International Journal of Computer Sciences and Engineering Systems
Vol. 9 No. 2 (December, 2015)

An Efficient Five Stage Fifth order Embedded Techniques for Connected Component Detector with Error Control

S. Senthilkumar
Department of Mathematics, National Institute of Technology,Tiruchirappalli-620 015, Tamilnadu, India
E-mail: ssenthilkumar1974@yahoo.co.in

Abstract: *In this paper a novel five-stage fifth order Runge-Kutta methods have been developed, compared and implemented based on Geometric Mean coupled with Contraharmonic Mean and Harmonic Mean. The function of the simulator is that it is capable of performing connected component detector for any kind as well as any size of input image. It is a powerful tool for researchers to examine the potential applications of CNN. Using the Embedded method, a versatile algorithm for simulating connected component detector CNN array is implemented. The aim of this article is focused on popular single step algorithms for solving non linear differential equations of cellular Neural Networks are discussed. Simulation results and comparison have also been presented to show the efficiency of the Numerical integration Algorithms. It is found that the fifth order RK-Embedded Harmonic Mean outperforms well in comparison with Contrharmonic Mean. A more quantitative analysis has been carried out to clearly visualize the goodness and robustness of the proposed algorithm.*

Keywords: *Numerical Integration Techniques, Cellular Neural Networks, Embedded Five Stage Fifth order Methods, Connected Component Detector, Ordinary Differential Equations*

1. STRUCTURE AND FUNCTIONS OF CELLULAR NEURAL NETWORK

The uniqueness of Cellular Neural Networks (CNNs) are analog, time-continuous, non-linear dynamical systems and formally belong to the class of recurrent neural networks. CNNs have been proposed by Chua and Yang [l, 2], and they have found that CNN has many important applications in signal and real-time image processing.

Roska *et al.* [3] have presented the first widely used simulation system which allows the simulation of a large class of CNN and is especially suited for image processing applications [20]. It also includes signal processing, pattern recognition and solving ordinary and partial differential equations etc.

It is of interest to state that embedded methods are actually two methods built into one. The first method is of order p and the second has order p + 1. The difference between these methods provides an error estimate for the first method with order p. Error estimates by these methods have been derived by Merson [4], Fehlberg [5]. Evans and Yaakub [6, 7] introduced a new embedded Runge-Kutta RK(4,4) method which is actually two different RK methods but of the same order p = 4. This embedded method has been developed using Runge-Kutta methods based on arithmetic mean (RKAM) and Contraharmonic Mean (RKCoM). Yaacob and Sanugi [9] adapted embedded Harmonic mean and Ponalagusamy and Senthilkumar presented in detail about the Comparison of RK-Fourth Orders of Variety of Means on Multilayer Raster CNN Simulation. Evans [23] introduced a new 4^{th} order Runge-Kutta Method for Initial Value Problems with Error Control. Sanugi [24] discussed about the numerical strategies for initial value type ordinary differential equations.

Evans and Yaakub [12] introduced a new fourth order Runge Kutta formula based on the Contra-Harmonic mean. Evans and Yaakub[10] adapted fifth order contra harmonic mean for initial value problems with error control. Chi-Chien Lee and Jose Pineda de Gyvez [11] introduced Euler, Improved Euler, Predictor-Corrector and Fourth-Order (quartic) Runge-Kutta algorithms in time-multiplexing CNN simulation.

It is known that the general p-stage Runge-Kutta method for solving $\dot{y}(x) = f(x, y(x))$ is defined by

$$y_{n+1} = y_n + h\sum_{i=1}^{p} b_i k_i$$

where

$$k_i = f\left(x_n + c_i h, y_n + h\sum_{i=1}^{p} a_{ij} k_i\right),$$

$$c_i = \sum_{j=1}^{p} a_{ij}; i = 1, 2, 3, \ldots, p$$

with p dimensional vectors c and b and the $(p \times p)$ matrix $A(a_{ij})$. The study of $RK(p, p + 1)$ methods with a built in error estimate had been proposed by Merson [4] and Fehlberg [5].

This can be presented in the array form as,

$$\begin{array}{c|ccccc}
0 & & & & & \\
c_2 & a_{21} & & & & \\
c_3 & a_{31} & a_{32} & & & \\
\cdot & \cdot & \cdot & \cdot & & \\
\cdot & \cdot & \cdot & \cdot & & \\
c_p & a_{p,1} & a_{p,2} & \ldots\ldots & a_{p,p-1} & \\
c_{p+1} & a_{p+1,1} & a_{p+1,2} & \ldots\ldots. & a_{p+1,p-1} & a_{p+1,p} \\
\hline
 & b_1 & b_2 \ldots\ldots & & b_{p-1}\ b_p & b_{p+1}
\end{array}$$

of order p + 1 and the method.

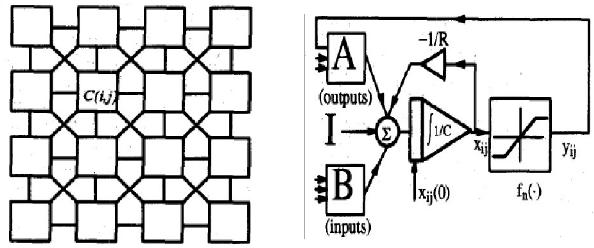
$$\begin{array}{c|ccccc}
0 & & & & & \\
c_2 & a_{21} & & & & \\
c_3 & a_{31} & a_{32} & & & \\
\cdot & \cdot & & \cdot & & \\
\cdot & \cdot & \cdot & \cdot & & \\
c_p & a_{p,1} & a_{p,2} & \ldots\ldots & a_{p,p-1} & \\
\hline
 & a_{p+1,1} & a_{p+1,2} & \ldots\ldots. & a_{p+1,,p-1} & a_{p+1,,p}
\end{array}$$

has order p. The values of y_{n+1} from a given value of y_n are obtained from the above two methods of order p+1 and p respectively and the difference of the results computed by those methods is used to determine the error estimate. In this article, the connected component CNN simulation problem is solved with different approach using fifth order RK-Embedded Contra-Harmonic Mean.

2. STRUCTURE AND FUNCTIONS OF CELLULAR NEURAL NETWORK

CNN is a hybrid of Cellular Automata and Neural Networks and it shares the best features of both worlds. Like Neural Networks, its continuous time feature allows real-time signal processing, and like Cellular Automata, its local interconnection feature makes VLSI realization feasible. Its grid-like structure is suitable for the solution of a high order system of first order non-linear differential equations on-line and in real-time. CNN is an analog nonlinear dynamic processor array shown Figure, l(a). The following are the features of CNN [12].

(i) Each analog processor is capable of processing continuous signals, in either continuous-time or discrete-time modes.



(a) (b)

Figure 1: CNN Structure and Block Diagram

(ii) The processors are placed on a 3D geometric cellular grid (several 2D layers) and are basically similar.

(iii) Interaction among processors is local and mainly translation invariant.

(iv) The mode of operation may be transient, equilibrium, periodic, chaotic, or combined with logic (without Analog to /Digital Conversion).

The general CNN architecture consists of $M*N$ cells placed in a rectangular array. The basic circuit unit of CNN is called a cell denoted by C_{ij}. It contains linear and nonlinear circuit elements. Any cell, C_{ij}, is connected only to its neighbor cells (adjacent cells interact directly with each other). This intuitive concept is called neighborhood and is denoted by N_{ij} and its size determines the degree of connectivity of the CNN. For most applications one restricts N to nearest-neighbors (neighborhood radius 1), the interaction with the nearby cells given by the spatially invariant parameter set a_{kl}, b_{kl}, and I, hereafter called template values. In this article, the capacitance c and the resistance R are assumed to be normalized to 1, and all quantities are dimensionless. The output y_{ij} of each cell is a piecewise linear function of its state x_{ij}, the function is called saturation function shown in fig. 3. Values of –1 will be represented by white cells, +1 by black cells. The total number of parameters needed to specify a CNN with neighborhood radius is at most 19, the templates $A = \{a_{kl}\}$ and $B = \{b_{kl}\}$ are 3 × 3 matrices and by employing the following notation to denote their corresponding entries.

$$A = \begin{matrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{matrix} \text{ and } B = \begin{matrix} b_1 & b_2 & b_3 \\ b_4 & b_5 & b_6 \\ b_7 & b_8 & b_9 \end{matrix} \quad (1)$$

The center entry of the A-template a_5 corresponds to the self-coupling (self feedback) of a cell and is also denoted by a_c. The whole template a $T = \{A, B, I\}$ consists of the feedback parameters A, the control parameters B, and bias I.

Cells not in the immediate neighborhood have indirect effect because of the propagation effects of the dynamics of the network. Three voltages describe the operation of the network. Each cell has a state $x_{ij}(t)$ input $u_{ij}(t)$ and output $y_{ij}(t)$ of the ij^th cell. The dynamics of each cell is governed by the differential equation, which corresponds to the analog circuit in Fig. 2.The state of each cell is bounded for all time $t > 0$ and, after the transient has settled down, a cellular neural network always approaches one of its stable equilibrium points. This last fact is relevant because it implies that the circuit will not oscillate. The dynamics of a CNN has both output feedback (A) and input control (B) mechanisms.

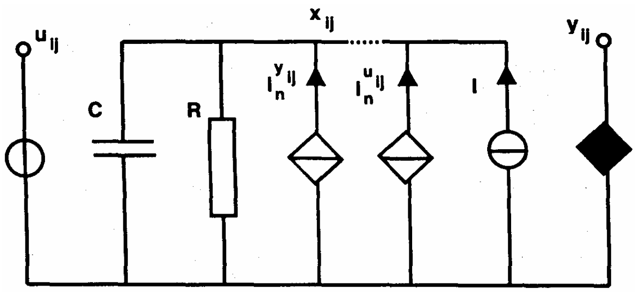


Figure 2: A CNN Cell

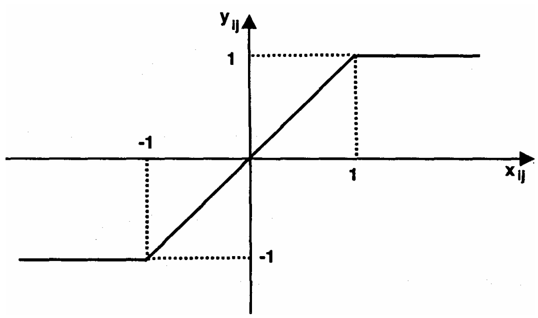


Figure 3: Output Non-Linearity

The first order nonlinear differential equation defining the dynamics of a cellular neural network cell can be written as follows.

$$c\frac{dx_{ij}(t)}{dt}=\frac{-1}{R}x_{ij}(t)+\sum_{c(k,l)\in N(i,j)}A(i,j;k,l)y_{kj}(t)+\sum_{c(k,l)\in N(i,j)}B(i,j;k,l)u_{kl}(t)+I\,,\quad 1\le i\le M;\ 1\le j\le N. \tag{2}$$

and the output equation is given by,

$$y_{ij}(t)=\frac{1}{2}\left[\left|x_{ij}(t)+1\right|-\left|x_{ij}(t)-1\right|\right],\quad 1\le i\le M;\ 1\le j\le N. \tag{3}$$

where x_{ij} is the state of cell $C(i, j)$, $x_{ij}(0)$ is the initial condition of the cell, c is a linear capacitor, R is a linear resistor, I is an independent current source, $\alpha(i, j; k, l)y_{kl}$ and $\beta\{i, j; k, l)u_{kl}$ are voltage controlled current sources for all cells $C(k,l)$ in the neighborhood $N(i,j)$ of cell $C(i,j)$, and y_{ij} represents the output equation.

From the equ. (1) it is observed that the summation operators of each cell is affected by its neighboring cells. $A(.)$ represents on the output of neighboring cells and is called as feedback operator, $B(.)$ in turn affects the input control and is known as the control operator. In particular, the entry values of matrices $A(.)$ and $B(.)$ are dependent on the application chosen by the user which are space invariant and are referred as cloning templates. A current bias I and cloning templates establishes the transient behavior of the cellular nonlinear network. A continuous-time cell implementation is shown in fig. 1(b) as an equivalent block diagram. CNNs have as input a set of analog values and its programmability is done via cloning templates. Thus, programmability is one of the most attractive properties of CNNs.

3. NUMERICAL INTEGRATION TECHNIQUES

The CNN is described by a system of nonlinear differential equations. Therefore, it is necessary to discretize the differential equation for performing behavioural simulation. For computational purposes, a normalized time differential equation describing CNN is used by Nossek *et al.*,[14]

$$f'(x(n\tau))=\frac{dx_{ij}(n\tau)}{dt}=-x_{ij}(n\tau)+\sum_{c(k,l)\in N_r(i,j)}A(i,j;k,l)y_{kl}(n\tau)+\sum_{c(k,l)\in N_r(i,j)}B(i,j;k,l)u_{kl}(n\tau)+I,\ 1\le i\le M;\ 1\le j\le N;$$

$$y_{ij}(n\tau)=\frac{1}{2}\left[\left|x_{ij}(n\tau)+1\right|-\left|x_{ij}(n\tau)-1\right|\right],\ 1\le i\le M;\ 1\le j\le N; \tag{4}$$

where τ is the normalized time. For the purpose of solving the initial-value problem, well established Single Step methods of numerical integration techniques are used in [8, 9]. These methods can be derived using the definition of the definite integral

$$x_{ij}((n+1)\tau)-x_{ij}(n\tau))=\int_{\tau_n}^{\tau_{n+1}}f'(x(n\tau))d(n\tau)\,. \tag{5}$$

3.1 Explicit Euler's Algorithm

Euler's method is the simplest of all algorithms for solving ordinary differential equations. It is an explicit formula which uses the Taylor-series expansion to calculate the approximation.

$$x_{ij}((n+1)\tau)=x_{ij}(n\tau))+\tau f'(x(n\tau)) \tag{6}$$

3.2 RK-Gill Algorithm

The RK-Gill algorithm discussed by Oliveria [9] is an explicit method which requires the computation of four derivatives per time step. The increase of the state variable x^{ij} is stored in the constant k^{ij}_{1}. This result is used in the next iteration for evaluating k^{ij}_{2} and repeat the same process to obtain the values of k^{ij}_{3} and k^{ij}_{4}.

$$k_1^{ij}=f'(x_{ij}(n\tau)),\ k_2^{ij}=f'(x_{ij}(n\tau)+\frac{1}{2}k_1^{ij})$$

$$k_3^{ij} = f'(x_{ij}(n\tau) + \left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right)k_1^{ij}) + \left(1 - \frac{1}{\sqrt{2}}\right)k_2^{ij}) \quad (7)$$

$$k_4^{ij} = f'(x_{ij}(n\tau) - \frac{1}{\sqrt{2}}k_2^{ij} + \left(1 + \frac{1}{\sqrt{2}}\right)k_3^{ij})$$

Therefore, the final integration is a weighted sum of the four calculated derivates is given below.

$$x_{ij}((n+1)\tau) = x_{ij}(n\tau) + \frac{1}{6}[k_1^{ij} + (2-\sqrt{2})k_2^{ij} + (2+\sqrt{2})k_3^{ij} + k_4^{ij}] \quad (8)$$

3.3 Fifth Order RK-Algorithm

The Fifth Order Runge-Kutta algorithm is an explicit method and discussed by Morris Badder [6,7]. It starts with a simple Euler method. The increase of the state variable x^{ij} is stored in the constant $k^{ij}{}_1$. This result is used in the next iteration for evaluating $k^{ij}{}_2$. The same procedure must be repeated to compute the values of $k^{ij}{}_3$, $k^{ij}{}_4$, $k^{ij}{}_5$ and $k^{ij}{}_6$.

$$k_1^{ij} = \tau f'(x_{ij}(n\tau)),\ k_2^{ij} = \tau f'\left(x_{ij}(n\tau) + \frac{1}{4}k_1^{ij}\right)$$

$$k_3^{ij} = \tau f'(x_{ij}(n\tau) + \left(\frac{1}{8}\right)k_1^{ij}) + \left(\frac{1}{8}\right)k_2^{ij}),$$

$$k_4^{ij} = \tau f'(x_{ij}(n\tau) - \frac{1}{2}k_2^{ij} + k_3^{ij})$$

$$k_5^{ij} = \tau f'(x_{ij}(n\tau) + \frac{3}{16}k_1^{ij} + \frac{9}{16}k_4^{ij})$$

$$k_6^{ij} = \tau f'(x_{ij}(n\tau) - \frac{3}{27}\ k_1^{ij} + \frac{2}{7}k_2^{ij} + \frac{12}{7}\ k_3^{ij} - \frac{12}{7}\ k_4^{ij} + \frac{8}{7}k_5^{ij}) \quad (9)$$

Therefore, the final integration is a weighted sum of the five calculated derivatives which is given below.

$$x_{ij}((n+1)\tau) = x_{ij}(n\tau) + \frac{1}{90}[7k_1^{ij} + 32k_3^{ij} + 12k_4^{ij} + 32k_5^{ij} + 7k_6^{ij}] \quad (10)$$

where *f(.)* is computed according to the given function.

3.4 RK-Embedded Contra-Harmonic Mean

The Fifth Order RK-Embedded Contra-Harmonic Mean [10] is given by,

$k_1 = f(y_n)$, $k_2 = f(y_n + 0.101727541\ hk_1)$

$k_3 = f(y_n - 0.5236574475\ hk_1 + 1.11653361910\ hk_2)$,

$$k_4 = f(y_n + 4.7450804540\,hk_1 - 4.2354437705\,hk_2 - 0.0096366835\,hk_3); \quad (11)$$

$$k_5 = f(y_n - 0.5736403905hk_1 + 0.9301175162hk_2 + 0.4667978567hk_3 + 0.1767250176hk_4)$$

Therefore, the final integration is a weighted sum of the four calculated derivates is given below.

$$y_{n+1} = y_n + \frac{h}{3}\left[\frac{k_1^2k_2^2}{k_1+k_2} + \frac{k_2^2k_3^2}{k_2+k_3} + \frac{k_3^2k_4^2}{k_3+k_4} + \frac{k_4^2+k_5^2}{k_4+k_5}\right] \quad (12)$$

4. DERIVATION AND ERROR ESTIMATE OF RKGCOM(5,5) METHOD

The fifth order Geometric Mean (GM) formula [25] is given by,

$$y_{n+1} = y_n + h(-2.1088641714\sqrt{k_1k_2} + 1.6117344951\sqrt{k_2k_3} + 1.1894300714\sqrt{k_3k_4} + 0.3076996050\sqrt{k_4k_5} \quad (13)$$

where

$k_1 = f(y_n)$, $k_2 = f(y_n - 0.2264469689\ hk_1)$

$k_3 = f(y_n - 0.163031141\ hk_1 + 0.1027569416\ hk_2)$,

$k_4 = f(y_n + 3.1992668228\ hk_1 - 0.4021478413\ hk_2 - 2.2971189815\ hk_3)$;

$k_5 = f(y_n - 14.488068506$ hk1 -0.6194404053 hk2 $+ 13.65666768673\ hk_3 + 2.3508320447\ hk_4)$ (14)

and the fifth order Contraharmonic Mean (CoM) formula in the form[10]

$$y_{n+1} = y_n + h\left[\left(\sum_{i=1}^{4} w_i\left(\frac{k_i^2 + k_{i+1}^2}{k_i + k_{i+1}}\right)\right)\right] \quad (15)$$

where

$\sum_{i=1}^{4} w_i = 1$, $w_1 = -0.1773157366$, $w_2 = 1.0254553152$,

$w_3 = -0.0779114700$, $w_4 = 0.2297718914$

$k_1 = f(y_n)$, k2 $= f(y_n + 0.101727541\ hk_1)$,

$k_3 = f(y_n - 0.523657447\ hk_1 + 1.11653361910\ hk_2)$,

$k_4 = f(y_n + 4.7450804540\ hk_1 - 4.2354437705\ hk_2 - 0.0096366835\ hk_3)$;

k5 = f(yn –0.5736403905 hk1 + 0.9301175162 hk2 $+ 0.4667978567\ hk_3 + 0.1767250176\ hk_4)$ (16)

Therefore, the final integration is a weighted sum of the four calculated derivates is given below.

$$y_{n+1} = y_n + h\left[w_1\frac{k_1^2k_2^2}{k_1+k_2} + w_2\frac{k_2^2k_3^2}{k_2+k_3} + w_3\frac{k_3^2k_4^2}{k_3+k_4} + w_4\frac{k_4^2+k_5^2}{k_4+k_5}\right] \quad (17)$$

is called RKGCoM(5,5). The difference between (15) and (17) ,i.e., $y_{n+1}^{GM} - y_{n+1}^{CoM}$ provides an error estimate for the approximation to the numerical solution. By applying the same procedure as in the RK(4,4) technique, it is possible to obtain an error estimate for the five stage explicit GM-CoM method of order five by implementing the local truncation error for the fifth order Geomentric mean Runge-Kutta method and the fifth order Contraharmonic mean technique.

For the fifth order Geomentric mean Runge-Kutta method we have

$$y_{n+1}^{GM} = y_n + LTE^{GM}$$

and for the Contraharmonic mean method

$$y_{n+1}^{CoM} = y_n + LTE^{CoM}$$

where y_{n+1}^{GM} and y_{n+1}^{CoM} are the numerical approximations at x_{n+1} obtained by the Geomentric mean and Contraharmonic mean methods respectively and y_{n+1}^{GM} and y_{n+1}^{CoM} are the local truncation errors of the fifth order Geomentric mean Runge-Kutta method and the fifth order Contraharmonic mean methods.

The error estimate is obtained by taking the difference between these two methods for the numerical approximations at x_{n+1} by

$$y_{n+1}^{GM} - y_{n+1}^{CoM} = LTE^{GM} - LTE^{CoM}$$

The local truncation error for the fifth order Geomentric mean Runge-Kutta method involves y derivates given by

$$\begin{aligned} LTE_{GM} &= h^6[-.0.00367007\, ff_y^5 + 0.00203894\, f^2 f_y^3 f_{yy} \\ &-0.0121412\, f^3 f_y f_{yy}^2 + 0.00339055\, f^3 f_y f_{yyy}^2 \\ &+0.00411231\, f^4 f_{yy} f_{yyy} - 0.0000951775\, f^4 f_y f_{yyyy} \\ &+0.0000893556\, f^5 f_{yyyyy}] + O(h^7) \end{aligned} \tag{18}$$

Similarly the local truncation error for the Contraharmonic mean method is given by

$$\begin{aligned} LTE_{CoM} &= h^6[0.0132485733\, ff_y^5 + 0.0202501069\, f^2 f_y^3 f_{yy} \\ &+0.0095106268\, f^3 f_y f_{yy}^2 - 0.0022879188\, f^3 f_y f_{yyy}^2 \\ &-0.0001379536\, f^4 f_{yy} f_{yyy} - 0.0003448339\, f^4 f_y f_{yyyy} \\ &-0.0000178190\, f^5 f_{yyyyy}] + O(h^7) \end{aligned} \tag{19}$$

Therefore the absolute difference between y_{n+1}^{GM} and y_{n+1}^{CoM} is given by

$$\begin{aligned} \left| \mathrm{LTE^{GM}\text{-}LTE^{CoM}} \right| &= h^6[-0.0095785033\, ff_y^5 \\ &-0.0182111669\, f^2 f_y^3 f_{yy} - 0.0216518268\, f^3 f_y f_{yy}^2 \\ &+0.0056784688\, f^3 f_y f_{yyy}^2 + 0.0042502630\, f^4 f_{yy} f_{yyy} \\ &+0.0002496564\, f^4 f_y f_{yyyy} + 0.0001071746\, f^5 f_{yyyyy} \end{aligned} \tag{20}$$

5. DERIVATION AND ERROR ESTIMATE OF RKGHM(5,5) METHOD

The fifth order Geometric Mean (GM) formula [25] is given by,

$$\begin{aligned} y_{n+1} &= y_n + h(-2.1088641714\sqrt{k_1k_2} \\ &+1.6117344951\sqrt{k_2k_3} + 1.1894300714\sqrt{k_3k_4} \\ &+0.3076996050\sqrt{k_4k_5}) \end{aligned} \tag{21}$$

where

$k_1 = f(y_n)$, $k_2 = f(y_n - 0.2264469689\, hk_1)$

$k_3 = f(y_n - 0.163031141\, hk_1 + 0.1027569416\, hk_2)$,

$k_4 = f(y_n + 3.1992668228\, hk_1 - 0.4021478413\, hk_2 - 2.2971189815\, hk_3)$;

$k_5 = f(y_n - 14.488068506\, hk_1 - 0.6194404053\, hk_2 + 13.65666768673\, hk_3 + 2.3508320447\, hk_4)$ (22)

and the fifth order Harmonic Mean (HM) formula in the form

$$y_{n+1} = y_n + h\left[\sum_{i=1}^{4} w_i \left(\frac{2k_i k_{i+1}}{k_i + k_{i+1}}\right)\right] \tag{23}$$

where

$k_1 = f(y_n)$, $k_2 = f(y_n + 1.2808930996\, hk_1)$,

$k_3 = f(y_n - 0.1216330944\, hk_1 + 0.3153843796\, hk_2)$,

$k_4 = f(y_n + 0.1914004985\, hk_1 + 0.1771161874\, hk_2 + 0.1314833141\, hk_3)$;

$k_5 = f(y_n + 0.8116923889\, hk_1 - 0.0668709652\, hk_2 - 0.12223829683\, hk_3 + 0.3775615446\, hk_4)$ (24)

Therefore, the final integration is a weighted sum of the four calculated derivates is given below.

$$\begin{aligned} y_{n+1} &= y_n + h(0.1990193382\left(\frac{2k_1k_2}{k_1+k_2}\right) \\ &-0.2535846961\left(\frac{2k_{21}k_3}{k_2+k_3}\right) + 0.5740453004\left(\frac{2k_3k_4}{k_3+k_4}\right) \\ &+0.4805200575\left(\frac{2k_4k_5}{k_4+k_5}\right) \end{aligned} \tag{25}$$

is called RKGHM(5,5). The difference between (15) and (23) $y_{n+1}^{GM} - y_{n+1}^{HM}$ provides an error estimate for the approximation to the numerical solution. By applying the same procedure as in the RK(4, 4) technique, it is possible to obtain an error estimate for the five stage explicit GM-HM method of order five by implementing the local truncation error for the fifth order Geomentric mean Runge-Kutta method and the fifth order Harmonic mean technique.

For the fifth order Geomentric mean Runge-Kutta method we have

$$y_{n+1}^{GM} = y_n + LTE^{GM}$$

and for the Harmonic mean method

$$y_{n+1}^{HM} = y_n + LTE^{HM}$$

where y_{n+1}^{GM} and y_{n+1}^{CoM} are the numerical approximations at x_{n+1} obtained by the Geomentric mean and Harmonic mean methods respectively and y_{n+1}^{GM} and y_{n+1}^{CoM} are the local truncation errors of the fifth order Geomentric mean Runge-Kutta method and the fifth order Harmonic mean methods.

The error estimate is obtained by taking the difference between these two methods for the numerical approximations at x_{n+1} by

$$y_{n+1}^{GM} - y_{n+1}^{HM} = LTE^{GM} - LTE^{HM}$$

The local truncation error for the fifth order Geomentric mean Runge-Kutta method involves y derivates given by

$$\begin{aligned} LTE_{GM} &= h^6[-.00367007\, ff_y^5 + 0.00203894\, f^2 f_y^3 f_{yy} \\ &-0.0121412\, f^3 f_y f_{yy}^2 + 0.00339055\, f^3 f_y f_{yyy}^2 \\ &+0.00411231\, f^4 f_{yy} f_{yyy} - 0.0000951775\, f^4 f_y f_{yyyy} \\ &+0.0000893556\, f^5 f_{yyyyy}] + O(h^7) \end{aligned} \quad (26)$$

Similarly the local truncation error for the Harmonic mean method is given by

$$\begin{aligned} LTE_{HM} &= h^6[-0.0000959103\, ff_y^5 + 0.00874182\, f^2 f_y^3 f_{yy} \\ &-0.0399141\, f^3 f_y f_{yy}^2 + 0.0163115\, f^3 f_y f_{yyy}^2 \\ &-0.00601939\, f^4 f_{yy} f_{yyy} - 0.00157171\, f^4 f_y f_{yyyy} \\ &+0.0000329614\, f^5 f_{yyyyy}] + O(h^7) \end{aligned} \quad (27)$$

Therefore the absolute difference between y_{n+1}^{GM} and y_{n+1}^{HM} is given by

$$\begin{aligned} \left| \mathrm{LTE^{GM}\text{-}LTE^{HM}} \right| &= h^6[-0.0035741597\, ff_y^5 \\ &-0.00670288\, f^2 f_y^3 f_{yy} - 0.0277729\, f^3 f_y f_{yy}^2 \\ &-0.01292095\, f^3 f_y f_{yyy}^2 + 0.0101317\, f^4 f_{yy} f_{yyy} \\ &+0.0016668875\, f^4 f_y f_{yyyy} + 0.0000563942\, f^5 f_{yyyyy} \end{aligned} \quad (28)$$

By following an argument suggested by Lotkin [16], if we assume that the following bounds for f and its partial derivatives hold for $x \in [a, b]$ and $y \in (-\infty, \infty)$, we have

$$\left| f(x,y) \right| < Q, \left| \frac{\partial^{i+j} f(x,y)}{\partial x^i \partial y^j)} \right| < \frac{P^{i+j}}{Q^{j-1}},\ i + j \le P \quad (29)$$

where P and Q are positive constants and p is the order of the method.

In this case $p = 6$. Hence using (29), we obtain

$$\left.\begin{aligned} &\left| ff_y^5 \right| < Q(\frac{P^{o+1}}{Q^{I-1}}) \\ &\left| f^2 f_y^3 + f_{yy} \right| < Q^2 (\frac{P^I}{Q^0})^3 \frac{p^2}{Q} \\ &\left| f^3 f_y f_{yy}^2 \right| < Q^3 P(\frac{P^2}{Q}) \\ &\left| f^3 f_y^2 f_{yyy} \right| < Q^3 P^2 (\frac{P^3}{Q^2}) \\ &\left| f^4 f_{yy} f_{yyy} \right| < Q^4 . \frac{P^2}{Q} \frac{P^3}{Q^2} \\ &\left| f^4 f_y f_{yyyy} \right| < Q^4 P \frac{P^4}{Q^3} \\ &\left| f^5 f_{yyyy} \right| < Q^5 \frac{P^5}{Q^4} \end{aligned}\right\} < P^5 Q \quad (30)$$

From Eqs. (20) and (29) we obtain

$$LTE_{GM} – LTE_{CoM} \le (0.0391\ 559342)\ \mathrm{P}^5\ .\ \mathrm{Qh}^6 \quad (31)$$

$$\left| y_{n+1}^{GM} - y_{n+1}^{CoM} \right| \le 0.0391559342 P^5 .Qh^6 \quad (32)$$

If we suppose that the tolerance $TOL = 0.00001$ than by setting $\left| y_{n+1}^{GM} - y_{n+1}^{CoM} \right| \le TOL$ then the error control and step size selection can be determined by Eq.(18) to give the formula

$$0.0391559342\ P^5\ Qh^6 < TOL \text{ or}$$

$$h < \left[\frac{TOL}{0.0391559342 P^5 Q} \right]^{1/6} \quad (33)$$

Similarly from Eqs. (29) and (30) We obtain

$$LTE_{GM} – LTE_{HM} \le (0.016429892)\ P^5\ .\ Qh^6 \quad (34)$$

$$\left| y_{n+1}^{GM} - y_{n+1}^{HM} \right| \le 0.016429892 P^5 .Qh^6 \quad (35)$$

If we suppose that the tolerance $TOL = 0.00001$ than by setting $\left| y_{n+1}^{GM} - y_{n+1}^{HM} \right| \le TOL$ then the error control and step size selection can be determined by Eq.(26) to give the formula

$$0.016429892 P^5 Qh^6 < TOL \ or \ h < \left[\frac{TOL}{0.016429892 P^5 Q} \right]^{1/6} \quad (36)$$

6. SIMULATION BEHAVIOR OF CONNECTED COMPONENT DETECTOR

The behavior of the four algorithms are compared by simulating the connected component detector is discussed by Matsumoto *et al.* [20, 21].

6.1 Connected Component Detector (CCD)

Matsumoto *et al.* [20,21] discussed CCD which has a very important applications in image processing, pattern recognition, data compression and in many other important features of transaction process. The initial pattern and cloning template chosen for simulation is shown in fig.4 because the CCD state variables frequently change sign during the transient time and is called as a "Worst case network" from the viewpoint of ringing of the variables. The obtained results are representative for many other simulations with different initial patterns and templates.

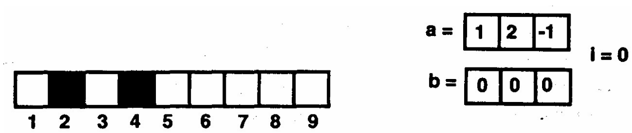


Figure 4: Initial Pattern and Cloning Template

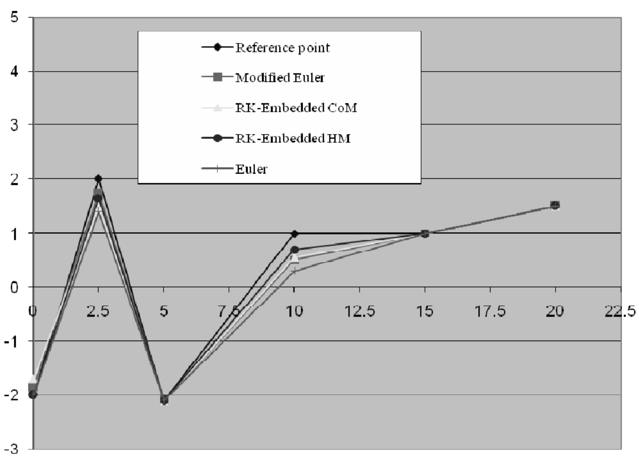


Figure 5: Simulation Results for Δt = 1 from t = 0 to t = 20

Fig. 5 shows the transient of the state variables of cell 6 obtained by the different algorithms. The reference function is constructed by using Fifth order RK-Embedded Contra-Harmonic algorithm with a much smaller step size (Δtref = 0.01). The larger error term of other algorithm is due to "run after characteristic". The zero crossings are shifted to the right and the extreme values are larger than in the original system. But, the RK-Gill algorithm reduces extreme values and the fifth order RK-Embedded Contra-Harmonic algorithm approximates the state variable best due to computational effort. If a step size value is exceeds 1.52 then the other RK-Embedded algorithms becomes unstable and the state variables starts to oscillate. As a result, it is desirable to know the maximum step size of all the algorithms to speed up the simulations. The results are shown in Fig. 6.

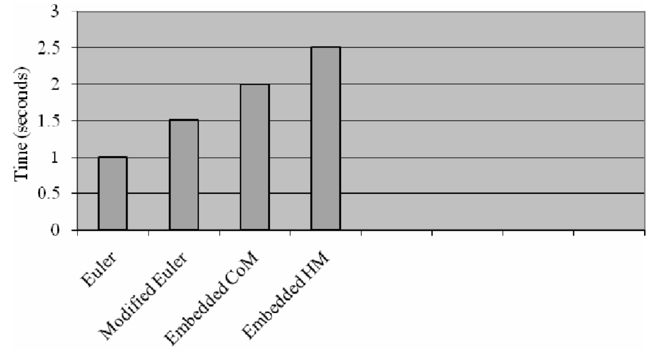


Figure 6: Maximum Stepsize Still Leads to more Convergence

The system starts converge even for very large step sizes, but the transient behaviour of the state variables is approximated roughly. If the computation time is compared considerably then the performance of an algorithm depends on the computational effort for a single time step as well as on the maximum step size still leading to convergence. From this it is ease to compare the time necessary for a single iteration step divided by the maximum step size. The normalized to the performance of the Modified other RK-Embedded algorithms are shown in fig. 7.

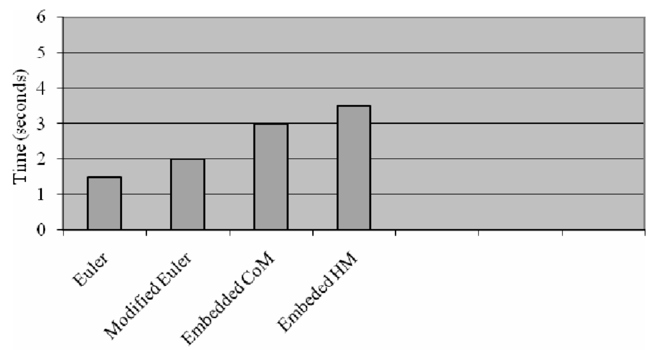


Figure 7: Performance Comparison for four Numerical Methods

The other RK-embedded algorithm fallout in the shortest simulation time. Hence, it is useful for simulations where high accuracy of the transient of the state variables is not obtained. This is particularly true for templates where the correct dynamics behaviour depends primarily on the sign of the derivatives and on their values. As a final point, the accuracy of the algorithm as a function of the stepsize Dt is depicted in fig. 8. for this an error term ε is defined as the maximum deviation of a state variable xij from its reference function $\hat{x}^{ij}$, by $\varepsilon^{ij} = \max_k \left| x^{ij}(t_k) - \hat{x}^{ij}(t_k) \right|$

Fig. 8 shows that the other RK-embedded algorithms are unsuitable for yielding low value of ε. The error function increases drastically even for a relative small size. Fifth order RK-Embedded Contra–Harmonic Mean order algorithm yield accuracy over a large range of the step size, however if a "threshold" is exceeded the error term grows. The non-monotonic behavior of ε is caused by shifting the timing instances at which the state variables are evaluated. Thus

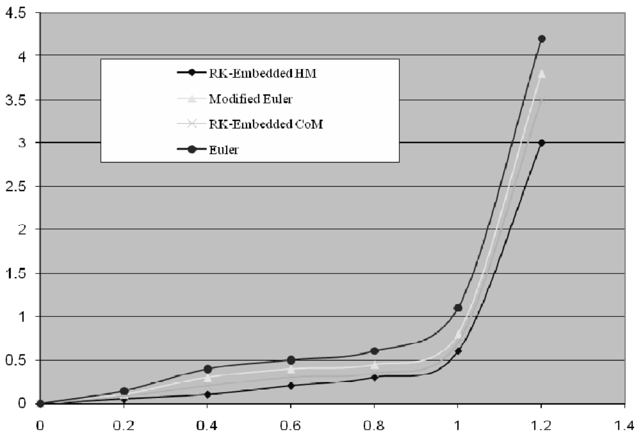


Figure 8: Computation Accuracy (ε) versus Stepsize(Δt)

one can jump over" worst case time instances and achieve locally a smaller ε if the step size is increased.

6.2 Bifurcation Behavior

In second example Seller [22] discussed about, a parameter dependent late is chosen and the system is simulated near a bifurcation value. Considering the template and the initial pattern in Fig. 10 the output of cell 3 white for a current $i = -3$ and black for a current $i = -2$. The bifurcation value between is determined by interval nesting in several imulations. The results depicted in Fig. 9 shows the bifurcation behaviour versus the stepsize Δt. The accuracy of i is with in 0.01 caused by the large error term the other RK-Embedded algorithm results in an erroneous bifurcation value. For large number of stepsize fifth order RK-Contra-Harmonic Mean algorithm yields accurate.

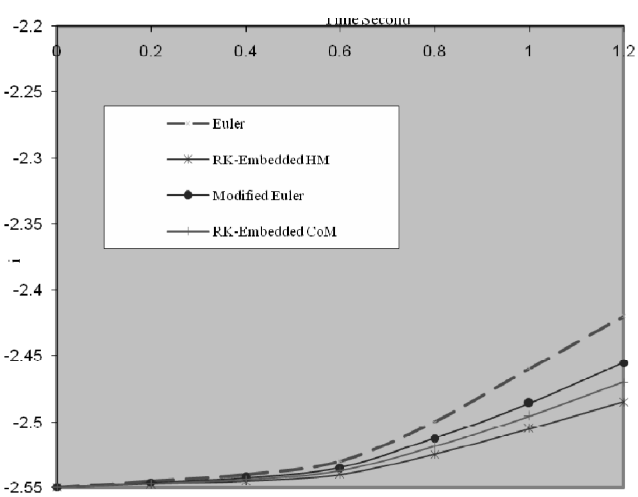


Figure 9: Bifurcation Point Vs Stepsize

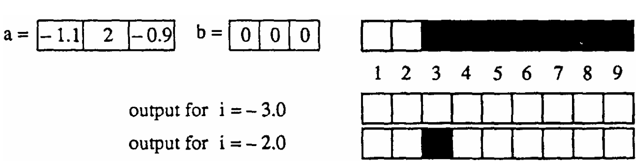


Figure 10: Cloning Template, Initial Pattern and Output Patterns

Table 1
Comparison of LTE and Error Estimation for RK-Fifth Order Embedded Means

Sl. No.	*RK-Embedded Method*	*Local Truncation Error [LTE]*	*Error Estimation*
1	RK-Embedded Harmonic Mean [Present Paper]	LTEGM – LTEHM ≤ (0.016429892) P5 . Qh6 = $\left\|y_{n+1}^{GM} - y_{n+1}^{HM}\right\|$ ≤ 0.016429892 .P^5 . Qh^6	ERREST = $\|Y_{GM} - Y_{HM}\|$ ×0.016429892
2	RK-Embedded Contra-Harmonic Mean [Present Paper]	$LTE_{GM} - LTE_{CoM} \leq$ (0.031559342) $P^5 . Qh^6 = \left\|y_{n+1}^{M} - y_{n+1}^{GM}\right\|$ ≤ 0.0391559342 .P^5 . Qh^6	ERREST = $\|Y_{GM} - Y_{GM}\|$ ≤ 0.0391559342

7. DISCUSSIONS AND CONCLUSION

The present article sheds some light on different numerical integration algorithms on the simulation of cellular neural network. It is pertinent to pin-point out here that using the fifth order RK-Embedded Harmonic Mean algorithm guarantees more accurate values compared to the other methods. As there is a trade-off between speed and accuracy of numerical integration techniques it is useful to implement different algorithms in CNN-simulators. Euler or classical RK-Fourth order algorithm is preferential for a very fast tool if only the correct final state is of importance. But in contrast, the unusual good convergence feature of this algorithm can be explained by the fact that the desired behaviour of CNNs depends primarily on the qualitative dynamics of the state variables. If the end user is interested in the transient of the state variables in detail the fifth order RK-Embedded Harmonic Mean is well suitable if the chosen stepsize of 0.5 gives a good of the transient behaviour. For the examination of bifurcation values only the fifth order RK-Embedded Harmonic Mean algorithm is recommended because of its high precision leading to reliable results in a large range of the stepsize.

REFERENCES

[1] Chua, L. O., Yang, L., Cellular Neural Networks: Theory, *IEEE Transactions on Circuits and Systems*, **35**, 1257–1272, 1988.

[2] Chua, L. O., Yang, L., Cellular Neural Networks: Applications, *IEEE Transactions on Circuits and Systems*, **35**, 1273–1290, 1988.

[3] Roska *et al.* CNNM Users Guide, Version 5.3x, Budapest (1994).

[4] Merson, R. H. (1957), An Operational Method for the Study of Integrating Processes. *Proc. Symp. Data Processing*, Weapon Research Establishment, Salisbury, S.Australia.

[5] Fehlberg, E. (1969), "Low Order Classical Runge Kutta Formulas with step-size Control and their Application to Some Heat Transfer Problems. NASA Tech. Report 315 (1969), *Extract Published in Computing*. 66-71 (1970).

[6] Evans, D. J. and Yaakub, A. R., A New Runge Kutta RK(4,4) Method, *Intern. Computer Math.*, **58**, 169-187, 1995.

[7] Yaakub, A. R. and Evans, D. J., A Fourth order Runge-Kutta RK(4,4) Method with Error Control, *Intern. J. Computer Math.*, 71, 383-411, 1999.

[8] R. Ponalagusamy and S. Senthilkumar, "A Comparison of RK-Fourth Orders of Variety of Means on Multilayer Raster CNN Simulation", *Trends in Applied Science and Research*, **3**(3): 242-252, 2008.

[9] Nazeerudin Yaacob and Bahrom Sanugi, 1998, "A New Fourth-Order Embedded Method Based on the Harmonic Mean", *Mathematika, Jilid,hml* 1-6.

[10] Evans, D. J. and Yaakub, A. R., "The Fifth order Runge-Kutta R. K. (5,5) Method with Error Control, *Intern. J. Computer Math.*, **79**, 1179-1185, 2002.

[11] Chi-Chien Lee, Jose Pineda de Gyvez, Single-Layer CNN Simulator, *International Symposium on Circuits and Systems*, 6, 217–220, 1994.

[12] L. O. Chua and T. Roska, "The CNN Universal Machine Part 1: The Architecture", in Int. *Workshop on Cellular Neural Networks and their Applications (CNNA)*, 1-10, 1992.

[13] K. K. Lai, and P. H. W. Leong, "Implementation of Time-Multiplexed CNN building Block Cell," *IEEE. Proc. Of Microwave*, 80-85, 1996.

[14] K. K. Lai, and P. H. W. Leong, "An Area Efficient Implementation of a Cellular Neural Network", IEEE, 51-54, 1995.

[15] D. J. Evans and N. Yaacob, "A Fourth order Runge-Kutta Method Based on the Heronian Mean Formula", *Intern. J. Computer Math.*, **58**, 103-115, 1995.

[16] Lotkin, M., *On the Accuracy of RK Methods*, MYAC, 5, 128-132, 1951.

[17] Ralston, R. H., Runge-Kutta Methods with Minimum Error Bonds, *Math. Comput.*, **16**, 431-437, 1957.

[18] Nossek, J. A., Seiler, G., Roska, T., Chua, L. O., "Cellular Neural Networks: *Theory and Circuit Design", Int. J. of Circuit Theory and Applications*, 20, 533-553, 1992.

[19] Yaakub, A. R. and Evans, D. J., A Fourth order Runge-Kutta R. K. (4,4) method with Error Control, *Intern. J. Computer Math.*, 57, 1995, 249-256.

[20] Matsumoto, T., Chua, L. O. and Yokohama, T., CNN Cloning Template Connected Component Detector", *IEEE Transactions on Circuits and Systems*, **37**, 633-635, 1990.

[21] Matsumoto, T., Chua, L. O. and Yokohama, T., CNN Cloning Template: Hole Filler, *IEEE Transactions on Circuits and Systems*, **37**, 635-638, 1990.

[22] Seiler, G., A Casecade of Bifurcation in a Family of Cellular Neural Networks, Report TUM-LNS-TR-90-5, *Technical University of Munich*, Archisstr, 21,D-800,Munich, 20, 533-553, 1990.

[23] D. J. Evans, "A New 4th order Runge-Kutta Method for Initial Value Problems with Error Control, *International Journal of Computer Mathematics*, **39**, 217-227, 1991.

[24] Sanugi, B. B., New Numerical Strategies for Initial Value Type Ordinary Differential Equations, Ph.D Thesis, Loughbrough University of Technology, U. K., 1986.

[25] B. B. Sanugi and N. B. Yaacob, A New Fifth Order Five-stage Runge-Kutta Method for Initial Value Type Problems in ODEs, *International Journal of Computer Mathematics*, **59**, 187-207, 1996.