beta_A(x_1), \beta_A(y_1)\}, \max\{\beta_B(x_2), \beta_B(y_2)\}\} \\
&= \max\{\max\{\beta_A(x_1), \beta_B(x_2)\}, \max\{\beta_A(y_1), \beta_B(y_2)\}\} \\
&= \max\{(\beta_A \times \beta_B)((x_1, x_2), (\beta_A \times \beta_B)(y_1, y_2)\}
\end{aligned}$$

Hence $A \times B$ is a IF d-Ideal of X.

5. HOMOMORPHISM OF IF D-ALGEBRA

Definition 5.1: A mapping $f: X \to Y$ of d-algebras is called a homomorphism if $f(x * y) = f(x) * f(y)$, for all $x, y \in X$. Note that if $X \to Y$ is a homeomorphism of d-algebra then $f(0) = 0$.

Let $f: X \to Y$ be a homomorphism of d-algebra for any IFS $A = (\alpha_A, \beta_A)$ in Y. We define a new IFS $A^f = (\alpha_A^f, \beta_A^f)$ in X by $\alpha_A^f(x) = \alpha_A(f(x)), \beta_A^f)(x)\ \beta_A(f(x))$ for all $x \in X$.

Theorem 5.2: Let $f: X \to Y$ be a homomorphism of d-algebra. If an IFS $A = (\alpha_A, \beta_A)$ is an IF d-ideal then an IFS $A^f = (\alpha_A^f, \beta_A^f)$ in X is an IF d-ideal of X.

Proof: (i) $\alpha_A^f(x) = \alpha_A(f(x)) \leq \alpha_A(0)\ \alpha_A(f(0)) = \alpha_A^f(0)$ and

$$\beta_A^f(x) = \beta_A(f(x)) \geq \beta_A(0) = \beta_A(f(0)) = \beta_A^f(0)$$

(ii) $\alpha_A^f(x) = \alpha_A(f(x)) \geq \min\{\alpha_A(f(x) * f(y))\ \alpha_A(f(\mathrm{y}))\}$

$$= \min\{\alpha_A(f(x * y))\ \alpha_A(f(y))\}$$

$$\alpha_A^f(x) = \min\{\alpha_A^f(x * y), \alpha_A^f(y)\}$$

and

$$\beta_A^f(x) = \beta_A(f(x)) \leq \max \beta_A(f(x) * f(y)), \beta_A(f(y))\}$$

$$= \max \beta_A(f(x * y)), \beta_A(f(y))\}$$

$$\beta_A^f(x) = \max\{\beta_A^f(x * y), \beta_A^f(y)\}$$

(iii) $\alpha_A(f(x) * f(y)) \geq \min\{\alpha_A(f(x)), \alpha_A(f(y))\}$

$$\alpha_A(f(x * y)) = \min\{\alpha_A^f(x), \alpha_A^f(y)\}$$

$$\alpha_A^f(x * y) = \min\{\alpha_A^f(x), \alpha_A^f(y)\}$$

and

$$\beta_A(f(x) * f(y)) \leq \max\{\beta_A(f(x)), \beta_A(f(y))\}$$

$$\beta_A(f(x * y)) = \max\{\beta_A^f(x), \beta_A^f(y)\}$$

$$\beta_A^f(x * y) = \max\{\beta_A^f(x), \beta_A^f(y)\}$$

Theorem 5.3: Let $f: X \to Y$ be an epimorphism of d-algebra. If $A^f = (\alpha_A^f, \beta_A^f)$ is IF d-algebra of X then $A = (\alpha_A, \beta_A)$ ais an IF d-ideal of Y.

Proof: Let $y \in Y$, there exists $x \in X$ such that $f(x) = y$. Then

(i) $\alpha_A(y) = \alpha_A(f(x)) = \alpha_A^f(x) \le \alpha_A^f(0) = \alpha_A(f(0)) = \alpha_A(0)$

and

$\beta_A(y) = \beta_A(f(x)) = \beta_A^f(x) \ge \beta_A^f(0) = \beta_A(f(0)) = \beta_A(0)$.

(ii) Let $x, y, \in Y$. Then there exist $a, b \in X$ such that $f(a) = x$ and $f(b) = x$, then

$$\begin{aligned}\alpha_A(x) = \alpha_A(f(a)) = \alpha_A^f(a) &\ge \min\{\alpha_A^f(a * b) = \alpha_A^f(b)\}\\ &= \min\{\alpha_A(f(a * b)), \alpha_A(f(b))\}\\ &= \min\{\alpha_A(f(a) * f(b)), \alpha_A(f(b))\}\\ &= \min\{\alpha_A(x * y), \alpha_A(f(y))\}\end{aligned}$$

and

$$\begin{aligned}\beta_A(x) = \beta_A(f(a)) = \beta_A^f(a) &\le \max\{\beta_A^f(a * b) = \beta_A^f(b)\}\\ &= \max\{\beta_A(f(a * b)), \beta_A(f(b))\}\\ &= \max\{\beta_A(f(a) * f(b)), \beta_A(f(b))\}\\ &= \max\{\beta_A(x * y), \beta_A(f(y))\}\end{aligned}$$

(iii) Let $x, y, \in Y$. Then there exist $a, b \in X$ such that $f(a) = x$ and $f(b) = x$, then

$$\begin{aligned}\alpha_A(x * y) = \alpha_A(f(a) * y(b)) = \alpha_A^f(a * b) &\ge \min\{\alpha_A^f(a), \alpha_A^f(b)\}\\ &= \min\{\alpha_A(f(a)), \alpha_A(f(b))\}\\ &= \min\{\alpha_A(x), \alpha_A(y)\}\end{aligned}$$

and

$$\begin{aligned}\beta_A(x * y) = \beta_A(f(a) * (b)) = \beta_A^f(a * b) &\le \max\{\beta_A^f(a), \beta_A^f(b)\}\\ &= \max\{\beta_A(f(a)), \beta_A(f(b))\}\\ &= \max\{\beta_A(x), \beta_A(y)\}.\end{aligned}$$

Definition 5.4: Let f be a map from a set X to Y. If $A = (\alpha_A, \beta_A)$ and $B = (\mu_B, \gamma_B)$ are IFS in X and Y respectively, then the pre image of B under f, denoted by $f^{-1}(B)$, is IFS in X defined by $f^{-1}(B) = f^{-1}(\mu_B), f^{-1}(\gamma_B)$ where $f^{-1}(\mu_B) = (\mu_B) f$.

Theorem 5.5: Let S be sub d-algebra of X and f: $S \to S$ be a map defined by $f(x) = x$, for all $x \in S$. If $A = (\alpha_A, \beta_A)$ is an IF d-ideal of X, then the pre image $f^{-1}(A)$, of A under f is an IF d-ideal of S.

Proof: (i) $f^{-1}(\alpha_A(0)) = \alpha_A(f(0)) = \alpha_A(0) \geq \alpha_A(x) = \alpha_A(f(x)) = f^{-1}(\alpha_A)(x)$

and

$$f^{-1}(\beta_A(0)) = \beta_A(f(0)) = \beta_A(0) \leq \beta_A(x) = \beta_A(f(x)) = f^{-1}(\beta_A)(x)$$

(ii) For all x, y, $\in S$, then

$$\begin{aligned} f^{-1}(\alpha_A(x)) = \alpha_A(f(x)) = \alpha_A(x) &\geq \min\{\alpha_A(x * y), \alpha_A(y)\} \\ &= \min\{\alpha_A(f(x * y)), \alpha_A(y)\}, \text{ since } S \text{ is sub d-algebra of } X. \\ &= \min\{f^{-1}(\alpha_A)(x * y), f^{-1}(\alpha_A)(y)\} \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\beta_A)(x) &= \beta_A(f(x)) = \beta_A(x) \geq \max\{\beta_A(x * y), \beta_A(y)\} \\ &= \max\{\beta_A(f(x * y)), \beta_A(y)\}, \text{ since } S \text{ is sub d-algebra of } X. \\ &= \max\{f^{-1}(\beta_A)(x * y), f^{-1}(\beta_A)(y)\}. \end{aligned}$$

(iii) For all x, y, $\in S$, then

$$\begin{aligned} f^{-1}(\alpha_A)(x * y) = \alpha_A(f(x * y)) = \alpha_A(x * y) &\geq \min\{\alpha_A(x), \alpha_A(y)\} \\ &= \min\{\alpha_A(f(x)), \alpha_A(f(y))\} \\ &= \min\{f^{-1}(\alpha_A)(x), f^{-1}(\alpha_A)(y)\} \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\beta_A)(x * y) &= \beta_A(f(x * y)) = \beta_A(x * y) \leq \max\{\beta_A(x), \beta_A(y)\} \\ &= \max\{\beta_A(f(x)), \beta_A(f(y))\} \\ &= \max\{f^{-1}(\beta_A)(x), f^{-1}(\beta_A)(y)\} \end{aligned}$$

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