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ON IMPROVING A WEIGHTED ADDITIVE MODEL FOR FUZZY GOAL PROGRAMMING PROBLEMS

ABSTRACT: *Fuzzy goal programming is a useful tool to deal with problems involving multi objective goals in a fuzzy environment. It is used positively to solve real life problems. To solve fuzzy goal programming problems several weighted additive models are proposed in the literature. A weighted additive model is formulated by Tiwari, Daharmar, and Rao (Fuzzy Sets and Systems 24 (1987) 27-34). This model is used in the literature and some further research has been carried out based on it. However, there is an oversight within the formulation of this model that sometimes yields suboptimal solutions. The oversight is also repeated in a new research which is done by Chen and Tsai (European Journal of Operational Research 133 (2001) 548-556). This paper explains the lack of precision within the formulation of Tiwari et al.'s model and a correction is suggested to enable it to achieve better solutions. This correction helps us to improve the model to deal with any kind of linear fuzzy goals easily. Illustrative examples are given to support the ideas.*

Keywords: *Fuzzy goal programming; Goal programming; Fuzzy programming*

1. INTRODUCTION

Goal programming (GP) is a well-known approach for solving multiple criteria decision making problems with several conflicting objectives. Charnes and Cooper [4] introduced GP in 1961. Since then, GP has been applied extensively in practice [7, 14]. GP models aim to minimize deviations of the objective values from aspiration (target) levels, specified by decision maker(s) [4, 13, 15]. However, determining precise aspiration levels for the objectives in real world problems often is a dicult task for decision maker(s). In fact, most of the real world problems take place in an imprecise environment. An objective with an imprecise aspiration level can be treated as a fuzzy goal [12]. Initially, fuzzy set theory was combined with GP by Narasimhan in 1980 and he presented a fuzzy goal programming (FGP) model [12]. Narasimhan used the basic notion of fuzzy subsets to solve FGP problems, where his method involved solving a set of 2K linear programming (LP) problems each containing 3K constraints, K denotes the number of fuzzy goals in the original problem. Hannan [6] simplified Narasimhan's method as an equivalent LP in 1981. After these pioneering works, extensive research in the field of FGP has been performed and applied to real life problems [1, 2, 9, 10]. To solve FGP problems different models based on different approaches are proposed [5, 6, 8, 11, 17, 18, 19]. In [3], a survey and classification of FGP models is presented. Among various methods for solving FGP problems, different weighted additive models are proposed [5, 6, 8]. One of the first weighted additive models is formulated by Tiwari, Daharmar, and Rao (TDR model) [16]. Some further research has been carried out based on it [5]. However, in the formulation of this model an oversight has happened. This paper discusses the oversight within the formulation. It is proved that the TDR model can yield suboptimal and therefore undesirable solutions. A correction within the formulation is suggested. It is shown that the suggestion allows the model to achieve a better solution. In this paper, a general form of FGP problem is introduced which includes all kinds of fuzzy goals. The fuzzy goals, which are not considered by the TDR model, are treated easily. Some examples are added to illustrate the discussions.

2. AN ADDITIVE MODEL FOR FUZZY GOAL PROGRAMMING

2.1 Fuzzy goal programming

In conventional GP models the decision maker is required to specify a precise aspiration level for each of the objectives. Sometimes, in real world problems the aspiration levels are not known precisely. In such situations fuzzy set theory can be employed [12, 20]. An objective with an imprecise aspiration level can be treated as a fuzzy goal. The possible fuzzy goals are considered in the following general form of FGP model [12, 16, 17].

$$OPT \quad G_i(X) \lesssim g_i \qquad i = 1, ..., i_0 \tag{1}$$

$$G_i(X) \gtrsim g_i \qquad i = i_0 + 1, ..., j_0 \tag{2}$$

$$G_i(X) \;\; \cong \;\; g_i \qquad i = j_0 + 1, \ldots, k_0 \tag{3}$$

$$G_i(X) \;\; \tilde{\in} \;\; [g_i^l, g_i^u] \qquad i = k_0 + 1, \ldots, K \tag{4}$$

$$X \;\; \in \;\; C_s, \qquad \text{‘[gl}$$

where

- OPT means finding an optimal decision X such that all fuzzy goals are satisfied [6, 12, 16],
- $G_i = \sum_{j=1}^{n} a_{ij} x_j, \;\; i = 1, \ldots, K,$
- g_i is the imprecise aspiration level for the ith fuzzy goal ($i = 1, \ldots, k_0$),
- g_i^l and g_i^u are the imprecise lower and upper bounds for the ith fuzzy goal respectively ($i = k_0 + 1, \ldots, K$),
- C_s is an optional set of hard constraints as found in LP,
- the symbol ‘~’ is a fuzzifier representing the imprecise fashion in which the goals are stated. In fact, the symbols $\lesssim$ ($\gtrsim$), $\cong$ and $\tilde{\in}$ refer to approximately lesser (greater) than or equal to, approximately equal to and approximately belong to respectively.

In fuzzy set theory membership functions identify fuzzy subsets [20]. Therefore, fuzzy goals can be identified as fuzzy sets defined over the feasible set with the membership functions. Piecewise linear membership functions are used more than other types of membership functions to express the fuzzy goals [16, 19, 21]. In this paper, for fuzzy goals (1)-(4) piecewise linear membership functions are defined respectively as follows [16, 19]:

$$\mu_i = \begin{cases} 1 & G_i(X) \leq g_i \\ \dfrac{U_i - G_i(X)}{U_i - g_k} & g_i \leq G_i(X) \leq U_i \qquad i = 1, \ldots, i_0 \\ 0 & G_i(X) \geq U_i \end{cases} \tag{5}$$

$$\mu_i = \begin{cases} 1 & G_i(X) \geq g_i \\ \dfrac{G_i(X) - L_i}{g_i - L_i} & L_i \leq G_i(X) \leq g_i \quad i = i_0 + 1, \ldots, j_0 \\ 0 & G_i(X) \leq L_i \end{cases} \tag{6}$$

$$\mu_i = \begin{cases} 0 & G_i(X) \leq L_i \\ \dfrac{G_i(X) - L_i}{g_i - L_i} & L_i \leq G_i(X) \leq g_i \quad i = j_0 + 1, \ldots, k_0 \\ \dfrac{U_i - G_i(X)}{U_i - g_i} & g_i \leq G_i(X) \leq U_i \\ 0 & G_i(X) \geq U_i \end{cases} \tag{7}$$

$$\mu_i = \begin{cases} 0 & G_i(X) \leq L_i \\ \dfrac{G_i(X) - L_i}{g_i^l - L_i} & L_i \leq G_i(X) \leq g_i^l \\ 1 & g_i^l \leq G_i(X) \leq g_i^u \quad i = k_0 + 1, \ldots, K \\ \dfrac{U_i - G_i(X)}{U_i - g_i^u} & g_i^u \leq G_i(X) \leq U_i \\ 0 & G_i(X) \geq U_i, \end{cases} \tag{8}$$

where L_i and U_i are the lower and upper limits of the maximum admissible violations for fuzzy goals [16]. They are either subjectively chosen by the decision maker [6, 12] or tolerances in a technical process [8, 9]. The above membership functions are depicted in Figure 1 respectively.

2.2. The TDR model

In [16], Tiwari et al. consider only fuzzy goals of types (1) and (2) with the membership functions of types (5) and (6) respectively. They use the usual addition as an operator to aggregate the fuzzy goals. Their model for solving an FGP problem with fuzzy goals of types (1) and (2) is as follows:

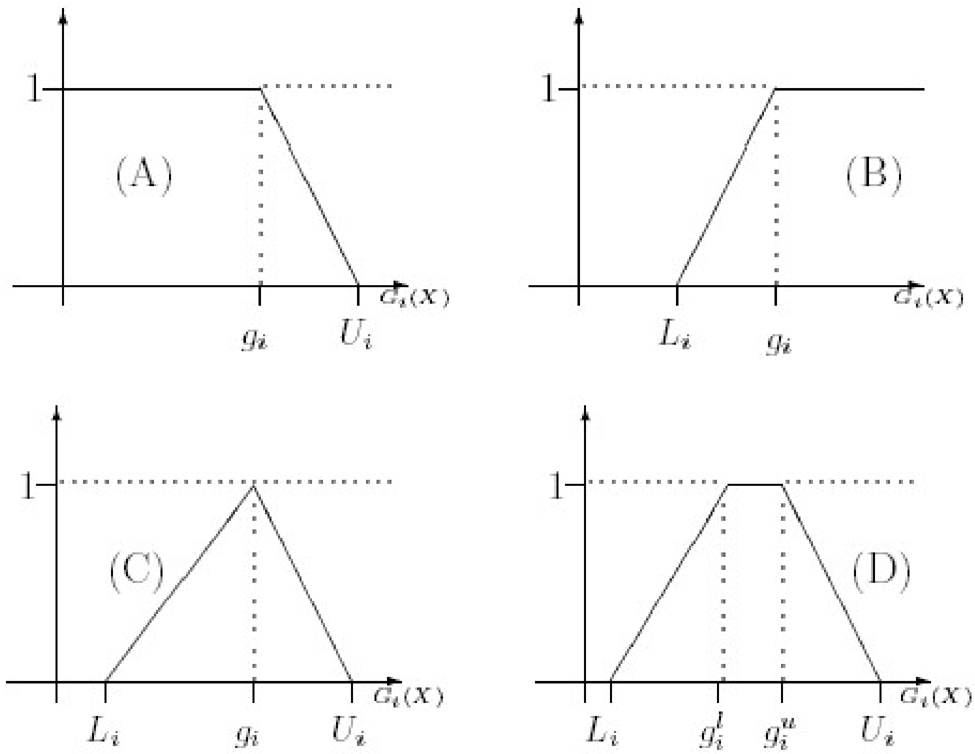

Fig. 1: Piecewise Linear Membership Functions

$$\begin{aligned} &\text{maximize } V(\mu) = \sum_{i=1}^{K} w_i \mu_i \\ &\text{s.t.} \\ &\mu_i = \frac{U_i - G_i(X)}{U_i - g_i} \qquad i = 1, \ldots, i_0 \\ &\mu_i = \frac{G_i(X) - L_i}{g_i - L_i} \qquad i = i_0 + 1, \ldots, K \\ &0 \le \mu_i \le 1 \qquad i = 1, \ldots, K \\ &X \in C_s, \end{aligned} \tag{9}$$

where wi is the relative weight of the ith fuzzy goal and $\sum_{i=1}^{K} w_i = 1$. $V(\mu)$ has been called the fuzzy achievement function or fuzzy decision function.

The following section discusses an oversight within model (9) and suggests a correction to improve the model.

3. IMPROVING THE TDR MODEL

It is obvious that for fuzzy goals $G_i(X) \lesssim g_i$ ($G_i(X) \gtrsim g_i$) solutions X which obtain for $G_i(X)$ lesser (greater) values than g_i are more desirable solutions. It can be seen from Figure 1 ((A) and (B)), since these solutions have the highest value of membership function (i.e. 1). However, the following theorem shows that in the TDR model these fuzzy goals are not allowed to achieve values strictly lower or greater than g_i.

Theorem 3.1: In model (9), $G_i(X) < g_i$ for $i = 1, ..., i_0$ and $G_i(X) > g_i$ for $i = i_0 + 1, ..., K$ never hold.

Proof. In model (9), $0 \le \mu_i \le 1$ for $i = 1, ..., K$ and

- for $i = 1, ..., i_0$, $\mu_i = \dfrac{U_i - G_i(X)}{U_i - g_i}$ thus $0 \le \dfrac{U_i - G_i(X)}{U_i - g_i} \le 1$. Hence
- for $i = i_0 + 1, ..., K$, $\mu_i = \dfrac{G_i(X) - L_i}{g_i - L_i}$ thus $0 \le \dfrac{G_i(X) - L_i}{g_i - L_i} \le 1$. Hence

  $0 \le G_i(X) - L_i \le g_i - L_i$ and $L_i \le G_i(X) \le g_i$.

Theorem 3.1 shows an oversight in the formulation of model (9). It can be seen from Figure 1 that for fuzzy goals (1) and (2) there exists a large possibility for μ_i to have a value of 1. However, μ_i in model (9) can have value of 1 only when $G_i(X) = g_i$. To eliminate the problem in model (9) and to improve the optimal solution, this paper proposes the following model.

$$\text{maximize} \quad Z = \sum_{i=1}^{K} w_i \mu_i$$

$$\text{s.t.}$$

$$\mu_i \le \frac{U_i - G_i(X)}{U_i - g_i} \qquad i = 1, ..., i_0$$

$$\mu i \le \frac{G_i(X) - L_i}{g_i - L_i} \qquad i = i_0 + 1, ..., K \tag{10}$$

$$0 \leq \mu_i \leq 1 \qquad i = 1, \ldots, K$$
$$X \in Cs.$$

In model (10), $G_i(X)$ is not restricted to have special values. It can have greater values than g_i as well as lower values. Theorem 3.2 shows model (10) always yields an optimum value that is as good as the TDR model.

Theorem 3.2: Suppose both models (9) and (10) have optimal solutions and $V(\mu^o)$ and Z^* are the optimal values respectively, then $V(\mu^o) \leq Z^*$.

Proof. Let (X^o, μ^o) be the optimal solution of model (9) with the optimum value $V(\mu^o)$. It is clear that (X^o, μ^o) is a feasible solution for model (10). Let Z^0 be the objective function value of model (10) for (X^o, μ^o) then $Z^o = V(\mu^o)$. Since model (10) is a maximization LP, every feasible solution has a lower or equal value than the optimum value. Therefore, $Z^o \leq Z^*$ and hence $V(\mu^o) \leq Z^*$.

Example 1 shows that model (10) can obtain a strictly greater optimum value than model (9).

Example 1. Tiwari et al. solved a numerical example to illustrate model (9) [16]. Their example is considered here. However, for the aim of this paper only weights of fuzzy goals are changed arbitrarily. Thus the FGP problem is:

$$\begin{aligned}
\text{OPT} \quad & G_1: & 4x_1 + 2x_2 + 8x_3 + 1x_4 &\lessapprox \text{‘}35 \\
& G_2: & 4x_1 + 7x_2 + 6x_3 + 2x_4 &\gtrapprox 100 \\
& G_3: & x_1 - 6x_2 + 5x_3 + 10x_4 &\gtrapprox 120 \\
& G_4: & 5x_1 + 3x_2 + 2x_4 &\gtrapprox 70 \\
& G_5: & 4x_1 + 4x_2 + 4x_3 &\gtrapprox 40 \\
& & 7x_1 + 5x_2 + 3x_3 + 2x_4 &\leq 98 && (11) \\
& & 7x_1 + x_2 + 6x_3 + 6x_4 &\leq 117 && (12) \\
& & x_1 + x_2 + 2x_3 + 6x_4 &\leq 130 && (13) \\
& & 9x1 + x_2 + 6x_4 &\leq 105 && (14) \\
& & x_1, x_2, x_3, x_4 &\geq && (15)
\end{aligned}$$

The tolerance limits of the five fuzzy goals are set as (55, 40, 70, 30, 10) respectively in [16]. In this example weights for fuzzy goals are set as (0.1, 0.1, 0.1, 0.6, 0.1) arbitrarily. Model (10) for solving this FGP is:

$$\text{maximize} \quad Z = 0.1\mu_1 + 0.1\mu_2 + 0.1\mu_3 + 0.6\mu_4 + 0.1\mu_5$$

s.t.

$$\mu_1 \le \frac{55-(4x_1+2x_2+8x_3+x_4)}{20} \tag{16}$$

$$\mu_2 \le \frac{4x_1+7x_2+6x_3+2x_4-40}{60} \tag{17}$$

$$\mu_3 \le \frac{x_1-6x_2+5x_3+10x_4-70}{50} \tag{18}$$

$$\mu_4 \le \frac{5x_1+3x_2+2x_4-30}{40} \tag{19}$$

$$\mu_5 \le \frac{4x_1+4x_2+4x_3-10}{30} \tag{20}$$

$$0 \le \mu_i \le 1 \qquad i = 1, ..., 5$$

Plus constraints (11) – (15),

with the optimal solution

$$(x_1^*, x_2^*, x_3^*, x_4^*) = (0, 13.12, 0, 15.31),$$

$$(\mu_1^*, \mu_2^*, \mu_3^*, \mu_4^*, \mu_5^*) = (0.67, 1, 0.09, 1, 1),$$

$$Z^* = 0.88.$$

If in constraints (16)-(20) '≤' replace with '=', then model (9) for solving the above FGP problem attains the optimal solution

$$(x_1^o, x_2^o, x_3^o, x_4^o) = (0, 9.75, 0, 15.88),$$

$$(\mu_1^o, \mu_2^o, \mu_3^o, \mu_4^o, \mu_5^o) = (0.98, 1, 0.6, 0.78, 0.97),$$

$$Z^o = 0.82.$$

Example 1 shows that the TDR model could yield suboptimal solutions and model (10) obtains sometimes strictly better solutions. Example 2 explains an invaild conclusion which is deduced based on the results of model (9).

Example 2. In a recent paper [5], the FGP problem in Example 1 is solved by model (9) with another set of weights. In [5, P. 552], (0.001,0.05,0.2,0.7,0.049) are considered as weights for the fuzzy goals. The optimal solution of model (9) with this set of weights is:

$$\begin{aligned}(x_1^o, x_2^o, x_3^o, x_4^o) &= (0, 8.26, 1.66, 16.12)\\(\mu_1^o, \mu_2^o, \mu_3^o, \mu_4^o, \mu_5^o) &= (0.45, 1, 1, 0.68, 0.99),\\Z^o &= 0.77.\end{aligned}$$

It can be seen that the fourth fuzzy goal has the greatest weight with respect to the other fuzzy goals. But, in the optimal solution of model (9) it is not strongly satisfied, since $\mu_4^o = 0.68.$ In [5], this is assumed as deficiency of model (9) and has tried to develop another model for dealing with this problem. However, the optimal solution of model (10) with this new set of weights is:

$$\begin{aligned}(x_1^*, x_2^*, x_3^*, x_4^*) &= (0, 12.3, 1.87, 15.45),\\(\mu_1^*, \mu_2^*, \mu_3^*, \mu_4^*, \mu_5^*) &= (0, 1, 0.4, 0.94, 1),\\Z^* &= 0.84.\end{aligned}$$

Now, the fourth fuzzy goal is strongly satisfied and it can be seen that the lack of precision within the formulation was the cause of this problem.

It should be reminded that in [5], to overcome the discussed problem, weights from the objective function of model (9) are omitted and $\mu_i \geq \alpha_i (i, ..., K)$ are inserted to the model, where i is the desirable achievement degree for the ith fuzzy goal and is specified by the decision maker. The model proposed in [5, P. 552] is as follows:

$$\begin{aligned}&\textit{maximize} \quad \sum_{i=1}^{K} \mu_i\\&\text{s.t.}\\&\mu_i \geq \alpha_i \qquad i = 1, ..., K \qquad (21)\end{aligned}$$

Plus all of the constraints of model (9).

However, model (21) has the same oversight as model (9) which is discussed in this section. A similar theorem to Theorem 3.1 can be used to prove that model (21) yields suboptimal solutions. It could be corrected in the same way as model (10).

An advantage of model (10) is that μ_i in the optimal solution still determines the degree of membership function for the ith fuzzy goal. Theorem 3.3 proves this fact. In this theorem it is supposed that all weights are strictly positive, otherwise a fuzzy goal with a zero weight could be omitted from the set of fuzzy goals.

Theorem 3.3: In the optimal solution of model(10), μ_i is equal to the degree of membership function for the ith fuzzy goal.

Proof. On the contrary suppose that (X^o, μ^o) is an optimal solution of model (10), where there exists at least one μ_i^o (*say* μ_t^o) which is not equal to the degree of membership function of the *t*th fuzzy goal. Without loss of generality assume that $1 \le t \le i_0$. Two cases are considered:

- $G_t(X^o) < g_t \Rightarrow \dfrac{U_t - G_t(X^o)}{U_t - g_t} > 1$

  But, $\mu_t^o \le \dfrac{U_t - G_t(X^o)}{U_t - gt}$ and $\mu_t^o \le 1$. Since there is no other bound on μ_t^o and model (10) is a maximization LP, μ_t^o should be 1 which in this case is equal to the degree of membership function of the tth fuzzy goal.

- $G_t(X^o) \ge g_t \Rightarrow \dfrac{U_t - G_t(X^o)}{U_t - gt} \le 1$

  Let $s_t = \dfrac{U_t - G_t(X^o)}{U_t - gt} - \mu_t^o$ then $s_t > 0$. Define (X^*, μ^*) as $X^* = X^o$, $\mu_i^* = \mu_i^o$ for $i \ne$ t and $\mu_t^* = \mu_t^o + s_t$. Then $\mu_t^* = \dfrac{U_t - G_t(X^*)}{U_t - g_t}$ and $\mu_t^* \le 1$. Also, the other constraints of model (10) are satisfied. Therefore, (X^*, μ^*) is a feasible solution.

  $Z^* = \sum_{i=1}^{K} w_i \mu_t^* = \sum_{i=1, i\ne t}^{K} w_i \mu_i^o + w_t \mu_t^* = \sum_{i=1}^{K} w_i \mu_i^o + w_t s_t > \sum_{i=1}^{K} w_i \mu_i^o$, which is a contradiction.

If $i_0 + 1 \leq t \leq K$ then it can be treated similarly and the proof is completed.

The following model represents an LP for solving general FGP problem presented in Section 1. Fuzzy goals of types (3) and (4), which are not considered by Tiwari et al. in the formulation of model (9), are incorporated into the model.

$$
\begin{array}{lllll}
\text{maximize} & Z & = & \sum_{i=1}^{K} w_i \mu_i & \\
\text{s.t.} & & & & \\
 & \mu_i & \leq & \dfrac{U_i - G_i(X)}{U_i - g_i} & i = 1, \ldots, i_0, j_0 + 1, .., k_0 \\
 & \mu_i & \leq & \dfrac{G_i(X) - L_i}{g_i - L_i} & i = i_0 + 1, \ldots, k_0 \\
 & \mu_i & \leq & \dfrac{U_i - G_i(X)}{U_i - g_i^u} & i = k_0 + 1, \ldots, K \\
 & \mu_i & \leq & \dfrac{G_i(X) - L_i}{g_i^l - L_i} & i = k_0 + 1, \ldots, K \\
 & 0 & \leq & \mu_i \leq 1 & i = 1, \ldots, K \\
 & & & X \in Cs. &
\end{array} \tag{22}
$$

As it is clear in Figure 1 (membership function (D)), fuzzy goals of type (4) have a membership function value of 1 inside an interval. Theorem 3.4 shows explicitly that model (22) yields this assumption.

Theorem 3.4 Suppose (X^*, μ^*) is the optimal solution of model (22). If for $i = k_0 + 1, \ldots K$, $g_i^l \leq G_i(X^*) \leq g_i^u$ then $\mu_i^* = 1$.

Proof.

(i) $G_i(X^*) \leq g_i^u \Rightarrow U_i - g_i^u \leq U_i - G_i(X^*) \Rightarrow \dfrac{U_i - G_i(X^*)}{U_i - g_i^u} \geq 1.$

(ii) $G_i(X^*) \geq g_i^l \Rightarrow G_i(X^*) - L_i \geq g_i^l - L_i \Rightarrow \dfrac{G_i(X^*) - L_i}{g_i^l - L_i} \geq 1.$

(i) and (ii) in addition to constraints $\mu_i^* \leq 1$ for $i = k_0+1, ..., K$ imply that the maximum value for μ_i^* is 1 and therefore $\mu_i^* = 1$.

Example 3. To demonstrate the proposed model, model (22), an example is given. The FGP problem in this example is similar to the one in Example 1. Only G_4 and G_5 in Example 1 are changed as follows:

$$G_4 : 5x_1 + 3x_2 + 2x_4 \;\tilde{=}\; 70$$

$$G_5 : 4x_1 + 4x_2 + 4x_3 \;\tilde{\in}\; [40, 45],$$

where $L_4 = 50$, $U_4 = 100$, $L_5 = 30$, $U_5 = 55$.

Set the weights arbitrarily as (0.2,0.1,0.15,0.35,0.2). The model (22) for solving this FGP problem is:

$$\text{maximize} \quad Z = 0.2\mu_1 + 0.1\mu_2 + 0.15\mu_3 + 0.35\mu_4 + 0.2\mu_5$$

s.t.

$$\mu_1 \leq \frac{55-(4x_1+2x_2+8x_3+x_4)}{20}$$

$$\mu_2 \leq \frac{4x_1+7x_2+6x_3+2x_4-40}{60}$$

$$\mu_3 \leq \frac{x_1-6x_2+5x_3+10x_4-70}{50}$$

$$\mu_4 \leq \frac{5x_1+3x_2+2x_4-50}{20}$$

$$\mu_4 \leq \frac{100-5x_1-3x_2-2x_4}{30}$$

$$\mu_5 \leq \frac{4x_1+4x_2+4x_3-30}{10}$$

$$\mu_5 \le \frac{55 - 4x_1 - 4x_2 - 4x_3}{10}$$

$$0 \le \mu_i \le 1 \qquad i = 1, ..., 5$$

Plus constraints (11) - (15),

with the optimal solution

$$(x_1^*, x_2^*, x_3^*, x_4^*) = (0, 11.25, 0, 15.62),$$

$$(\mu_1^*, \mu_2^*, \mu_3^*, \mu_4^*, \mu_5^*) = (0.84, 1, 0.38, 0.75, 1),$$

$$Z^* = 0.79.$$

4. CONCLUDING REMARKS

In this paper a weighted additive model by Tiwari et al. for solving FGP problems is discussed. An oversight within the formulation of it is explicitly proved. A correction for eliminating the lack of precision within that model is suggested and proved that the proposed model always yields an optimum value at least as good as Tiwari et al.'s model. In addition, an invalid conclusion in a further research [5] based on Tiwari et al.'s model is discussed. Using the new proposed model shows that such invalid conclusion was due to the original problem in Tiwari et al.'s model. Also, it is shown that the same oversight is repeated in a developed model on Tiwari et al.'s model in [5]. Finally, two other fuzzy goals, which are not considered by Tiwari et al., are incorporated into the new proposed model. Improvements to Tiwari et al.'s model are given in this paper since it was one of the first proposed weighted additive models for solving FGP problems and is used in further researches such as in [5].

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