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SOLVING NONLINEAR EQUATION OF MOTION OF A SPHERE ROLLING DOWN AN INCLINED PLANE IMMERSED IN A NEWTONIAN FLUID BY USING PARAMETER PERTURBATION METHOD

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Abstract: *In this research, the unsteady motion of a rolling spherical particle on an inclined surface immersed in a Newtonian fluid is studied for a particular type of the drag force and a wide range of Reynolds number by using parameter perturbation method (PPM). An analytical solution for velocity, acceleration and position of the particle is derived, using parameter perturbation method. Equation of motion was solved generally and for some practical conditions. The present investigation shows the effectiveness of PPM and exhibits a new application of this method for nonlinear problems.*

Keywords: *Drag coefficient, Spherical particle, Acceleration motion, Inclined plane, Parameter perturbation method (PPM).*

1. INTRODUCTION

The motion of objects in fluids is present in various manufacturing processes such as sediment transport and deposition in pipelines, alluvial channels, and power process [1-6]. Many studies on the moving spherical particles in low and high concentrations can be found in the literature, *e.g.* [7-9].

A particle falling or rolling down a plane in a fluid under the influence of gravity will accelerate until the gravitational force is balanced by the resistance forces that include buoyancy and drag. The constant velocity reached at that stage is called the "terminal velocity" or "settling velocity". Knowledge of the terminal velocity of solids falling in liquids is required in many industrial applications. Typical examples include hydraulic transport slurry systems for coal and ore transportation, thickeners, mineral processing, solid-liquid mixing, fluidization equipment, drilling for oil and gas, geothermal drilling. The resistive drag force depends upon drag coefficient. Particles Drag coefficient and terminal velocities are most significant design parameters in engineering applications. There have been several attempts to relate the drag

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coefficient to the Reynolds number. The most comprehensive equation set for predicting CD from Re for Newtonian fluids has been published by Clift *et al*., [10], Khan and Richardson [11], Chhabra [12] and Hartman and Yutes [13]. Comparing between most of these relationships for spheres, reveals quite low deviations [13].

Most of above mentioned applications involve the description of the particle position, velocity and acceleration during time e.g. classification, centrifugal and gravity collection or separation, where it is often necessary to determine the trajectories of particle accelerating in a fluid for proposes of design or improved operation [14]. For some industrial problems such as flow in the rolling ball viscometer which entails the measurement of the rolling velocity of a tightly fitting sphere in an inclined tube, transport of solid particles in inclined pipelines or sedimentation of solid particles in inclined open channels, we need information about the motion of particles rolling down an inclined plane. This topic is received less attention in the technical literature. Jan and Chen [15] developed a CD–Re correlation for a single spherical particle rolling down a smooth plane in an incompressible Newtonian media for range of $0.1 \le \mathrm{Re} \le 105$. In their work, inclination angle was varied between $2° \le \theta \le 10°$. They used this correlation to numerically solve the equation of motion for a sphere rolling down a smooth inclined plane by their own experimental works. Jan and Chen established their correlations for three regimes:

$$\begin{cases} CD = 322/\mathrm{Re} & \text{if} \quad \mathrm{Re} < 10 \\ CD = 10^{[3.02 - 1.89 \log \mathrm{Re} + 0.422 \log (\mathrm{large})]} & \text{if} \quad 10 < \mathrm{Re} < 20000 \\ CD = 0.74 & \text{if} \quad \mathrm{Re} > 20000 \end{cases} \tag{1}$$

Where Reynolds number is defined as follows:

$$\mathrm{Re} = \frac{\rho u D}{\mu} \tag{2}$$

NOMENCLATURE

a, b, c, d	Constants	*P*	Embedding parameter
A	General differential operator	Re	Reynolds number
Acc	Acceleration [m/s2]	*t*	Time [s]
B	Boundary operator	*u*	Velocity [m/s]
CD	Drag coefficient	α, β	Constants
D	Particle diameter [m]	μ	Dynamic viscosity [kg/ms]
G	Acceleration due to gravity [m/s^2]	ρ	Fluid density [kg/m^3]
PPM	Parameter perturbation method	ρ_s	Particle density [kg/m^3]
M	Particle mass [kg]	θ	Inclination angle [°]
N	Nonlinear part of equation	Γ	Boundary of domain
L	Linear part of equation	Ω	Domain

In Eq. (2), ρ, *u*, *D* and μ denote the fluid density, particle velocity, particle diameter and fluid viscosity, respectively. Chhabra and Ferreira [16] used Eq. (1) to generate one correlation for range of $0.1 \le \mathrm{Re} \le 105$ in the following structure:

$$C_D = \alpha + \frac{\beta}{\mathrm{Re}}, \tag{3}$$

where α and β are constants. They recommended a relationship with 11% average relative deviation:

$$C_D = 0.816 + \frac{321.906}{\mathrm{Re}}. \tag{4}$$

Figure 1 demonstrates the variations of *CD* versus Re for Eq. (4) with experimental points from Jan and Chen [15], in a log–log diagram

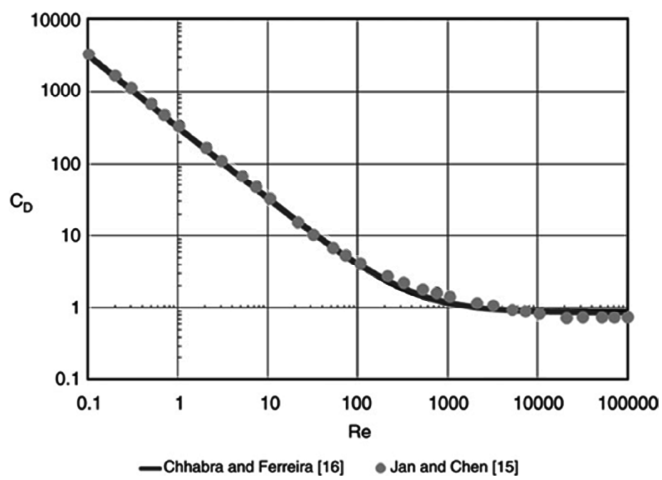

Figure 1: Drag Curve for a Sphere Rolling Down a Smooth Plane

Eq. (3) is of the same form as that used by Rumps [17], Ferreira [18] and Oseen[19] for free fall of spherical particles such as:

$$C_D = 0.5 + \frac{24}{\mathrm{Re}}, \tag{5}$$

which was presented by Ferreira [18] for vertically falling sphere. Comparing Esq. (4) and (5), it could be found that the drag coefficient for a sphere rolling down a smooth plane is much larger than that for vertically free fall.

The aim of current study is the analytically investigation of acceleration motion of a spherical particle rolling down an inclined boundary with drag coefficient in the form of Eq. (3), using parameter perturbation method (PPM). Investigation and solution of falling objects' equation is a new application for PPM which was used for some other engineering problems [20-44].

For instance in the fields of fluid mechanics and heat transfer, Ganji and Rajabi [26] employed Perturbation Method (PM) and PPM to solve the nonlinear radiation heat transfer equation. They showed the capability of PPM in comparison to PM while no small parameter was existed. Ganji and Ganji [30] solved for temperature distribution in the thermal boundary on a flat plate using PPM and verified the results with numerical solution.

Ganji and Sadighi [36] used PPM for two practical applications that arose in “heat transfer” and “porous media”. They solved strongly nonlinear equations and compared their results with numerical solutions. Their outcomes proved the ability of PPM. Ramiro *et al.*, [42] applied PPM to solve a 2-D heat convection problem. They considered an ax symmetric impinging jet. Their results present the accurate variation of temperature in comparison with numerical solution. Recently, Mahmud *et al.*, [44] used PPM for a deformable channel with wall suction and injection while the domain was considered to be filled with porous medium. They demonstrated that PPM could be used for complex fluid mechanics problems with reliable accuracy.

2. PROBLEM DEFINITION

Consider a small, spherical, non-deformable particle of diameter D, mass m and density ρ_s rolling down a smooth plane in an infinite extent of an incompressible Newtonian fluid of density ρ and viscosity μ. The velocity of the particle at any instant of time is represented by u. Figure 2 demonstrates a schematic figure of current problem. Neglecting surface force and lift friction force, the equation of motion is obtained as follows [16]:

$$m\left(1.4+2\frac{\rho}{\rho_s}\right)\frac{du}{dt}=mg\left(1-\frac{\rho}{\rho_s}\right)\sin\theta-\frac{1}{8}\pi D^2\rho C_D u^2 \tag{6}$$

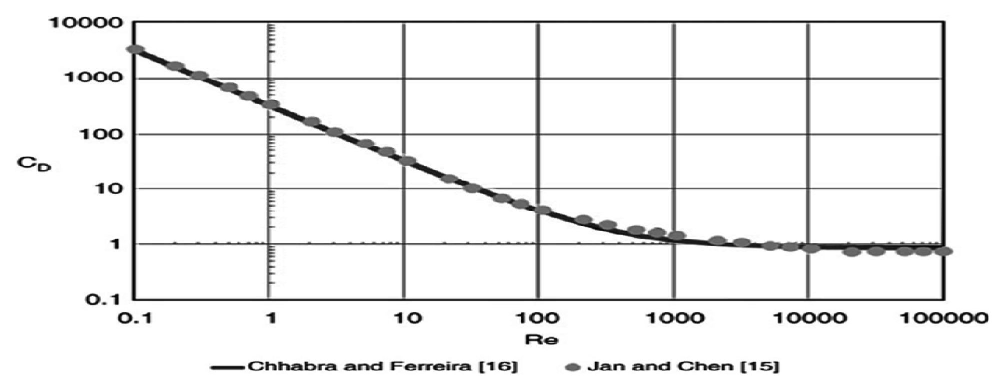

Figure 2: Schematic Picture of a Spherical Particle Rolling Down a Smooth Plane in a Newtonian Fluid

where *CD* C_D represents the drag coefficient. In the right hand side of the Eq. (6), the first term indicates the buoyancy affect and the second one corresponds to resistant drag force, The main difficulty in solution of Eq. (6) lies in the nonlinear term which is generated due to nonlinear nature of the drag coefficient, C_D. Substituting Eq. (3) in Eq. (6) and rearranging parameters, Eq. (6) could be rewritten as follows:

$$a\frac{du}{dt}+bu+cu^2-d=0, \quad u(0)=0 \tag{7}$$

wehere

$$a=m\left(1.4+2\frac{\rho}{\rho_s}\right) \tag{8}$$

$$b=\frac{\alpha}{s}\pi D^2\rho \tag{9}$$

$$c=\frac{S}{s}\pi D\mu \tag{10}$$

$$d=mg\left(1-\frac{\rho}{\rho_s}\right)\sin\theta\,. \tag{11}$$

Eq. (7) is a nonlinear ordinary differential equation which could be solved by numerical techniques such as Runge-Kutta method. We employ PPM and verify our results with numerical solution using fourth order Runge-Kutta method.

3. ANALYSIS OF THE PARAMETER PERTURBATION METHOD

Obtaining closed form solution of nonlinear fluid engineering problems is hardly possible. Therefore, particular approximate analytical solutions are developed by using various analytical and numerical techniques. Perturbation methods play a central role in solving nonlinear fluid mechanics problems.

The parameter perturbation method (PPM) is one of the well-known methods to solve various nonlinear equations that are established in 1999 by He [20–25]. This method has been used by many authors in [26–41] and the references therein to handle a wide variety of scientific and engineering applications: linear and nonlinear, and homogeneous and inhomogeneous as well. It was shown by many authors that this method provides improvements over existing techniques. Therefore in the present work we examine the nonlinear equation of particle motion and attempt to obtain its solution using the PPM. The parameter perturbation method is a combination of the classical perturbation technique and parameter technique. To explain the basic idea of the PPM for solving nonlinear differential equations we consider the following nonlinear differential equation:

$$u(t):=\varepsilon v(t)+l \tag{12}$$

$$v(t)=v_0(t)+\varepsilon v_1(t)+\varepsilon^2 v_2(t)+\mathcal{O}(\varepsilon^3)\,. \tag{13}$$

4. APPLICATION

4.1. Substituting Eq. (12) in (13), grouping similar powers of epsilon and setting each coefficient equal to zero the following set of equations are obtained:

General equation:

$$\varepsilon^0 : 2clv_0(t) + a\left(\frac{d}{dt}v_0(t)\right) + bv_0(t) + \frac{-d + bl + cl^2}{2} = 0 \tag{14}$$

$$t = 0, \quad v_0(t) = 0 \tag{15}$$

$$\varepsilon^1 : 2clv_0(t) + a\left(\frac{d}{dt}v_1(t)\right) + bv_1(t) + cv_0(t)^2 = 0 \tag{16}$$

$$t = 0, \quad v_1(t) = 0 \tag{17}$$

$$\varepsilon^2 : 2bv_2(t) + a\left(\frac{d}{dt}v_2(t)\right) + 2cv_0(t)v_1(t) + 2clv_2(t) = 0 \tag{18}$$

$$t = 0, \quad v_2(t) = 0 \tag{19}$$

$$\varepsilon^2 : bv_3(t) + a\left(\frac{d}{dt}v_3(t)\right) + 2cv_0(t)v_2(t) + 2clv_3(t) + cv_1(t)^2 = 0 \tag{20}$$

$$t = 0, \quad v_3(t) = 0 \tag{21}$$

By continuing the above terms, higher accuracy will be gained. Solving Eqs. (14), (16), (18), and (20), considering appropriate initial conditions, we have:

$$v_0 = \left[-\frac{e^{\frac{(2cl+b)t}{a}}(-d + bl + cl^2)}{(2cl+b)\,\varepsilon} + \frac{-d + bl + cl^2}{(2cl+b)\,\varepsilon}\right]e^{-\frac{(2cl+b)t}{a}} \tag{22}$$

$$v_1(t) = \frac{c\,(-d + bl + cl^2)^2\left[-\frac{ae^{\frac{(2cl+b)t}{a}}}{2cl+b} - 2t + \frac{ae^{\frac{(2cl+b)t}{a}}}{2cl+b}\right]e^{-\frac{(2cl+b)t}{a}}}{(2cl+b)^2\,\varepsilon^2\,a} \tag{23}$$

$$\begin{aligned} v_2(t) = {} & \frac{e^{\frac{-(2cl+b)t}{a}}(-d + bl + cl^2)^3c^2}{(2cl+b)^5\,\varepsilon^3} + \frac{1}{a^2\varepsilon^3(2cl+b)^5}\Bigg(e^{\frac{-(2cl+b)t}{a}} \\ & \Big(2t^2b^2 + 2bat + 4be^{\frac{-(2cl+b)t}{a}}at + 8bt^2cl + 4atcl + a^2e^{\frac{-(2cl+b)t}{a}} \\ & - 2a^2e^{\frac{(2cl+b)t}{a}} + 2a^2e^{\frac{-(2cl+b)t}{a}} + 8e^{\frac{-(2cl+b)t}{a}}atcl + 8t^2c^2l^2\Big) \\ & (-d + bl + cl^2)^3c^2\Bigg) \end{aligned} \tag{24}$$

More duration of particle motion is covered. By increasing series terms (powers of p-terms), the accuracy of PPM solution is improved and a larger period of acceleration motion of the particle is covered. In the current study, we increased the number of terms until the terminal velocity of particle is reached.

4.2. Real Combination of Sphere-Fluid

Mentioned method was applied for real combination of solid–fluid. A single Aluminum spherical particle of 3 mm diameter was assumed to roll down a smooth inclined plane in an infinity medium of olive oil, 75% glycerin solution and water. Required physical properties of selected materials are given in Table 1. Inserting above properties into Eqs. (8)- (11) And using Eq. (4), different combination are gained which are classified in Table 2.

Table 1
Physical Properties of Materials

Material	*Density [kg/m^3]*	*Viscosity [kg/m.s]*
Olive oil	913.0	0.0840
75% Glycerin	1178.2	0.0182
Water	998.0	0.0010
Aluminum	2702.0	–

Table 2
Selected Coefficient of Eq. (6)

Solid	*Fluid*	*a*	*b*	*c*	*d/sin* θ
Aluminum	Olive oil	0.000007929539600	0.03185493252	0.002778200055	0.00002482439441
	75% Glycerin	0.000008679401924	0.006901902047	0.003585186533	0.00002114444506
	Water	0.000008169880089	0.0003792253872	0.003036849567	0.00002364492347

By substituting the above coefficients in Eq. (7), and for four different inclination angles, twelve different nonlinear equations are achieved. Inclination angles were selected to be 2°, 6°, 20° and 40°. Parameter perturbation method was applied to gained equations and results were compared with numerical method. Figures. 3-6 depict the variation of rolling velocity of the particle versus time for different inclination angles and fluids.

Presented results demonstrate an excellent agreement between PPM and numerical solution. For a given inclination angle, by increasing the fluid viscosity, terminal velocity and acceleration duration are decreased. Effect of Inclination angle on velocity of particles was investigated using PPM and is shown in Figs. 7-8. Results show that increasing of inclination angle increases the terminal velocity as well as acceleration duration and displacement. Also by augmentation of viscosity, the dependence of terminal time on inclination angle is decreased. Outcomes illustrated that higher acceleration is obtained for a larger inclination angle. Acceleration of particles tends to zero after a while due to constant value of terminal velocity. To show the effect of inclination angle the displacement of particle rolling down in water was obtained for instant time during rolling procedure. Figures 9 demonstrates the positions of the particles for the same time steps at different inclination angles.

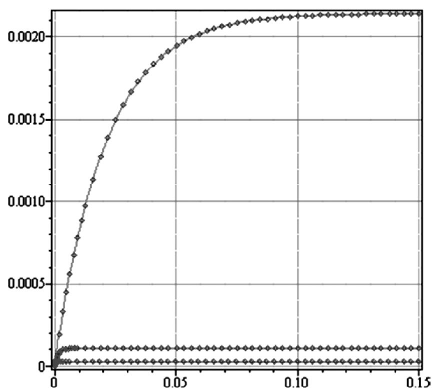

Figure 3: Water, Oilve Oil, 75% Glycerinl Velocity Variation for Different Fluids (θ = 2°)

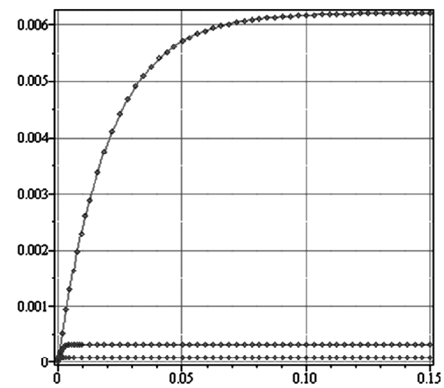

Figure 4: Water, Oilve Oil, 75% Glycerinl Velocity Variation for Different Fluids (θ = 6°)

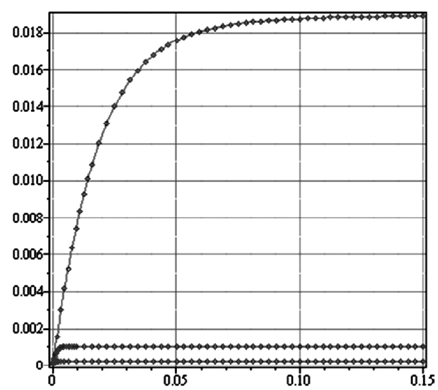

Figure 5: Water, Oilve Oil, 75% Glycerin Velocity Variation for Different Fluids (θ = 20°)

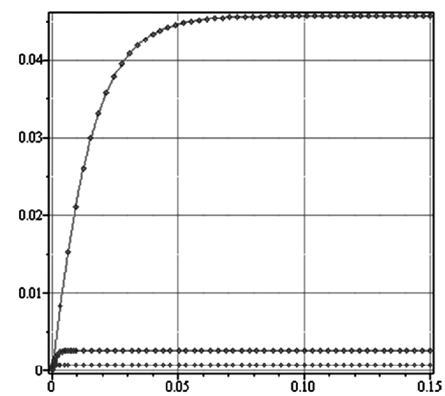

Figure 6: Wate, Oilve Oil, 75% Glycerin Velocity Variation for Different Fluids (θ = 60°)

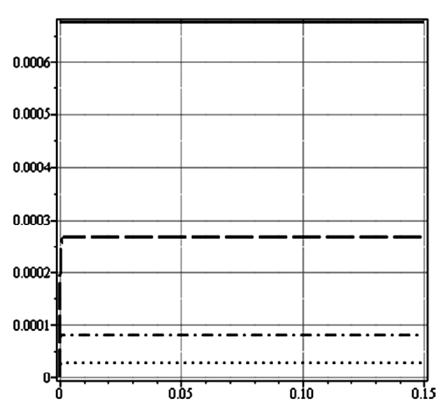

Figure 7: Velocity Variation for Oilve Oil and Different Inclination Angle (Solid Line: θ = 60°, Dash Line: θ = 20°, Dash-Dot Line: θ = 6°, Dot Line: θ = 2°)

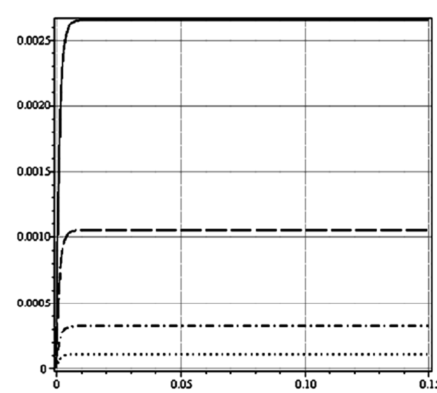

Figure 8: Velocity variation for 75% solution of Glycerin and Different Inclination Angle (Solid Line: θ = 60°, Dash Line: θ = 20°, Dash-Dot Line: θ = 6°, Dot Line: θ = 2°)

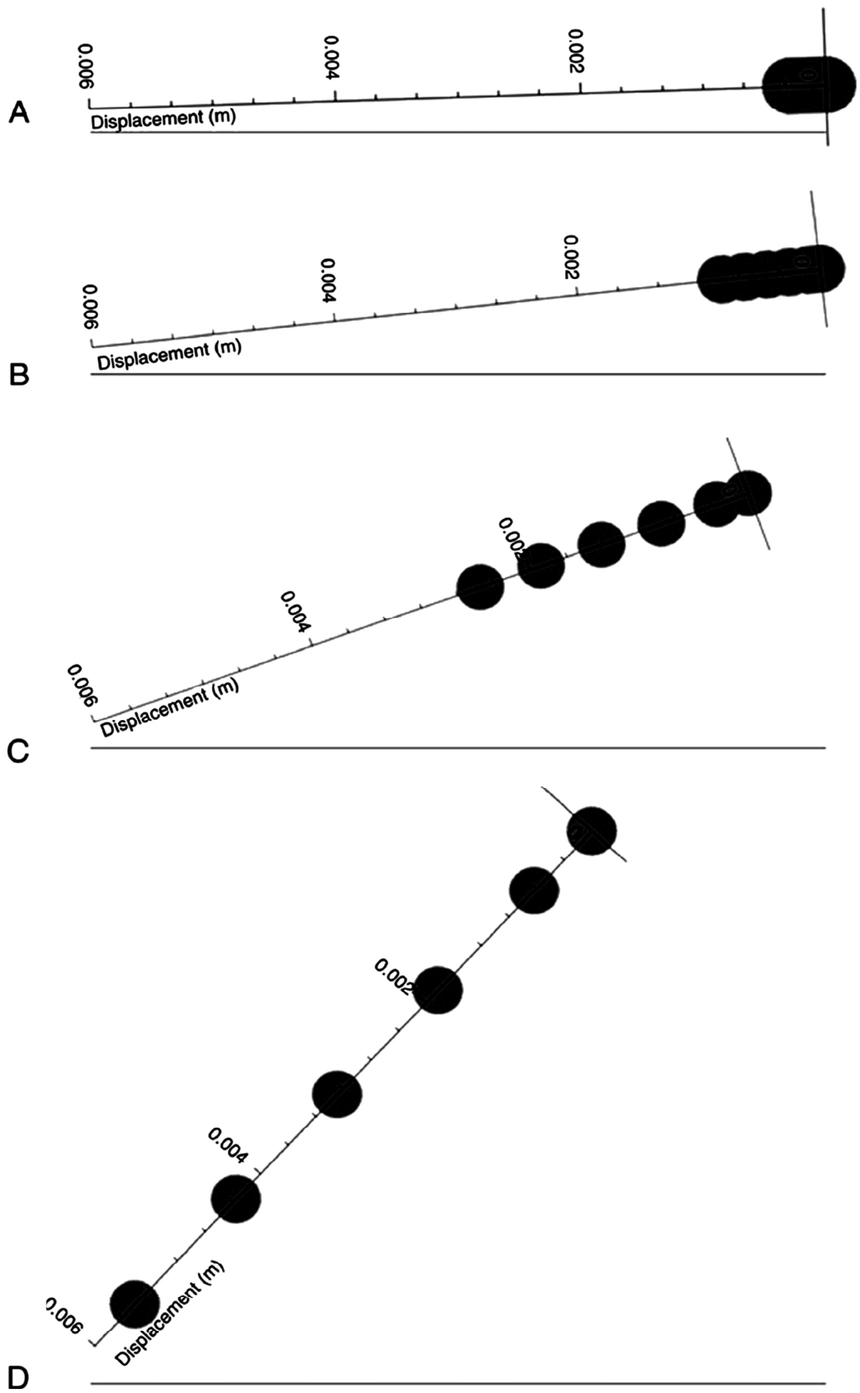

Figure 9: Positions of Rolling Particle for Different Inclination Angle, (Fluid: Water). Time Step = 0.03 (s), (a) $\theta = 2°$, (b) $\theta = 6°$, (c) $\theta = 20°$, (d) $\theta = 60°$

Figure 9 clearly illustrates how inclination angle affects the displacement of particles while other conditions are equivalent.

5. CONCLUSIONS

The parameter perturbation method has been applied to the problem of rolling sphere down an inclined plane submerged in Newtonian fluid with nonlinear drag coefficient and used to solve the corresponding equation of motion for velocity of the sphere. For various cases, the problem has been solved and instantaneous velocity, acceleration and position has been obtained. The numerical comparison shows an excellent agreement between the results of analytical analysis and numerical analysis obtained by employing Runge-Kutta method. The results indicate the applicability and efficiency of the PPM to the aforementioned problem. The simplicity of the method gives considerable advantage to the approach over similar methods. PPM can be applied to a range of hydraulic and sedimentation problems in engineering mechanics.

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