

THERMAL ANALYTICAL INVESTIGATION OF NATURAL CONVECTION IN POROUS AND SOLID FINITE-LENGTH FIN WITH INSULATED TIP

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Abstract: This work employed a simple analytical method to study the performance of porous and solid fins in a natural convection environment called homotopy perturbation. Also energy balance and Darcy's model used to formulate the heat transfer equation. To study the thermal performance, a type case considered is finite-length fin with insulated tip.

In through of analysis, the effects of different parameters such as Rayleigh number (Ra), darcy number (Da), thermal conductivity ratio (Kr), and Length thickness ratio on the temperature distribution along the fin are considered, and appears on newly parameter called porosity parameter (S_p). The effects of the S_p , convection parameter (m) on the dimensionless temperature distribution and heat transfer rate are discussed. The fins used in this article are Titanium, Nickel and Steel and heat transfer convectivity discussed for air and water.

Keyword: Porous, Finite-length fin with insulated tip, Heat transfer, Natural convection, Homotopy perturbation method (HPM).

1. INTRODUCTION

High rate of heat transfer with reduced size and cost is in demand for a number of engineering applications such as heat exchangers, economizers, superheaters, conventional furnaces, gas turbines, etc. Some engineering applications also require lighter fin with higher rate of heat transfer where they use high thermal conductivity metals in applications such as airplane and motorcycle applications. However, cost of high thermal conductivity metals is also high. Thus, the optimum situation is enhancement of heat transfer by increasing the heat transfer rate and decreasing the size, weight and cost of fin.

Fins are frequently used in many heat transfer applications. Extensive research has been done in this area and many references are available especially for heat transfer in porous fins. Described below are a few papers relevant to the study described herein.

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Kiwan and Al-Nmr [1] conducted thermal analysis of natural convection porous fins. They grouped all the geometric and flow parameters that influence the temperature distribution in to one parameter called Sh . Three cases of fin types were considered: the infinite fin, finite fin with insulated tip and finite fin with un-insulated tip. Further investigation of S_h effect for all cases revealed that increasing S_h by increasing either Da or Ra increases the heat transfer from the fin. They also found that there is limit to increasing both Kr and L/t that effect the heat transfer rate from the porous fin. Nguyen and Aziz [2] compared the heat transfer rates from convecting-radiating fins for different profile shapes. They used finite difference approach to study this performance. They used rectangular, trapezoidal, triangular and concave parabolic shapes to compare heat transfer rates. Taludkar and Mishra [3] studied the effect of combined radiation and convection heat transfer in a porous channel bounded by isothermal parallel plates. Both the radiation Nusselt number and convection Nusselt number are reciprocal to porous medium shape parameter (PMSP). Mueller and Abu-Mulaweh [4] studied the efficiency of horizontal single pin fin subjected to free convection and radiation heat transfer. Mokheimer [5] investigated locally variable heat transfer coefficient on the performance of extended surfaces subject to natural convection. Kang and Look [6] presented optimum designs of a thermally asymmetric convecting and radiating rectangular annular fin. Razelos and Kakatsios [7] determined the optimum dimensions of convecting-radiating heat transfer fins. Yu and Chen [8] performed a study on optimization of circular fins with variable hermal parameters. They considered rectangular profile circular fin for optimization. Yih [9] studied the radiative effect on natural convection over an isothermal vertical cylinder embedded in a porous medium. The obtained results are for dimensionless temperature profiles for various values of ξ with constant values of surface temperature excess ratio H and conduction-radiation parameter Rd . Rao and Venkateshan [10] conducted experiments to study the natural convection and radiation in horizontal fin arrays. Kobus and Oshio [11] studied the mixed convection and radiation effects on a vertical fin array. Popiel *et al.*, [12] analyzed the combined effects of natural convection and radiation on fin efficiency. Poulikakos and Bejan [13] optimized the fin geometry for minimum entropy production. Snider and Kraus [14] presented an analysis for the optimization of fin geometry.

Heat transfer equations are such phenomena which mostly occur non-linearly; hence solving them has been one of the most time consuming and difficult affairs among researchers of heat transfer. Therefore, many researchers and scientist of both heat transfer and mathematics have recently paid much attention to find and develop approximate solutions. Perturbation method is a traditional method which has got some limitations (clearly mentioned throughout this work). To overcome the difficulties and limitations of the above method many new ones have recently been introduced, two of which are Homotopy Perturbation Method (HPM) and Variational Iteration Method (VIM). That in this work used Homotopy Perturbation Method (HPM). This method was first introduced by He [15]. Because this method continuously deforms a difficult problem into a simple one, which is easy to solve, this method has been used by many authors such as Ganji [16-19] and the references therein to handle a wide variety of scientific and engineering applications such as linear and nonlinear, homogeneous equations as well.

In this study, the basic idea of the simple analytical method that called homotopy perturbation method (HPM) is introduced and then is applied to solve the nonlinear equations. The effects

of convection heat transfer in porous media are considered. The geometry considered is that of a rectangular profile fin. The porous fin allows the flow to infiltrate through it and solid–fluid interaction takes place. This study is performed using energy balance and Darcy’s model to formulate heat transfer equation. To study the thermal performance, one type of cases are considered finite- length fin with insulated tip. The effects of the porosity parameter (Sh), convection parameter (m) on the dimensionless temperature distribution and heat transfer rate are discussed. The results suggest that the convection transfers more heat than a similar model without convection.

NOMENCLATURE			
C_p	Specific heat	W	Width of the fin
Da	Darcy number, K/t^2	x	Axial coordinate
G	gravity constant	X	Dimensionless axial coordinate,
Gr	Grashof number	Greek Symbols	
k	Thermal conductivity	α	Thermal diffusivity
Kr	Thermal conductivity ratio, (k_{eff}/k_p)	β	Coefficient of volumetric thermal expansion
K	Permeability of porous fin	Δ	Temperature difference
L	Length	ε	Porosity or void ratio
Nu	Nusselt number, (hL/k_p)	σ	Stephen–Boltzmann constant,
m	convection parameter	θ	Dimensionless temperature,
Pr	Prandtl number, (ν/α)	θ_b	Base temperature difference, $(T_b - T_\infty)$
q	Heat transfer rate	ν	Kinematic viscosity
Ra	Rayleigh number, $Gr \times Pr$	ρ	Density
Sh	Porous parameter	Subscripts	
$T(x)$	Temperature at any point	s	Solid properties
T_b	Temperature at fin base	f	Fluid properties
t	Thickness of the fin	eff	Porous propertie
Bi	Biot Number, (hL_c/kb)		
$V_{w(x)}$	Velocity of fluid passing through the fin any point		

2. GOVERNING EQUATIONS

As shown in Fig. 1, a rectangular fin profile is considered.

The dimensions of the fin are Length L , width w and thickness t . The cross section area of the fin is constant. This fin is porous to allow the flow of infiltrate through it.

The following assumptions are made to solve this problem.

- The porous medium is isotropic and homogenous.
- The porous medium is saturated with single-phase fluid.
- The surface radiant exchange is neglected.

- Physical properties of both fluid and solid matrix are constant.
- The temperature inside fin is only function of x .
- There is no temperature variation across the fin thickness.
- The solid matrix and fluid are assumed to be at local thermal equilibrium with each other.
- The interactions between the porous medium and the clear fluid can be simulated by the Darcy formulation.

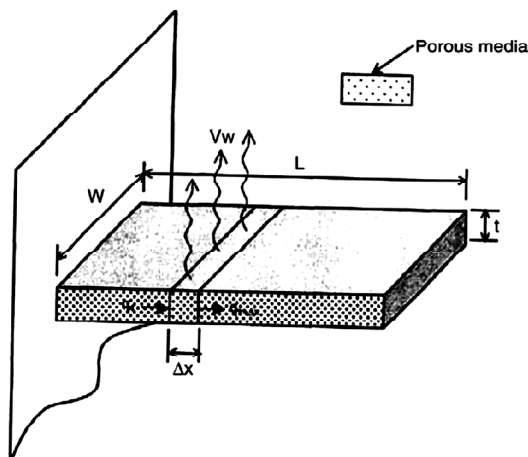


Figure 1: Schematic Diagram of Fin Profile Under Consideration

$$q(x) = -\int ktv \frac{dT}{dx} \rightarrow \left[\begin{array}{c} \uparrow v_w \\ \uparrow q_c = 2h_p \Delta x (T - T_\infty) \end{array} \right] \rightarrow q(x+\Delta x) = -\int ktv \frac{dT}{dx} - \int ktv \frac{d^2 T}{dx^2} \Delta x$$

Figure 2: Energy Balance in Fin Profile

Apply an energy balance to the slice segment of the fin of thickness Δx shown in Fig. 1, requires that

$$q(x) - q(x + \Delta x) = \dot{m}c_p(T(x) - T_\infty) + h(p.\Delta x)(T(x) - T_\infty). \quad (1)$$

The mass flow rate of the fluid passing through the porous material can be written as,

$$\dot{m} = \rho v_w \Delta x W. \quad (2)$$

From the Darcy's model we have:

$$v_w = \frac{g k \beta}{\nu (T - T_\infty)} \quad (3)$$

Substitutions of Eqs. (2) and (3) into Eq. (1) yields,

$$\frac{q(x) - q(x + \Delta x)}{\Delta x} = \frac{\rho c_p g k \beta w}{\nu} (T(x) - T_\infty)^2 + hp(T(x) - T_\infty). \quad (4)$$

As, $\Delta x \rightarrow 0$ Equation (4) becomes

$$\frac{dq}{dx} = \frac{\rho c_p g k \beta w}{\nu} (T(x) - T_\infty)^2 + hp(T(x) - T_\infty). \quad (5)$$

From Fourier's Law of conduction, we have

$$q = -k_{eff} A \frac{dT}{dx}. \quad (6)$$

Where A is the cross-sectional area of the fin ($A = W.t$) and k_{eff} is the effective thermal conductivity of the porous fin given by $k_{eff} = \phi k_f + (1 - \phi) k_s$. Substitution Eq. (6) into Eq. (5) gives,

$$\frac{d^2T}{dx^2} - \frac{\rho c_p g k \beta}{tk_{eff} \nu} (T(x) - T_\infty)^2 - \frac{hp}{k_{eff} A} (T(x) - T_\infty) = 0. \quad (7)$$

Hence, with applying energy balance equation at steady state condition as shown in Fig. 2, and Introducing non-dimensional temperature function Where, $\theta = \frac{T(x) - T_\infty}{T_b - T_\infty}$ and $X = \frac{x}{L}$ into Eq. (7) we have;

$$\frac{d^2\theta}{dx^2} - L^2 S_h \theta^2 - m^2 \theta = 0. \quad (8)$$

Porous parameter, $S_h = \frac{Dax.Ra}{k_r} \left(\frac{L}{t}\right)^2$ and Convection parameter, $m = \left(\frac{hp}{k_s A}\right)^{1/2}$.

Here, S_h is a porous parameter that indicates the effect of the permeability of the porous medium as well as buoyancy effect so Higher value of S_h indicate higher permeability of the porous medium or higher buoyancy forces. m is a convection parameter that indicates the effect of surface convecting of the fin.

Here is the summary of case to be considered for this research.

Case: finite-length fin with insulated tip ($\theta(0) = 1, d\theta/dx|_{x=1} = 0$).

3. ANALYSIS OF HE'S HOMOTOPY PERTURBATION METHOD

In this Letter, we apply the homotopy perturbation method [20, 21, 22-28] to the discussed problem. To illustrate the basic ideas of this method, we consider the following nonlinear differential equation,

$$A(u) - f(r) = 0, \quad r \in \Omega, \quad (9)$$

with the boundary condition of:

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad r \in \Gamma, \quad (10)$$

where A is a general differential operator, B a boundary operator, $f(r)$ a known analytical function and Γ is the boundary of the domain Ω . A can be divided into two parts, which are L and N , where L is linear and N is nonlinear. Eq. (6) can therefore be rewritten as follows:

$$L(u) + N(u) - f(r) = 0, \quad r \in \Omega, \quad (11)$$

Homotopy perturbation structure is shown as follows:

$$H(v, p) = (1 - p) [L(v) - L(u_0)] + p [A(v) - f(r)] = 0, \quad (12)$$

where,

$$v(r, p) : \Omega \times [0, 1] \rightarrow R. \quad (13)$$

In Eq. (12), $p \in [0, 1]$ is an embedding parameter and u_0 is the first approximation that satisfies the boundary condition. We can assume that the solution of Eq. (6) can be written as a power series in p , as following:

$$v = v_0 + pv_1 + p^2v_2 + \dots = \sum_{i=0}^n v_i p^i, \quad (14)$$

and the best approximation for solution is:

$$u = \lim_{p \rightarrow 1} v = v_0 + v_1 + v_2 + \dots \quad (15)$$

4. THE APPLICATION OF HPM

In this section, we will apply the HPM to nonlinear ordinary differential Eq. (8) with a boundary condition (24). According to the HPM, we can construct homotopy of Eq. (8) as follows:

$$(1 - P) (\theta''(x) - \theta(x)) + p (\theta''(x) - \theta^2(x) - \theta(x)) = 0. \quad (16)$$

We consider θ as follows:

$$\theta(x) = \theta_0(x) + \theta_1(x) + \theta_2(x) + \dots = \sum_{i=0}^n \theta_i(x). \quad (17)$$

From Eq. (14), if the two terms approximations are sufficient, we will obtain with substituting θ from Eq. (17) into Eq. (16) and some simplification and rearranging based on powers of p -terms, we have:

$$\begin{aligned} p^0 : -\theta_0(x) + \theta_0''(x) &= 0 \\ \theta_0(0) = 1, \theta_0(1) &= 0. \end{aligned} \quad (18)$$

$$\begin{aligned} p^1 : \theta_1''(x) - \theta_1(x) - \theta_0^2(x) &= 0 \\ \theta_1(0) = 0, \theta_1(1) &= 0. \end{aligned} \quad (19)$$

Solving Eqs. (18)-(19) with boundary conditions, we have:

$$\theta_0(x) = \frac{e^{-1} \times e^x}{e + e^{-1}} + \frac{e^1 \times e^{-x}}{e + e^{-1}}, \quad (20)$$

$$\theta_1(x) = \frac{1}{(1+e^2)^2(1+e^2)(4Ln(e)^2-1)} (-e^{x+6} - e^{x+4} - 2e^{-x+4} - 8e^4Ln(e)^2 + 8e^{x+4}Ln(e)^2 + 8e^{x+2}Ln(e)^2 - 8e^2Ln(e)^2 + 2e^2 + 2e^4 - 2e^{x+2} + e^{-x+2} + e^{-2x+6} + e^{-2x+4} + e^{2x} + e^{2x+2} - e^x). \quad (21)$$

The solution of this equation, when $p \rightarrow 1$, will be as follows:

$$\theta(x) = \theta_0(x) + \theta_1(x) + \dots \quad (22)$$

5. RESULTS AND DISCUSSION

The governing Eq. (8) is a non-Linear second order ordinary differential equation. The equation is solved by using the homotopy perturbation method. Depending on the tip condition of the fin, we have three different types of cases, that in this research we only study finite-length fin with insulated tip.

Table 1
The Values of h, k and Constants to Use in Research

$h_{\text{air}} = 25 \frac{w}{m^2.k}$	$h_{\text{water}} = 100 \frac{w}{m^2.k}$	$k_{ti} = 7.44 \frac{w}{m.k}$	$k_{st} = 16.27 \frac{w}{m.k}$	$k_{Ni} = 91.74 \frac{w}{m.k}$
$L = 5 m$	$t = 0.025 m$	$w = 1$	$k_s = 204 \frac{w}{m.k}$	$h_s = 15 \frac{w}{m^2.k}$

5.1. Finite-Length Fin with Insulated Tip

For this case, the second boundary condition at $x = L$ will be $d\theta/dx|_{x=L} = 0$. If we introduce the dimensionless axial coordinate $X = x/L$, the second boundary condition becomes $d\theta/dx|_{x=1} = 0$

Therefore, we may write:

$$d\theta/dx = 0 \quad \text{when} \quad x = 1$$

The boundary conditions may be written as;

$$\theta(0) = 1, \quad d\theta/dx|_{x=1} = 0. \quad (23)$$

Eq. (8) was solved using these two boundary conditions given by Eq. (23) for different values of S_h and m . Figure 1 shows the variation of dimensionless temperature distribution with the axial distance along the fin when the value of S_h is varying and of m was kept constant. From Fig. 3 we can see that the value of dimensionless temperature decreases along the fin Length. It should be noted that as the value of S_h increases, the temperature decreases rapidly and the fin quickly reaches the surrounding temperature. As the value of S_h increases, the fins cool down rapidly.

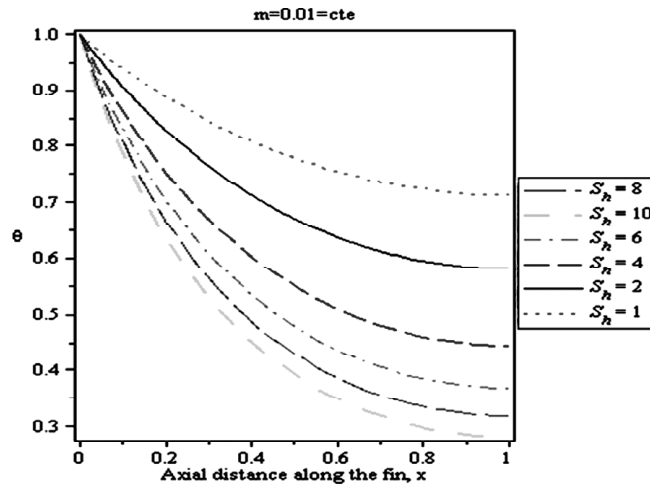


Figure 3: The Distribution of Axial Non-Dimensional Temperature Along the Finite Fin for Different Values of S_h

Figure 4, Shows the results for the effect of variation of m on dimensionless temperature distribution. The values of m were varied from 0.04, 0.2 and 0.4, while keeping $S_h = 1$. From Fig. 4, it is observed that if hair and different k be considered, variation of m has minor effect on temperature distribution of fin. As m increases the temperature of a given axial location decrease.

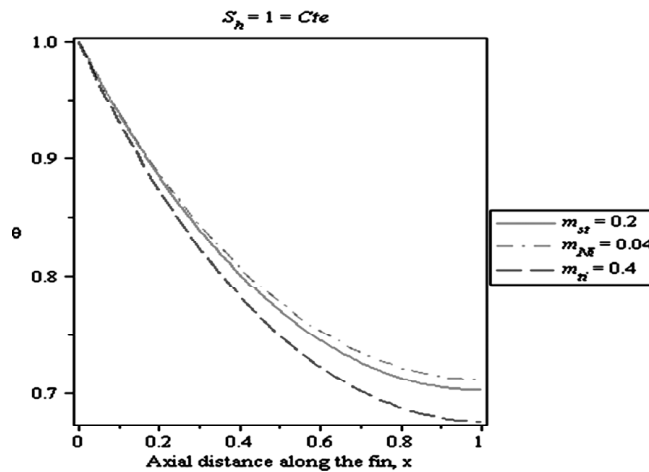


Figure 4: The Distribution of Axial Non-Dimensional Temperature Along the Finite Fin for Different Values of m & h_{air}

Figure 5, shows the results for the effect of variation of m on dimensionless temperature distribution. The values of m were varied from 0.15, 0.8 and 1.2, while keeping $S_h = 1$. From Fig. 5, it is observed that if h_{water} and different k be considered, variation of m has more effect on temperature distribution of fin.

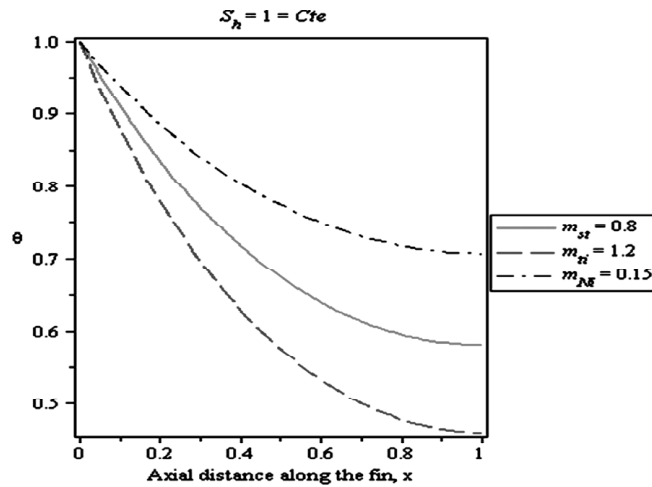


Figure 5: The Distribution of Axial Non-Dimensional Temperature Along the Finite Fin for Different Values of m & h_{water}

From Fig. 6 By comparison Figs 4, 5 we can see that by increase h , the convection parameter m will increase. Hence, by increase of m and $S_h = 1$, the temperature distribution rapidly descending process to go and fin quickly reaches the surrounding temperature. As the value of m increases, the fins cool down rapidly.

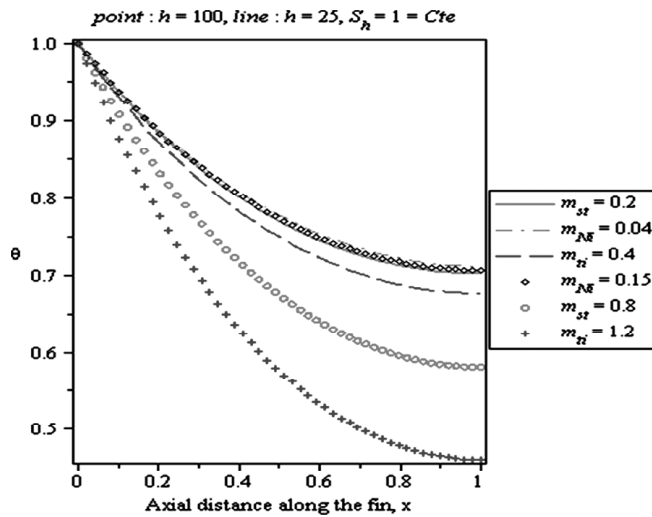


Figure 6: The Distribution of Axial Non-Dimensional Temperature Along the Finite Fin for Different Values of m

5.2. Solid fin

All condition of solid fin, equal will be by said condition in first paper. By applying energy balance equation at steady state condition, we have;

$$\frac{d^2\theta}{dx^2} - m^2\theta(x) = 0 . \tag{24}$$

The boundary conditions may be written as:

$$\theta(0) = 1, \quad d\theta/dx|_{x=1} = 0 . \tag{25}$$

Eq.24 was solved using these two boundary conditions given by Eq. (25) for different values of m .

Figure 7, shows the variation of the dimensionless temperature distribution at the base of the fin with the variation of the m parameter. From Fig. 7, it is observed that if hair and different k be considered, variation of m has minor effect on temperature distribution of fin.

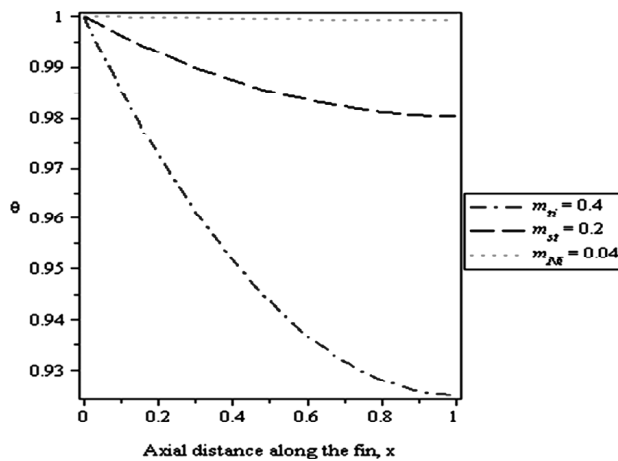


Figure 7: The Distribution of Axial Non-Dimensional Temperature Along the Finite Fin for Different Values of m

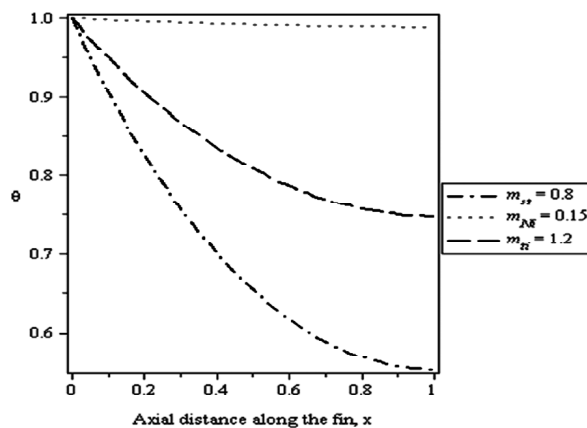


Figure 8: Compare the Distribution of Axial Non-Dimensional Temperature Along the Finite Fin for Different Values of m

Figure 8, shows the variation of the dimensionless temperature distribution at the base of the fin with the variation of the m parameter. From Fig. 8, it is observed that if h_{water} and different k be considered, variation of m has more effect on temperature distribution of fin. By comparison Figs. 6, 7 we can see that by increase h , the convection parameter m will increase. By increase of m , the temperature distribution rapidly descending process to go.

Figure 9, to compare temperature distribution in porous and solid fins.

From Fig. 9 observed that by $h_{\text{air}} = 25 \text{ w/m}^2\cdot\text{k}$ and different k in according to porosity parameter, velocity of temperature decrease in porous fin was more than solid fin, and fin quickly reaches the surrounding temperature and the fins cool down rapidly.

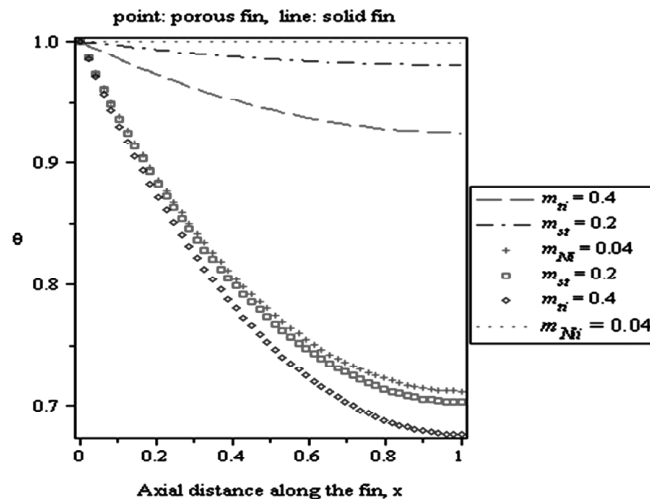


Figure 9: Compare Temperature Distribution in Porous and Solid Fins for Different Values of m

Maximum possible heat transfer rate obtained using porous fin is:

$$q_p = -k_{\text{eff}} A_b \left(\frac{dT}{dx} \right)_{x=0} \quad (26)$$

Maximum possible heat transfer rate obtained using solid fin is

$$q_s = h A_s (T_b - T_{\infty}) \quad (27)$$

Therefore, ratio of porous fin to solid fin heat transfer rate is;

$$\frac{q_p}{q_s} = \frac{-k_{\text{eff}} A_b \left(\frac{dT}{dx} \right)_{x=0}}{h A_s (T_b - T_{\infty})} \quad (28)$$

Writhing equation in dimensionless temperature and axial distance,

$$\frac{q_p}{q_s} = -\frac{k_{eff} A_b}{h L A_s} \left(\frac{d\theta}{dx} \right)_{x=0} \quad (29)$$

Figure 10 shows porous fin to solid fin heat transfer rate with convection and without convection. The figure also shows the variation ratio of porous fin to solid fin heat transfer rate with Kr . Variation of ratio of porous fin to solid fin heat transfer rate increases as Kr increases for both the cases. However comparing both the cases of with convection and without convection heat transfer rate is more with convection than without convection as shown in the figure.

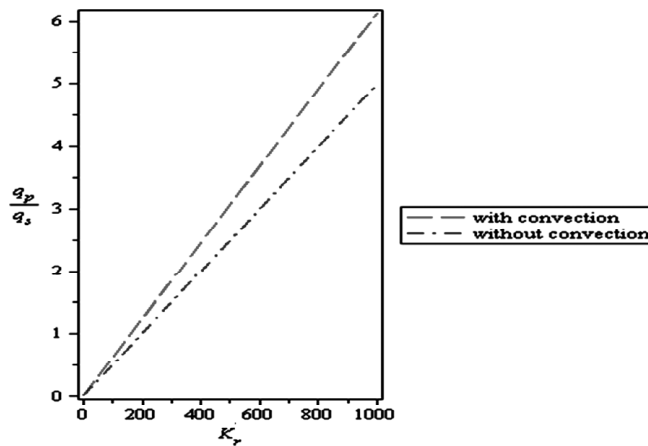


Figure 10: The Variation of the Ratio of Porous Fin to Solid Fin Heat Transfer Rate with Kr

6. CONCLUSION

This work introduces a simple method to analysis the performance of a porous fin and solid fin. It is found that the problem of heat transfer through the porous fin is governed by a second order nonlinear, ordinary, differential equation. It is also found that all geometric and flow parameters influencing the temperature distribution have been grouped in two parameters called S_h and m . This thermal analysis was performed on one type of fin case: the finite-length fin with insulated tip. The effect of these all two parameters has been investigated. It was found that increasing S_h by increasing either Da or Ra increases the heat transfer from fin. Increasing m ($h_{air} = 25 \text{ w/m}^2.k$ and different k) has minor effect on temperature distribution of fin. In addition, increasing the parameter m ($h_{water} = 100 \text{ w/m}^2.k$ and different k) has more effect on temperature distribution of fin. The ratio of heat transfer rate for porous fin to solid fin is compared for both the cases of with convection and without convection.

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